Dynamic modeling challenges in strengthening existing structures with advanced composites

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ABSTRACT: This paper highlights, discusses, and illustrates the dynamic modeling challenges in the field of strengthening of existing structures with advanced composite materials. While this strengthening method has made a significant way from an emerging technology three decades ago to a commonly accepted one today, it is still involved with critical modeling challenges. Most of those are found in the dynamic regime, particularly when the evolution of inter-laminar failure processes is concerned. The paper highlights those challenges, examines the features that trigger their evolution, and discusses their dynamic modeling. This includes the translation of physical phenomena that are spread over four orders of magnitude of characteristic length scale and up to six orders of magnitude of characteristic time scale into a sound mathematical model. The paper also discusses the numerical aspects of the modeling challenges as well as the experimental techniques required for capturing dynamic mechanisms that evolve in the sub-millisecond range. The discussion is illustrated with analytical, numerical, and experimental observations. Concluding remarks on challenges that still have to be faced and their impact on the resilience of the strengthened structure close the paper.

1 INTRODUCTION

The development of the modern technique of strengthening and retrofitting existing structures with advanced composite materials in the last three decades paved the way to many new and advantageous structural applications. Those range from the classical retrofitting of deficient or degraded reinforced concrete members to the upgrade of existing steel structures, rehabilitation of timber structures, and even improving the ability of masonry structures to resist gravity, wind, or seismic loads. At the same time, the unique combination of a commonly deteriorated or deficient existing structural element with layers of new high performance composites gives rise to new and unique physical phenomena and, inevitably, to new failure mechanisms.

The retrofitted structural member is a layered element that combines existing and supplemental layers. As such, one of the “new” families of physical phenomena that govern its behavior includes debonding, delamination, or inter-laminar disintegration of the bonded layers. Those mechanism, which govern almost any type of layered, laminated, adhesively bonded, or sandwich structure, have been reported since the very first investigations of the new structural configuration but they still define a critical modeling, analysis, and design challenge. In particular, the consideration of the interfacial failure mechanism in layered structures and the translation of its dominant physical features into analytical, numerical, or computational models is a challenge that still has to be faced.

One of the most prominent feature of the new mode of physical behavior is its brittle, abrupt, unstable, and in many cases catastrophic character. Many experimental investigations report on a catastrophic phenomenon that evolves without a clear and effective forewarning, and brings the structural element to a complete failure without any possibility to capture or intervene in the process (see, for example, Rabinovitch and Frostig 2003). In fact, the evolution of the failure mode happens so fast, that experimental techniques that could actually capture the dynamic process in its progression and extract data from the dynamic mechanism are scarce.

The aforementioned features define the behavior of the layered structure and particularly its mode of failure as dynamic by nature. Yet, the majority of the research effort allocated to the investigation of the layered structure and the strengthening method adopted static concepts and static methodologies. The static consideration of the debonding failure, its assessment, and its physical modeling using analytical or numerical tools is documented in hundreds of works in the literature. The scope of the dynamic investigations is significantly more limited. In fact, the dynamic nature of the physical phenomenon, its modeling, and its consideration in terms of quantitative assessment, prevention, or arrest, define an open question with many scientific and practical implications. Furthermore, it defines a broad spectrum of modeling challenges that still have to be faced.

One of the first works that raised the question of dynamic modeling was presented by Jerome and Ross (1997) who used a dynamic finite element (FE) model of the FRP plated beam. Linear and high order models (in terms of the consideration of the displacement field in the adhesive layer) that faced the dynamic challenge are reported in Soleiman Fallah et al (2008) and Hamed and Rabinovitch (2005). Perera and Bueso-Inchausti (2010) used spectral elements with nonlinear constitutive laws for the dynamic investigation of the fully bonded structure. Bruno (2009) faced the challenge of dynamic modeling of the crack propagation in fiber-reinforced composites using a combination of beam and interface layers. Bruno et al (2013) studied the debonding mechanism using a 2D FE model based on ALE formulation and a moving mesh. Sun et al (2015) quantified the role of cracking and IC debonding in the frequency domain. Chen et al (2015) used a dynamic 2D FE analysis and modeled the FRP layer using truss elements. Discrete dynamic events reflected the dynamic features involved with cracking or debonding. Huo et al. (2016) experimentally studied the bond behavior under impact loads and proposed strain rate dependent factors for scaling a static shear-slip law for the dynamic case. Experimental investigation on the effect of loading rate on the bond characteristics and debonding strength of FRP plated steel beams was presented in Al-Zubaidy et al (2012a) and Al-Zubaidy et al (2012b), respectively. Al-Zubaidy et al (2013) further investigated the dynamic regime by means of dynamic pull-out tests of bonded laminates. Shen et al (2015) experimentally studied the dynamic rate dependent aspects of the debonding phenomena in a double-lap shear specimen subjected to a dynamic pull-out load.

Rabinovitch (2012) studied the dynamic debonding mechanism using a model that combined high order kinematics and a cohesive interface and focused on failure that occurs few millimeters within the concrete substrate. Rabinovitch (2014a,b) presented an augmented static and dynamic models, respectively, that allowed a more inclusive consideration of the stress field in the adhesive layer and the variation of the interfacial conditions from one interface to another. Mulian and Rabinovitch (2015) developed the FE form of this model. The augmented analytical effort was experimentally validated in Mulian and Rabinovitch (2016) by detecting the movement of the debonding front in the sub-ms time scales.

The survey of the literature reveals that the effort allocated to the understanding, quantification, and mainly modeling the failure of the layered structure and, particularly, the dynamic aspects of this failure mechanism are still limited. The pioneering investigations that addressed the dynamic nature revealed interesting and relevant observations on the phenomena but they still define analytical, numerical, and experimental questions that have to be answered. Those are highlighted and discussed in this paper.

The objective of this paper is to highlight, explore, and gain insight into the current modeling challenges associated with existing structural elements retrofitted with advanced composites and, particularly, into the ones associated with the dynamic nature of their failure mechanisms. While the method of strengthening and retrofitting existing structures with advanced composites is consistently making its way to the consensus, the failure mechanisms that govern this structural form and their dynamic nature still have to be explored. Altogether, the paper aims to combine the dynamic perspective with the current and future modeling challenges associated with the layered element and thus to gain insight into their impact on the resilience of the retrofitted structure.

The methodology adopted in the paper takes a dynamic standpoint and combines analytical, computational, and experimental considerations in the attempt to highlight the open modeling questions associated with structural system. The following section discusses the fundamental dynamic phenomena that are at the focus of the investigation, raises the open questions that stem from those phenomena, and highlights the gaps of knowledge that still exist in the field. Then, an illustrative example that examines the particular case of beams strengthened for flexure with externally bonded composites is discussed. The discussion of this case highlights the dynamic physical modeling of the structural system, the principles that are derived based on this analysis, and their implementation through analytical, numerical, and experimental tools. The paper closes with some concluding remarks and a short discussion of potential directions of future research.

2 DYNAMIC PHENOMENA AND THE OPEN QUESTIONS THEY RAISE

The handling of structural mechanisms, and particularly ones that are brittle and associated with the failure of the structural system, is naturally divided into two parts. The first part focuses on the conditions that trigger the mechanism whereas the second part, which is in the focus of this paper, concerns the mechanism itself. In terms of analytical assessment of the system, the objective of the first part is to identify the conditions that lead to the failure mechanism and thus to allow safe use of the structural element under load level that do not exceed the critical ones. The second part examines the nucleation, initiation, evolution, propagation, and stability of the mechanism. The understanding of these processes is the key for the development of concepts for handling and manipulating this mechanism with the motivation to gain control of the negative impact of the mechanism, to convert it into a more favorable one, and to improve the resilience of the structure.

Focusing back on the case of the debonding failure in layered structures, the assessment of the evolution of the conditions that lead to the failure mechanism can probably be achieved using static methodologies. For the second phase of handling the failure process and pointing at means for its manipulation, a comprehensive dynamic consideration is a necessity. This necessity is based on a spectrum of physical phenomena that were observed in many experimental studies and that are dynamic by nature. Those include the abrupt evolution of a debonding failure and the associated rapid change to the structural scheme. The latter yields a rapid unloading of the

FRP layer, amplified loading of the substrate element, and the prominent evolution of inertial effects, noise, vibrations, release of the stored elastic energy, and propagation of stress waves in the element. The combination of these processes may yield a destructive mode and global failure of the entire element.

The dynamic nature of the evolution the debonding failure mechanism raise many questions, most of them still open:

- What are the mechanical processes that govern the debonding failure mechanism?
- What is the role of the inertial forces?
- What is the role of strain rate effects?
- What stands behind what is commonly referred to as “instability”?
- What happens when the response reaches an unstable phase?
- What are the implications of this instability in terms of structural response?
- What is the interactive role of the dynamic features in such instabilities?
- What is the role of the dynamic features in the determination of the material or interfacial properties of the layered element?
- How can dynamic tests be used for the calibration of such material or interfacial properties?
- How do the dynamic features change across geometric scales (if at all)?
- What lessons can be learnt from the dynamic features regarding the attempt to arrest the debonding failure?
- What happens when we look into 2D plate-like problems where the debonding involves change of shape and not only change of size of the debonded area?
- And the most critical question that has to be asked in the context of the dynamic nature: how does all this affect the resilience of the element?

The above questions define a spectrum of analytical, numerical, and experimental challenges. Facing these challenges, which are discussed in the next section, is essential for understanding the mechanism that govern the dynamic response and for its safe, effective, and robust use in practice.

3 DYNAMIC CHALLENGES

The challenges that stem from the above notions and open questions can be divided into three categories. The first one refers the physical modeling of the layered structure. The second category refers to the numerical and quantitative tools that are needed for the handling of the unique problem. In particular, it refers to the application of computational tools that are based on the FE method. The last, and probably the most challenging category, refers to experimentation and to the integration of the modeling and numerical tools with the experimental ones.

3.1 Category I: Physical Modeling

The first class of challenges is concerned with the physical modeling of the structure. Specifically, it concerns the conversion of the physical behavior observed in the lab or in the field into a mathematical model that can assess, quantify, describe, and clarify the physical behavior. The physical modeling has to take into account the characteristic features of the structural element. Among those, one can list:

- The layered layout of the element.
- The presence of distinct layers and the development of the independent modeling tools for each layer.
- The presence of interfaces that connect the layers together or govern the process of disintegration.
- The independent modeling of each interface according to its nature and to the nature of the materials it joins.
- The combination of distinct and sometimes significantly diverse material properties. Specifically, the combination of relatively stiff with relatively compliant materials and the tendency of the compliant material to give rise to a spectrum of localized phenomena. The physical modeling has to determine which of all those phenomena is indeed important, which can be neglected, and what must be accounted for.
- The spectrum of characteristic length scales that range from the ones governing the crack tip fields and the local geometry to ones that play a role on the global grid. The difference between those may be as large as four orders of magnitude.
- The spectrum of characteristic time scales that range from the μs scale governing the crack tip kinematics to the second scale governing the global dynamics of a realistic structural element. The difference between those may be as large as six orders of magnitude.

Along with the characteristic features of the structure, the physical modeling has to face the fundamental aspects of the structural response. One such aspect is the dynamic response and the need to identify the dynamic effects that govern the physical behavior. The dynamic aspects range from inertial effects to rate dependency of materials, interfaces, and failure criteria involved. The dynamic features are essential for capturing the behavior associated with transients, propagation of pressure waves, release of the stored elastic energy, and phenomena that propagate with velocities that are in the order of magnitude of the speed of sound. Another fundamental aspect that has to be taken into account in the physical modeling phase is the modeling of the nonlinearities involved. Those include material, geometrical, interfacial, contact, and friction nonlinearities. The dynamic accumulation of damage, either within a transient incident or from one incident to another, the inelastic response of the materials, and the degradation of the interfaces are additional nonlinear features that may play a role in the dynamic response. The combination of the above features defines the physical modeling as one of the more demanding aspects of the dynamic consideration.

3.2 Category II – Quantitative and Computational Frameworks

A second class of challenges associated with the layered structure and its dynamic and nonlinear response is attributed to building the quantitative and computational numerical frameworks for the handling of the above features. The physical combination of distinct layers and interfaces and the scatter of length, elastic, and time scales become a critical challenge when numerical or computational methods are considered. They become an even more significant challenge when the finite element method is considered as the computational framework. In this case, the combination of the above features and yet the demand to keep the computational effort reasonable is a significant obstacle to using most standard, 3D elasticity based FE tools.

An additional aspect that play a critical role in the conversion of the concepts of the physical modeling into a numerical framework is the need to capture a localized and, at the same time, traveling phenomenon. The evolution of the failure mechanism and its representation through the kinematics of the debonding front moves on the spatial and temporal grids and requires special techniques for localization and shift of computational efforts (Bruno et al 2013, Funari and Lonetti 2017). Again, when it comes to numerical methods that are based on spatial and/or temporal discretization, this feature becomes a significant challenge.

3.3 Category III: Experimentation

A third class of challenges that stem from the physical behavior is attributed to experimentation. In particular, it refers to the establishment of an experimental method and the development of experimental techniques that are able to capture, record, and quantify the dynamic process that governs the debonding failure in the layered structure. Such framework is essential for providing sound experimental observations for the development of the physical modeling tools and for their validation but it is also essential for the shift form the lab to the real world. This includes setting forth a sound experimental procedure for the calibration of the material and interfacial properties that govern the behavior, developing a framework for shifting from the local scale of the crack tip to the scale of the structural element and the scale of the entire structure, and finally, converting the analytical, numerical, and experimental observations into engineering practice.

4 AN ILLUSTRATIVE CASE: DYNAMIC DEBONDING IN FRP PLATED BEAMS

To illustrate the above challenges, discuss some potential means to face them, and explore some of the dynamic aspects of the problem at hand, the debonding mechanism in FRP plated beams is looked at. The discussion of the illustrative case, which aims at highlighting the open questions in the field and, if possible, point at potential answers, follows the three categories discussed above. First, the physical modeling and the translation of the notions regarding the dynamic behavior into a mathematical model are discussed. Then, the computational concepts are discussed and finally the experimental methodologies are demonstrated. The discussion of the illustrative case of debonding in FRP plated beams follows the analytical work outlined in Rabinovitch (2012, 2014a, 2014b), the numerical tools developed in Mulian and Rabinovitch (2015), and the experimental investigation reported in Mulian and Rabinovitch (2016, 2017). Yet, the discussion is brought here in a nutshell and the reader is referred to those works for further details.

4.1 Physical Modeling

The fundamentals of the physical modeling phase for the dynamics of the FRP plated beam address the features, characteristics, and challenges outlined above. The main ones, which are discussed on detail in Rabinovitch (2014a,b) are outlined next.

4.1.1 Layered Layout

The basic feature of the physical modeling is the consideration of the FRP plated beam as a layered element. The layered element combines three physical layers through two interfaces. The physical layers include the original beam that is being strengthened (the substrate beam), the FRP layer, and the adhesive layer. Those layers are designated by the scripts "*c*", "*a*", and "*frp*", respectively. Adjacent physical layers are linked together: the adhesive-substrate interface (designated by the script "*ac*") and the adhesive-FRP interface (designated by the script "*af*").

4.1.2 Physical layers

The physical layers are characterized by a finite thickness and they are governed by continuum mechanics. In that sense, they differ from the interfaces, which are thickness-less mathematical entities that are governed by interfacial laws. Each of the three physical layers is independently modeled using its own physical modeling tools. In the present illustrative consideration, the layers are modeled using different beams theories. In in more advanced considerations, which also take 3D effects into account, they can be modeled using plate or shell theories, see, for example, Elmalich (2012a,b, 2015), Feldfogel and Rabinovitch (2016, 2017). The level of refinement of the theory adopted for each layer is determined in accordance with the main physical phenomena that govern its behavior. For example, for the substrate layer and the FRP layer, in most cases it is sufficient to use conventional beam theories such as the one attributed to Bernoulli and Euler or, if shear deformations are expected to become important, the one attributed to Timoshenko. The modeling of the adhesive layer, on the other hand, requires a more refined theory, mainly because in most cases, the adhesive material is much softer than the adjacent ones and yet it is placed on the critical path of stress transfer between those layers. This combination of material and geometrical features yields a rich deformation and stress fields in the adhesive layer and, in accordance, necessitates the use of sufficiently rich kinematic assumptions. The latter has to take into account distortion of the cross section, its ability to change its height under loads, and the evolution of a sufficiently rich stress field in which all stress components vary through the thickness of the layer.

4.1.3 Interfaces

While the physical layers are modeled using continuum theories, which differ based on the physical features that have to be taken into account, the interfaces are modeled by mathematically linking the displacement jumps across the interfaces with the evolution of interfacial tractions. The type of modeling may range from simple linear links that, under the relevant definition of properties, may reflect full compatibility with minimized displacement jumps, delamination conditions where the layers are free to move, and the spectrum of states between those extremities. In more advanced formulations, the link between the layers is modeled using cohesive interfaces that automatically model this spectrum through nonlinear traction-separation laws. Those nonlinear relations generally reflect compatibility with a limited stiffness under low levels of interfacial displacement jumps, an increased level of the interfacial tractions up to a critical relative displacement, and then decay up to the state that reflects debonding. The development of interfacial laws under certain global modeling assumptions has gained much attention and it is well documented in the literature. Formulations that range from shear-slip relations to ones that account for the tangential and normal effects as well as to their coupling are reported. It is, however, important to note that in all cases, those relations are phenomenological by nature and therefore rely on experimental calibration and determination of their properties.

4.1.4 Dynamics

The dynamic aspects naturally cover a spectrum of physical phenomena. The leading one is probably the inertial response associated with the rapid changes to the structural layout and the resulting accelerations. At certain intervals along the process, the inertial effects may become more dominant than the static components of the response. Additional effects that are solely attributed to the dynamic response are also significant players in the response. Those include, but not limited to, rate dependency of the materials and interfaces, viscoelastic and visco-plastic behavior, and more general mechanisms of energy dissipation. While the modeling of inertial effects is well defined within the scope of Newton's laws, the physical modeling of those additional effects defines challenges that still have to be faced. Just to illustrate that, one may consider the rate dependency of the behavior of the cohesive interface. On the one hand, such rate dependency may introduce a pseudo-stiffening effect with rate dependent factorization of components of the traction-separation laws (Geissler and Kaliske 2010, Geissler et al 2007, Corigliano et al 2003, 2006). On the other hand, it may be claimed that the debonding velocity and mainly the velocity of the debonding front are insensitive to some physical parameters (Rabinovitch 2012, 2014b, Mulian and Rabinovitch 2015). The combination of the two features raises questions regarding the dynamic scaling of this parameter, for example with the size of the element, which may critically affect the ability to calibrate the properties of the cohesive interface based on laboratory scaled specimens. This observation implies that the introduction of the inertial effects is rather "straight forward" but the consideration of rate dependency involved a spectrum of research challenges.

4.1.5 Nonlinearities

The dynamic debonding phenomenon is governed by a range of strong nonlinearities, which, in turn, have to be taken into account in the physical modeling phase. The most prominent one is the interfacial nonlinearity that dictates the debonding mechanism. This nonlinear physical feature is modeled using the interfaces and the shift from a fully bonded state where the layers move together to a debonded state where they move independently. An outcome of the interfacial debonding mechanism is the evolution of secondary interfacial nonlinearities such as contact and friction at the delaminated regions. A third tier of nonlinearities naturally concerns the response of the material involved. This is particularly relevant when brittle materials such as concrete, masonry, or natural stone or elsto-plastic material such as steel are involved. The material nonlinearity may take a local form in terms of plasticity at the close vicinity of the delamination front or a global form involving significant parts of the structural element. In both cases, it may affect the conditions that trigger the dynamic process (e.g. cracks distributed along the length of the beam) as well as the process itself.

A forth tier of nonlinearities is attributed to geometrical ones and particularly to geometrical nonlinearity in the FRP layer. When the thin layer debonds, loses its support, and, at the same time, resists complex transient loading scenarios that may involve compression, it may buckle or wrinkle. In turn, the geometrical nonlinearities may affect back the interfacial and dynamic mechanisms yielding a coupled nonlinear problem.

The consideration of the accumulation of damage is fifth critical aspect of the nonlinearities involved in the physical behavior of the debonding beam. This feature refers to the material response and to the interfacial response. In both cases, the evolution of vibrations or the exposure of the structure to a series of loading incidents necessitates the consideration of the accumulation of damage and a sound methodology for the assessment of the current state of the materials and interfaces.

4.2 Analytics and realization of the physical modeling concepts

An illustrative model that follows the aforementioned physical modeling tools (or at least the main ones) is outlined in Rabinovitch (2014b). The geometrical layout of the element considered in the model is outlined in Figure 1. The model adopts the layered layout concept and applies the classical beam theory to the substrate ("*c*") and FRP ("*frp*") physical layers. As for the adhesive layer ("*a*"), the demand of using a refined theory and richer stress and displacement fields associated the unique geometry and material properties of the layer motivates the use of high order kinematic assumptions. The vertical and longitudinal displacement fields in the layer are modeled in that framework by means of second and third order polynomials. While other kinematic assumptions that represent a sufficiently rich displacement fields also apply, this polynomial representation, which is inspired by the closed form solution for the degenerated case of static behavior and zero longitudinal stiffness, is probably the simplest one that still captures the fundamental features of the response. Those include distortion of the cross sections, vertical compressibility, coupling of the responses through Poisson's effect and through equilibrium condition, a spectrum of isotropic or orthotropic constitutive behavior, and mainly evolution of a rich normal (axial and vertical) and shear stress fields. In addition, it allows the interfacial tractions to change from one interface to another and thus to reflect the physical distinction between them and the tendency of the failure mechanism to evolve across on interface or even to kink form one interface to another as a response to the augmented stress state.

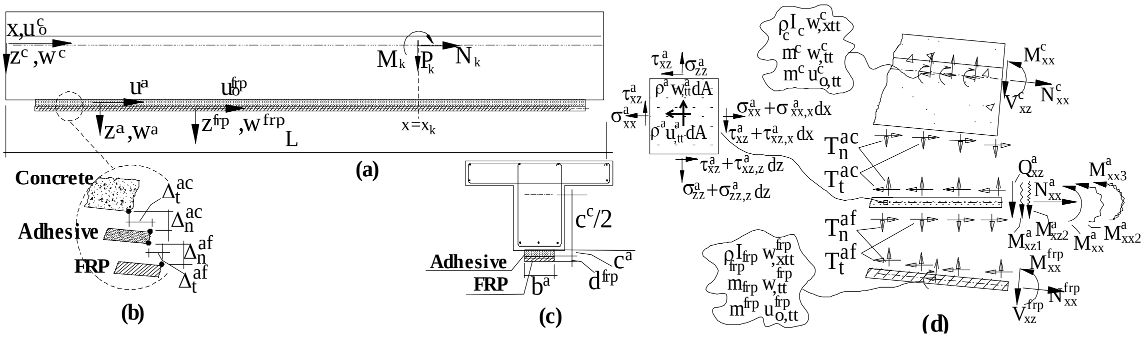

Figure 1: Notations and sign conventions (Rabinovitch 2014a): (a) Geometry and displacements; (b) cross section; (c) displacement jumps across the cohesive interfaces; (d) stress resultants, inertial effects, and dynamic equilibrium in the adhesive layer.

The interfaces in the model are introduced using cohesive laws that define the interfacial tractions as functions of the interfacial displacement jumps. Such laws that may range from simple uncoupled ones to more general ones that account for coupling of the tangential and normal effects, contact effects, and even accumulation of damage as well as laws that reflect the physical behavior by differ in their functional forms (Dimitri et al 2014). The cohesive laws used in Rabinovitch (2014a,b) and latter in Rabinovitch and Mulian (2015,2016,2017) adopt the formulation developed in Volokh and Needleman (2002). Those cohesive laws, which are independently applied to the adhesive-FRP and adhesive-substrate interfaces, represent the entire spectrum of bond conditions as well as the coupling of the shear and normal effects while using continuous and differentiable functions and only two constants (specific work of separation, characteristic length scale). In their basic form, the cohesive laws used here are based on a hyperelastic formulation. The consideration of accumulation of interfacial damage as well as contact and friction effects and their introduction into the model are therefore designated as challenges for further study. For generality, the cohesive laws are introduced into the following formulation in a generic form $\mathbf{T}=\mathbf{T}(\mathbf{\Delta}(x,t))$ where $\mathbf{T}=[T_n(x,t),\ T_t(x,t)]$ is the traction vector, $\mathbf{\Delta}=[\Delta_n(x,t),\ \Delta_t(x,t)]$ is the vector of displacement jumps, and the subscripts n, t designate the normal (peeling, separation) and tangential (shear, slip) directions, respectively.

The dynamic effects are introduced by means of the simpler inertial forces. Again, this first (and, in some cases, sufficient) step into the dynamic regime designate the consideration of rate dependency of the materials and interfaces, viscoelastic and visco-plastic behavior, and other dissipative mechanisms for future research. Finally, the consideration of nonlinearities takes into account the interfacial and geometrical ones whereas the materials are assumed linear and elastic. This step designates the consideration of material nonlinearity, inelasticity, and accumulation of damage as aspects that still have to be investigated.

The concepts discussed above are translated into an analytical model, see Rabinovitch (2014b). The equations of motion of the model take the following form:

$$b^a T_t^{ac}(x,t) + N_{xx,x}^c(x,t) + n - m^c u_{0,tt}^c(x,t) = 0 \quad (1)$$

$$b^a T_n^{ac}(x,t) + M_{xx,xx}^c(x,t) + \left(N_{xx}^c(x,t)\, w_{0,x}^c(x,t)\right)_{,x} + b^a c^c T_{t,x}^{ac}(x,t)/2 + q - m^c w_{0,tt}^c(x,t) + I_m^c w_{0,xxtt}^c(x,t) = 0 \quad (2)$$

$$N_{xx,x}^{frp}(x,t) - b^a T_t^{af}(x,t) - m^{frp} u_{0,tt}^{frp}(x,t) = 0 \quad (3)$$

$$-b^a T_n^{af}(x,t) + M_{xx,xx}^{frp}(x,t) + \left(w_{,x}^{frp}(x,t)\, N_{xx}^{frp}(x,t)\right)_{,x} + b^a c^{frp} T_{t,x}^{af}(x,t)/2 - m^{frp} w_{0,tt}^{frp}(x,t) + I_m^{frp} w_{0,xxtt}^{frp}(x,t) = 0 \quad (4)$$

$$-b^a T_t^{ac}(x,t) + b^a T_t^{af}(x,t) + N_{xx,x}^a(x,t) - m^a u_{0,tt}^a(x,t) - I_m^a u_{2,tt}^a(x,t) = 0 \quad (5)$$

$$-Q_{xz}^a(x,t) + M_{xx,x}^a(x,t) + \frac{b^a c^a}{2} T_t^{af} t(x,t) + \frac{b^a c^a}{2} T_t^{ac}(x,t) - I_m^a u_{1,tt}^a(x,t) - J_m^a u_{3,tt}^a(x,t) = 0 \quad (6)$$

$$-2M_{xz1}^a(x,t) + M_{xx2,x}^a(x,t) + \frac{b^a (c^a)^2}{4}\left(T_t^{af}(x,t) - T_t^{af}(x,t)\right) - I_m^a u_{0,tt}^a(x,t) - J_m^a u_{2,tt}^a(x,t) = 0 \quad (7)$$

$$-3M_{xz2}^a(x,t) + M_{xx3,x}^a(x,t) + \frac{b^a (c^a)^3}{8}\left(T_t^{ac}(x,t) + T_t^{af}(x,t)\right) - J_m^a u_{1,tt}^a(x,t) - K_m^a u_{3,tt}^a(x,t) = 0 \quad (8)$$

$$-b^a T_n^{ac}(x,t) + b^a T_n^{af}(x,t) + Q_{xz,x}^a(x,t) - m^a w_{0,tt}^a(x,t) - I_m^a w_{2,tt}^a(x,t) = 0 \quad (9)$$

$$-R_{zz}^a(x,t) + M_{xz1,x}^a(x,t) + \frac{b^a c^a}{2} T_n^{af}(x,t) + \frac{b^a c^a}{2} T_n^{ac}(x,t) - I_m^a w_{1,tt}^a(x,t) = 0 \quad (10)$$

$$-2M_{zz1}^a(x,t) + M_{xz2,x}^a(x,t) - \frac{b^a (c^a)^2}{4}\left(T_n^{ac}(x,t) - T_n^{af}(x,t)\right) - I_m^a w_{0,tt}^a(x,t) - J_m^a w_{2,tt}^a(x,t) = 0 \quad (11)$$

where the inertia terms are given by:

$$m^i = \rho^i b^i c^i;\ I_m^i = \frac{\rho^i b^i {c^i}^3}{12};\ J_m^i = \frac{\rho^i b^i {c^i}^5}{80};\ K_m^i = \frac{\rho^i b^i {c^i}^7}{448} \quad (12)$$

In the above equations, ρ^i, b^i, and c^i ($i=c,a,frp$) are the density and cross sectional properties of each layer ($i=c,a,frp$); u^c, w^c, u^{frp}, and w^{frp} are the longitudinal and vertical displacements in the substrate and FRP layers; $w_0^a..w_2^a$, $u_0^a..u_3^a$ are the high order displacements in the adhesive layer; $N_{xx}^c(x,t)$, $M_{xx}^c(x,t)$, $N_{xx}^{frp}(x,t)$, $M_{xx}^{frp}(x,t)$, $N_{xx}^a(x,t)$, $M_{xx}^a(x,t)$, $M_{xx2}^a(x,t)$, $M_{xx3}^a(x,t)$, $Q_{xz}^a(x,t)$, $M_{xz1}^a(x,t)$, $M_{xz2}^a(x,t)$, $R_{zz}^a(x,t)$, and $M_{zz1}^a(x,t)$ are generalized stress resultants; the comma represents a partial derivative; and T^j_n, T^j_t ($j=ac,af$) are the tractions across each cohesive interfaces. For brevity, the constitutive relations and the boundary and initial conditions are not presented here and the reader is referred to Rabinovitch (2014b) for further reading.

4.3 Numerical consideration

Facing the modeling challenges associated with the dynamic failure mechanism is naturally involved with a spectrum of nonlinearities. The main ones, which are discussed in the preceding section, include geometrical nonlinearities associated with large deformations, interfacial nonlinearities associated with the debonding mechanism and the evolution of friction and contact effects, and material nonlinearities associated with the inelastic response of the materials and the accumulation of damage. On top of that, the behavior is governed by dynamic transients and a scatter of characteristic length of time scales. This ensemble of physical phenomena challenges the ability to derive analytical solutions to any model that incorporates even part of them. This paves the way to numerical solutions and indeed, when it comes to quantitative solutions of the analytical model derived in Rabinovitch (2012) or Rabinovitch (2014a), a numerical approach based on the multiple shooting method for the solution is space and Newmark's method for the time integration was applied. While this approach achieved it goal, and while other numerical methods apply, it is noted that the most common numerical approach used today in structural analysis is the finite element (FE) method.

The application of the finite element method to the investigation and analysis of the dynamic debonding failure in FRP plated beams is a considerable challenge. This is attributed to three aspects, all related to scatter of characteristic scales. The first one refers to scatter of length scales and particularly to the fact that the characteristic length scale of the debonding phenomenon is of the order of the thickness of the thinnest layer involved. In the FRP plated beam, this may be two, three, or sometimes even four orders of magnitude smaller than the global dimensions of the structure that is being analyzed with the lower end referring to laboratory tests and the higher end to realistic applications. On top of that, the movement of the debonding front along the beam during the debonding process limits the ability to locally refine the mesh near critical points on the global grid. Alternatively, this physical feature points at using moving mesh solutions (Bruno et al 2013, Funari 2017). The need to capture phenomena in the sub-mm scale along structural elements that may be tens of meters long and yet to keep the computational efforts reasonable is a significant challenge.

The second aspect is the scatter of elastic scales. The layered or adhesively bonded structure combines materials with elastic properties that may differ by two or three orders of magnitude. When it comes to numerical analysis, and particularly FE, analysis, this spectrum of elastic scales may lead to extensive computational costs. Furthermore, the presence of interfaces linking layers of significantly different thickness and significantly different elastic properties may lead to local divergence of the FE solution, particularly near the regions that are of most interest and impact on the nucleation, evolution, and progress of the failure mechanism.

The third aspect is the scatter or time scales. While the characteristic time scales of the global response of the FRP plated beam (e.g. the vibration period attributed to the first natural frequency of such beam) are in the order of 10^{-1}-10^{-2} s, the characteristic time scales of the dynamic debonding process are in the order of 10^{-4}-10^{-5} s. This ratio, on the one hand, and the demand to capture local transient phenomena on the global temporal grid on the other hand, challenge any numerical method, including the FE one.

One of the approaches proposed in attempt to face the above challenges, to overcome at least some of the above obstacles, and yet to take advantage of the generality, geometrical flexibility, and applicability of the FE method, abandons the classical 3D elasticity based finite elements and focuses on FE formulations that are based on analytical layered beam or plate theories. In other words, this approach uses the aforementioned concepts of physical modeling to derive a mathematical model for the layered beam and then uses the FE method to solve the resulting equations. The application of the relevant kinematic assumptions to each layer and the modeling of the interaction between them allow to eliminate one dimension of the problem and to avoid meshing through the thickness of the layers. In terms of FE formulation, this may lead to a reduction of about two orders on magnitude in the number of degrees of freedom and an even more significant reduction in the computational complexity. Examples of the application of such concept to FRP plated beams is found in Madah and Rabinovitch (2012), Ben Dror and Rabinovitch (2015), and Mulian and Rabinovitch (2015). Examples of its application to the more complex case of plate and plate-like structures is found in Elmalich and Rabinovitch (2012a,b, 2015) and Feldfogel and Rabinovitch (2016,2017). An illustrative example that represents the finite element that is inspired by the formulation given in equations (1)-(12) and derived for the dynamic debonding analysis of the FRP plated beam is derived in Mulian and Rabinovitch (2015). This beam element includes 13 DOFs per node. 3 DOFs are attributed to the axial, vertical, and cross section rotational displacements of the substrate beam element; 3 are attributed to the same type of displacements but in the FRP layer; and 7 are attributed to the adhesive layer. Those 7 DOFs build the parabolic field of the vertical displacement and the cubic field of the axial displacement. Altogether, they define the high order kinematics adopted for that layer in attempt to capture its rich displacement and stress fiekds. More details about this element as well as its validation and verification appear in the aforementioned references.

4.4 Experimental investigations

The most challenging aspect of the handling of the dynamic debonding phenomena, its quantification, and its modeling, is the experimental consideration of the process. The main feature that contributes to the complexity of facing the experimental challenge is the time scales involved. Preliminary analysis (Rabinovitch 2012, 2014b) indicated that the velocity of the movement of the debonding front along the beam is in the order of 10^3 m/s. For laboratory scaled test specimens, this means that the duration of the entire debonding process is bounded by about 10^{-4}-10^{-3} s. The temporal resolution required for capturing this process may be in the order of 10^{-5} s or even 10^{-6} s. In terms of experiments, this necessitates a new experimental approach and new experimental tools for capturing the dynamic features of the physical phenomena.

An illustrative example of such new experimental approach, which focuses its lens on the dynamic nature of the process, is outlined in Mulian and Rabinovitch (2016, 2017). The experimental approach examines a small-scale FRP plated beam specimen subjected to 4 point bending in a displacement control mode. Along with the conventional monitoring of the load and the corresponding displacement, the dynamic evolution of the debonding process is detected by means of high-speed photography. The experimental setup, which is discussed in detail in Mulian and Rabinovitch (2016), is illustrated in Figure 2. A series of snapshots taken during the dynamic process appears in Figure 3. The photography rate adopted for the detection of the dynamic process is 88,000 frames per second and the time interval between one snapshot and another is $11.36 \cdot 10^{-6}$ s. The time coordinate, which is shifted to the first snapshot, appears on the upper right corner of each snapshot. The series of snapshots outlined in Figure 3 and the use of modern digital image processing tools allow detecting the movement of the debonding front in time. This is reflected by the black cross marks in the figure. More details regarding the image processing procedure, as well as quantitative results for two additional test specimens, appear in Mulian and Rabinovitch (2016, 2017).

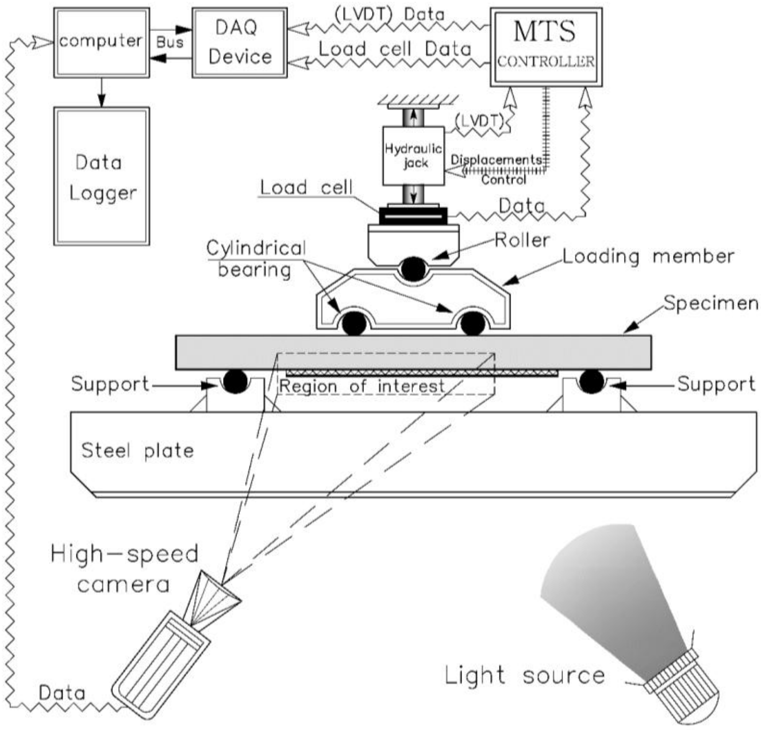

Figure 2 – Dynamic test setup (Mulian and Rabinovitch (2016))

The geometrical calibration used in the image processing of the experimental snapshots also allows quantifying the kinematics of the crack evolution. The curves showing the location of the crack front versus time and the assessment of the debonding crack tip velocity based on a finite difference approximation of the temporal derivative of the traction Vs. time curve appear in Figure 4. The results in Figure 4 are also compared with the ones that correspond to Specimen SB1 studied in Mulian and Rabinovitch (2016) and to the analysis, which is presented in Mulian and Rabinovitch (2016) and is relevant to the specimen studied here as well (both specimens have the same layout).

The detection of the movement of the debonding front in time, the analysis of the data, the assessment of the debonding velocities, which are in the order of 1000 m/s with temporal peaks the exceed 1500 m/s, and the numerical results clearly point at the dynamic nature of the process. The examined test specimen changes its form from a fully bonded element to an effectively debonded one, and changes the mechanism it resists loads from composite action of two components to a simple bending of one in less than $0.15 \cdot 10^{-3}$ s.

The rapid change in the sub-ms time scale and the involved velocities in the order of 1000 m/s demonstrate that the dynamic process takes the physical phenomena to regime that it completely different from what can be considered under static conditions. This regime is governed by inertial forces and rate dependent effects that necessitate completely different analytical, numerical, and experimental standpoints. As such, the dynamic consideration, its direct impact on the resilience of the layered structure, and its implications on the failure tolerance of full-scale FRP plated elements define a brand new set of challenges that has to be faced in the process of understanding and managing the behavior of the structural element.

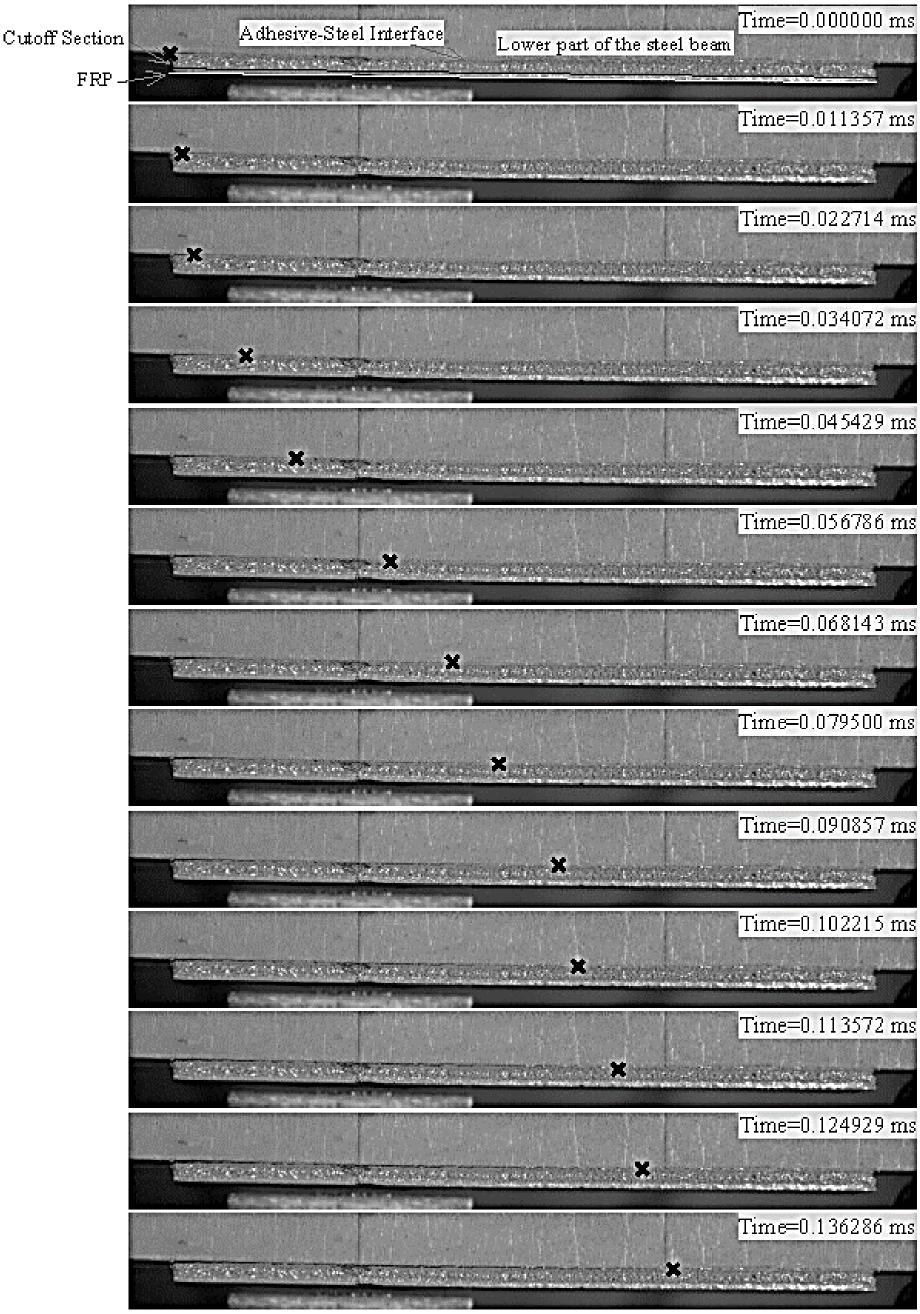

Figure 3: Debonding Front kinematics: A series of snapshots of the lower part of the beam, the adhesive layer and the FRP plate taken at intervals of $11.36 \cdot 10^{-6}$ s during the debonding process. The cross marks designate the crack tip.

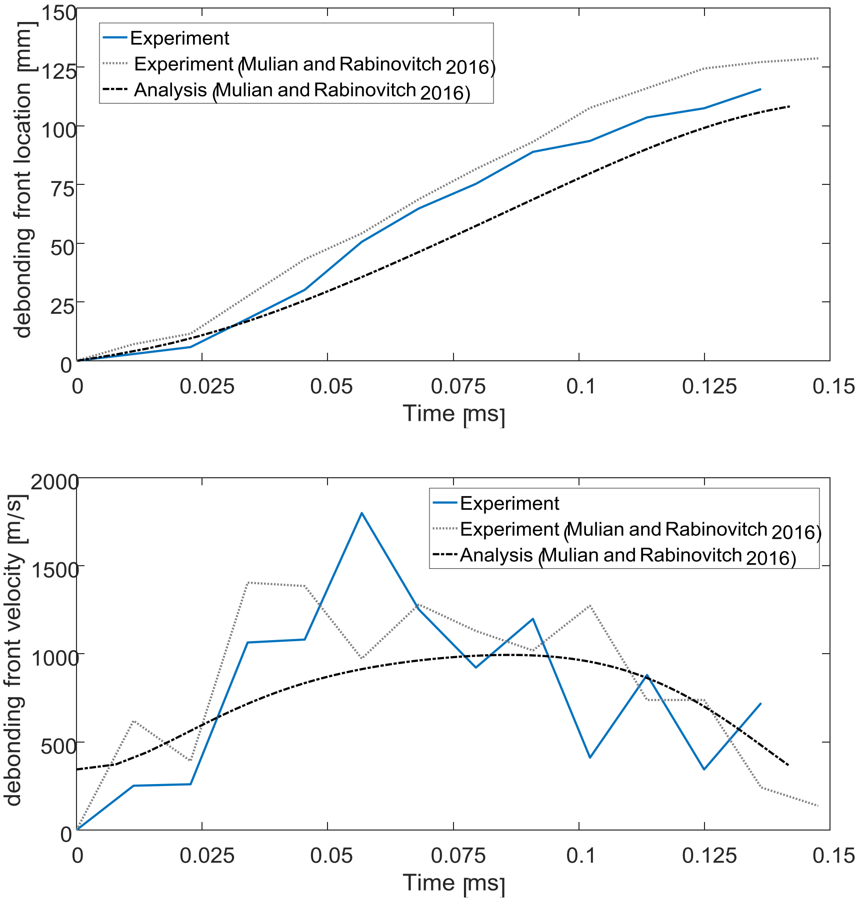

Figure 4: Debonding front kinematics: (a) location of the debonding front Vs. time; (b) velocity of the debonding front Vs. time. Time coordinate shifted to the initiation of the debonding process. Legend: —— Experiment; ······ Experimental results in Mulian and Rabinovitch 2016; -·-·-·- Numerical results in Mulian and Rabinovitch 2016.

5 CONCLUDING REMARKS

This paper has illustrated and discussed the challenges associated with the dynamic modeling of the behavior of FRP plated structural elements. In particular, it has focused its lenses on the dynamic aspects of the critical and still least understood debonding failure mechanism of the element. The discussion has brought up modeling challenges associated with the conversion of the physical behavior of the unique element into a sound mathematical model that incorporates the main features of the problem at hand. Those include the layered layout; the modeling of the physical layers and the interfaces that join them together; the scatter of geometric, elastic, and time scales; the inertial and rate dependent effects that take over when the process shifts into its dynamic regime; as well as the material, geometrical, and interfacial nonlinearities that govern the debonding problem.

Altogether, the unique physical features of the dynamic phenomenon define a thought-provoking analysis and modeling challenge. This challenge is relevant to the field of strengthening of existing structures with externally bonded layers of FRP but also to a broader spectrum of layered, sandwiched, laminated, and adhesively bonded structures. Along with that, they define unique numerical challenges that are amplified by the coupling of length and time scales over several orders of magnitude, variation of elastic properties, and, at the same time, physical phenomena that take place at the μs range and sub-mm scale but affect the global reiliance of the entire structural element.

The discussion in the paper has further highlighted the impact of the unique physical characteristics on the application of the FE method to the problem at hand and discussed a numerical analysis approach that combines the advantages of the physical modeling tools with those of the FE method. Finally, the paper has highlighted some of the fundamental challenges associated with the experimental detection of the dynamic response. In that context, the results of an experimental technique that is based on high-speed photography and digital image processing tools have demonstrated and quantified the unique features: debonding front velocities of the order of 1000 m/s and processes that change the structural nature of the examined specimen in less than $0.15 \cdot 10^{-3}$ s.

The use of FRP for the strengthening of existing structures have shifted form an emerging technology three decades ago to an established and well accepted one today. Yet, the application of the method still depends on the sound understanding of the physical behavior of the element and particularly it failure mechanism. The aspects, features, and characteristics discussed in this paper in the context of a dynamic perspective highlight some of the open challenges in this field. Facing these challenges is essential for the safe, sound, and resilient use of the layered structure.

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