SMAR 2017

Nonlinear Modeling of Bar Buckling and Rupture in RC Columns Under Cyclic Loads

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ABSTRACT: Reinforced concrete (RC) frame structures constitute a popular lateral-load resisting system in earthquake-prone regions. Modern design standards require that such structures are detailed to ensure a response mechanism dominated by flexural inelastic deformations under earthquake loading. For rare, extreme loading events, the response is significantly affected by the inelastic buckling of the reinforcing steel, and by the eventual rebar rupture due to low-cycle fatigue. The quantitative determination of the safety offered by RC frame structures against earthquake-induced collapse critically hinges on the availability of analytical tools which can account for the salient features of material response. Ideally, the tools should be algorithmically efficient, to enable systematic parametric analyses. This study presents a beam-based modeling approach for the analysis of RC frame members under cyclic loads, that can capture the effect of inelastic buckling and rupture of reinforcing steel bars. The approach uses force-based elements with a fiber sectional model and a corotational formulation to account for the geometric nonlinearity effect on the response of columns. A recently proposed phenomenological uniaxial model for steel reinforcement, capable of describing inelastic buckling and rupture due to low-cycle fatigue, is used for the reinforcing steel fibers. The strain penetration effect is also accounted for in the analyses. The modeling approach is validated with the results of experimental tests on RC columns under cyclic loads. A sensitivity study is also pursued to elucidate the impact of rebar buckling and strain penetration on the analytical results.

1 INTRODUCTION

Modern reinforced concrete (RC) frames are designed to develop inelastic flexural deformations in the member end regions. During earthquakes, longitudinal reinforcing bars of frame members may experience buckling due to the compressive stresses applied after the previous occurrence of inelastic tensile strains. Reinforcing bar buckling is also affected by the interaction between rebars and concrete. Specifically, the bars are initially restrained by the cover concrete until the latter crushes. At later stages of loading, buckling may be exacerbated by the outward pressure exerted by the core concrete on the rebars (Moehle, 2014). Experimental studies on cyclic behavior of single reinforcing bars revealed that buckling length affects the stress-strain response (Monti and Nuti, 1992; Gomes and Appleton, 1997; Rodriguez et al., 1999). There are also instances where the magnitude of the inelastic strains and the repeated cyclic loading lead to rupture of bars due to low-cycle fatigue. The analytical determination of the performance of RC structures under earthquake loading must rely on models which account for the effect of rebar buckling and rupture.

A number of studies have attempted to analytically capture the effect of rebar buckling on the response of RC components. Zong et al. (2013) conducted 3-D finite element simulations for circular RC columns and developed beam-on-springs model by isolating reinforcing bars. Massone and Lopez (2014) proposed a concentrated plasticity model with four plastic hinges accounting for initial imperfections. A hybrid finite element model consisting distributed plasticity and finite element models predict reinforcement buckling was proposed by Feng et al. (2015). Su et al. (2015) tested rectangular and circular columns and proposed a simplified model to estimate buckling length for circular columns. Kashani et al. (2016) studied modeling of rebar buckling in RC columns using nonlinear fiber-based elements and concluded that material models are needed considering inelastic buckling and rupture of rebars. Recently, a uniaxial material model has been developed by Kim and Koutromanos (2016) capable of representing rebar buckling and fracture.

This study presents a nonlinear beam-based modeling approach to simulate RC columns with significant reinforcing bar buckling. Reinforcing bars susceptible to buckling are modeled explicitly with a uniaxial material model accounting for inelastic buckling and rupture proposed by proposed by Kim and Koutromanos (2016). Moreover, the model considers strain penetration effects resulted by longitudinal reinforcement slip from anchorage of beam to foundation. Experimental cyclic response of one circular, one rectangular RC columns with significant buckling of reinforcing bars are validated by using beam-based models.

2 DESCRIPTION OF MODELING SCHEME

The analysis approach uses constitutive models and element formulations implemented in the open-source program *OpenSees* (McKenna et al. 2000). The column members are modelled with nonlinear beam elements using the force-based formulation of Neuenhofer and Filippou (1998). The specific formulation automatically (strongly) satisfies the equilibrium equations in the interior of the element. This entails the need to weakly satisfy compatibility at the element level. The weak satisfaction of compatibility requires iterations at the element level. The geometric nonlinearity (i.e., the P-Delta effects) is accounted for by means of a corotational element formulation.

The generalized stress-strain law, i.e. the relation between the sectional stress resultants (axial forces and moments) and the generalized strains (axial deformations and curvatures) in the beam models is described by means of a fiber model (Neunhofer and Filippou 1998). Specifically, the section is discretized into a number of fibers, each fiber provided with an appropriate uniaxial stress-strain law. Given the sectional axial deformation and curvature, the axial strain of each fiber can be calculated. The stress of each fiber is then obtained, by means of the corresponding stress-strain law. Finally, the sectional stress resultants are obtained by summing the corresponding contributions of all fibers comprising the section.

It is well-established (e.g., Neuenhofer and Filippou 1998) that force-based formulations allow the use of a single element for the representation of each member in a frame structure. In this case, experience suggests that the minimum number of quadrature (integration) points along the length of the element must be equal to 5. The use of multiple integration points over the length of an element for cases that involve softening response due to, e.g., concrete crushing or rebar rupture, may create issues due to the localization of inelastic deformations (Coleman and Spacone, 2001)). To prevent loss of objectivity for such cases, regularization techniques (e.g., Coleman and Spacone, 2001) or non-local element formulations need to be used. To circumvent the need for regularization, the present study – which focuses on cantilever columns – adopts a different procedure. Specifically, a column member is modeled using two elements. The length of the bottom element is set equal to the plastic hinge length of the column, i.e. the region where the majority of inelastic deformations are expected to occur when the column is subjected to cyclic lateral loads.

3 MATERIAL MODELS

3.1 Material Model for Concrete

The uniaxial constitutive law for concrete is the one formulated by Lu and Panagiotou (2014), schematically summarized in Fig. 1a. The specific material model can account for the strength and stiffness degradation associated with tensile cracking and compressive crushing. The model is also capable of accounting for the increased compressive strength and ductility of confined concrete. The strength and deformability of confined concrete are determined using the Equations by Mander et al. (1988).

3.2 Material Model for Reinforcing Steel

The reinforcing steel is modeled with the uniaxial stress-strain law by Kim and Koutromanos (2016). The specific law can account for the hysteretic response of the material, and includes a criterion to detect rebar buckling, based on first principles of mechanics. The monotonic stress-strain curve of the material model can account for the yield plateau, strain-hardening regime and rupture of the material, as shown in Fig. 1b, while the hysteretic law, which is schematically summarized in Fig. 1c, can account for the nonlinear kinematic hardening effect. The buckling criterion and post-buckling response of the material are governed by a dimensionless parameter L/d, i.e. the ratio of the length L of the bar segment that undergoes buckling over the bar diameter d. In the present study, the ratio L/d is determined using the procedure for rectangular and circular columns by Dhakal and Maekawa (2002) and Su et al. (2015), respectively. The specific procedure accounts for the buckling of longitudinal reinforcement in RC members.
To account for the rebar rupture, a criterion based on the accumulation of a continuous quantity D is adopted. Specifically, given the true material stress, f, and the plastic strain rate, $\dot{\varepsilon}_p$, the evolution of D is governed by the following rate equation.

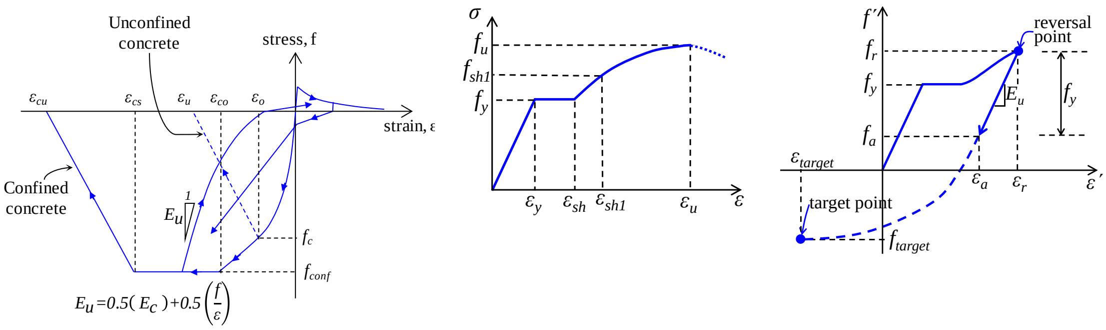
$$\dot{D} = \begin{cases} \left(\dfrac{f}{f_y}\right)^{2t} \dot{\varepsilon}_p \;,\; if\; f > 0 \\ 0 \quad otherwise \end{cases} \tag{1}$$

where t is a material constant. Rupture, i.e. material failure, occurs when the value of D becomes equal to D_{cr}, which is also a parameter of the material model. The criterion of Equation (1) is used to capture rupture under both monotonic and cyclic loading. This allows to calibrate the value of parameter D_{cr} using data from monotonic tension tests on rebars.

3.3 Accounting for the strain penetration effect

The modeling scheme also accounts for the strain penetration effect at locations where flexural inelasticity is expected to occur for column members. This is accomplished by introducing additional vertical elements at the base of a model, as shown in Fig. 2a for the support of a cantilever column. The top nodes of these vertical elements are connected to the base node of the column through rigid beam elements, while the bottom nodes of the vertical elements are fully restrained. This means that the axial deformations of the vertical truss elements are coupled to the vertical translation and rotation of the base node of the beam model. The vertical elements develop very high stiffness in compression, so that the slip affects only rebars under tensile axial stress.

The length L_b of the additional elements is determined so that the total slip due to strain penetration when the rebars yield equals the value obtained with the expression presented in Zhao and Sritharan (2007). More specifically, given the value s_y, of the total slip at yield, L_b can be obtained by stipulating that the uplift due to the additional vertical elements – which, at the onset of yield of the vertical reinforcement, will have an axial strain equal to ε_y – is equal to s_y, as explained in Fig. 2b.

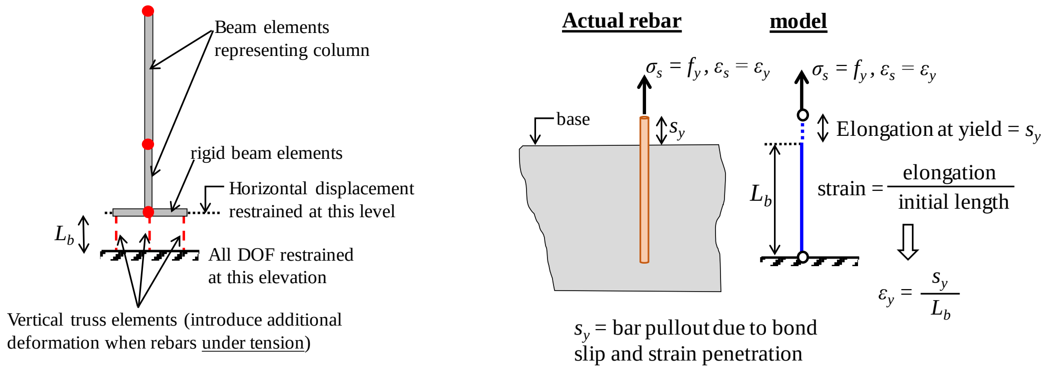


a) Stress-strain law for concrete

b) monotonic stress-strain curve for reinforcing steel

c) hysteretic stress-strain response of reinforcing steel

Figure 1: Uniaxial stress-strain laws for concrete and reinforcing steel.



a) Elements to account for strain penetration

b) determination of length L_b of vertical truss elements accounting for strain penetration

Figure 2: Accounting for strain penetration effect at the column base.

4 VALIDATION OF ANALYSIS METHODOLOGY

The analysis methodology is validated using the results of two previously conducted experimental tests of RC columns under cyclic lateral load. For both cases, the columns were subjected to a cyclic lateral displacement history with increasing amplitude, until the occurrence of severe strength degradation due to confined core crushing and rebar rupture. The values of material strength parameters for the concrete and reinforcing steel used in the analyses are listed in Tables 1 and 2, respectively. The values of the material parameters have been calibrated using the results of material tests that accompanied the column tests. Since the purpose of the validation analyses is to evaluate the predictive capabilities of the modeling scheme, no effort was made to fine-tune the material parameters so that the agreement between the experimental tests and the analytical models.

4.1 *Analysis of Column with Rectangular Cross-Section*

The first validation analysis is for an RC column specimen tested by Bae and Bayrak (2008), schematically depicted in Fig 3a. The specimen had a rectangular cross-section, which was made of concrete with a compressive strength of 36.5 MPa. The longitudinal reinforcement had a yield stress of 400 MPa, while the yield stress of the transverse reinforcement was 455 MPa. The specimen was subjected to a constant axial force of 2652 kN corresponding to an axial load ratio of 20%.

The analysis model is capable of satisfactorily capturing the hysteretic force-displacement response of the specimen, as shown in Fig. 4a. The stress-strain response of one of the outer rebars of the column section is presented in Fig. 4b. Rebar rupture was obtained in the analysis, as deduced from the loss of resistance in the reinforcing steel stress-strain plot of the same figure. The analysis also predicted spalling of the cover concrete (which was manifested as a complete loss of compressive strength in sectional fibers corresponding to the cover) and crushing of the confined core (the latter occurred in the form of compressive softening of sectional fibers corresponding to the core).

4.2 Analysis of Column with a Circular Cross-Section

An additional validation analysis is presented for a column with a circular cross-section tested by Lehman and Moehle (2000). The geometry of the column specimen is presented in Fig. 3b. The compressive strength of the concrete was equal to 31 MPa, and the yield strength of the longitudinal and transverse reinforcement was equal to 512 MPa and 606.8 MPa, respectively. The column was subjected to a constant axial load of 653.86 kN, corresponding to an axial load ratio of 7.2%. The analysis was capable of satisfactorily capturing the global hysteretic response of the specimen, as shown in Fig. 5a. The column specimen incurred strength degradation due to rebar fracture. Such fracture was also obtained for several of the fibers of the analytical model, as indicated from the stress-strain curve of one of these fibers in Fig. 5b. Rebar rupture was obtained in the model during the first loading cycles at a drift level of 7.3%.

Table 1: Material parameters for concrete

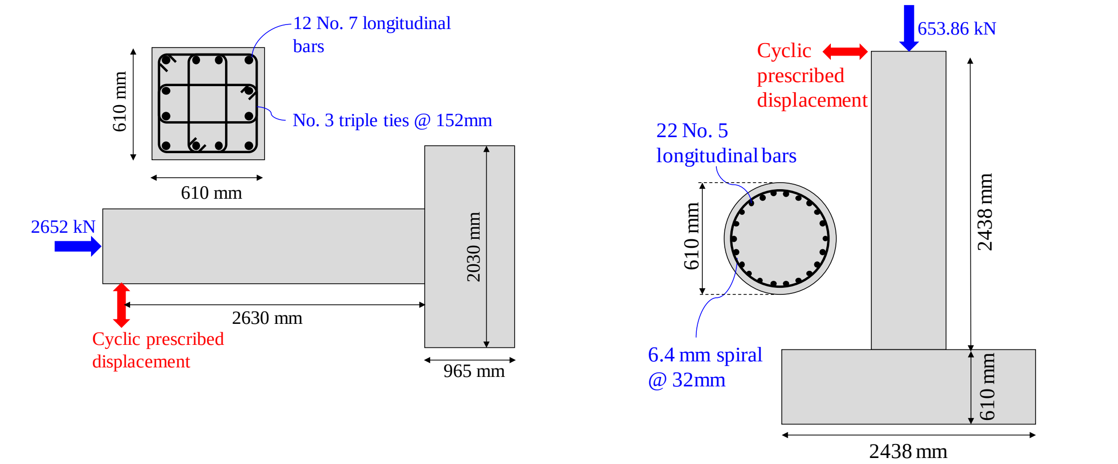
Specimen		E_c (MPa)	f_c (MPa)	f_t (MPa)	ε_o	ε_u	f_{conf} (MPa)	ε_{co}	ε_{cu}	M
Bae and Bayrak (2008)	Cover	30208	36.5	2.17	0.002	0.006	n.a.	n.a.	n.a.	0.042
	Core	30208	36.5	2.17	0.002	n.a.	43.0	0.004	0.022	0.042
Lehman and Moehle (2000)	Cover	26353	31.0	3.48	0.002	0.006	n.a.	n.a.	n.a.	0.071
	Core	26353	31.0	3.48	0.002	n.a.	45.0	0.008	0.020	0.071

Note: *E_c is the initial modulus of elasticity of concrete, M is a parameter affecting the tensile strength degradation of reinforced concrete.*

Table 2: Material parameters for reinforcing steel

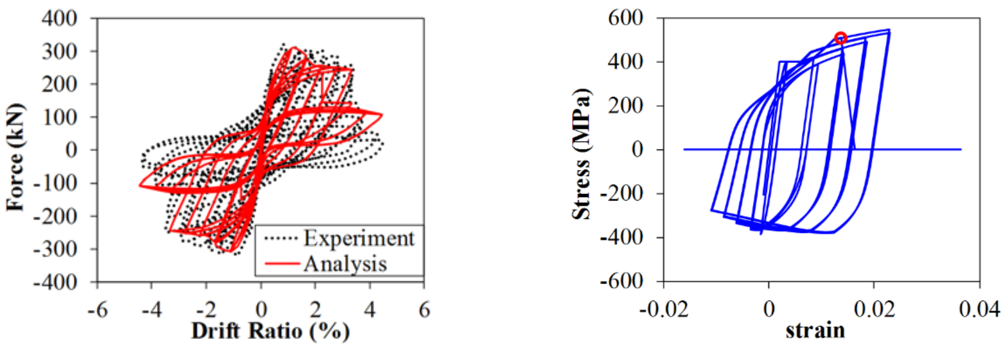
Specimen	E_s (MPa)	f_y (MPa)	ε_{sh}	ε_{sh1}	f_{sh1} (MPa)	ε_u	f_u (MPa)	*L/d*	D_{cr}
Bae and Bayrak (2008)	200000	400	0.006	0.04	584	0.12	645	12.0	0.20
Lehman and Moehle (2000)	200000	512	0.022	0.05	631	0.15	746	10.6	0.50

Note: *E_s is Young's modulus, f_y is rebar yield stress, ε_{sh} is strain at onset of strain hardening, ε_{sh1} and f_{sh1} are the strain and stress of an intermediate point on hardening regime of the monotonic curve, f_u is ultimate strength and ε_u is the strain corresponding to the ultimate strength.*



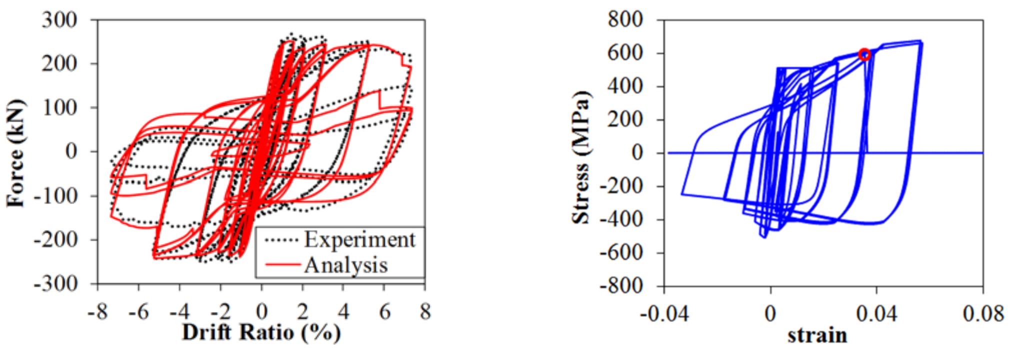
a) Specimen tested by Bay and Bayrak (2008) b) Specimen tested by Lehman and Moehle (2000)

Figure 3: Experimental tests used for the validation of the modeling scheme



a) Column hysteretic response b) Stress-strain response of a bar that ruptured (instant of rupture marked in red circle)

Figure 4: Results of validation analysis for column tested by Bae and Bayrak (2008).

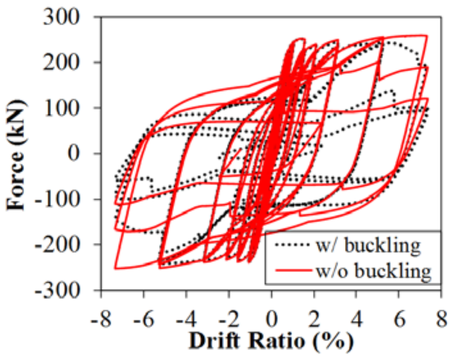


a) Column hysteretic response b) Stress-strain response of a bar that ruptured (instant of rupture marked in red circle)

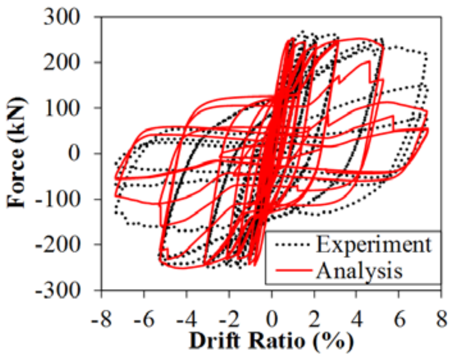
Figure 5: Results of validation analysis for column tested by Lehman and Moehle (2000).

5 PARAMETRIC STUDY

To investigate the effect of rebar buckling and strain penetration on the analytically obtained results, the analysis of the specimen by Lehman and Moehle (2000) is repeated, this time without including the effect of rebar buckling in the constitutive model for reinforcing steel. An additional repetition of the specific analysis is conducted for the case where the effect of strain penetration is neglected in the model. As deduced from Fig. 6a, neglecting rebar buckling in the model has negligible effect on the analytical results up to a drift level of 2%. For larger drift values, neglecting rebar buckling leads to larger energy dissipation (attested by the larger area of the corresponding hysteresis loops of the load-displacement curve) and to a later occurrence of rebar rupture and the associated strength degradation. Figure 6b compares the analytically obtained load-displacement curves for the cases with and without the strain penetration effect. Neglecting the strain penetration effect is found to lead to slightly greater energy dissipation (i.e., slightly greater area of hysteretic loops) and to earlier occurrence of rebar rupture.



a) Effect of rebar buckling



b) Effect of strain penetration

Figure 6: Parametric analyses to evaluate the effect of modelling assumptions for the column specimen tested by Lehman and Moehle (2000).

CONCLUSIONS

This study has used nonlinear beam elements and uniaxial constitutive models to evaluate the cyclic hysteretic response of RC columns. The model for reinforcing steel can account for rebar buckling and rupture due to low-cycle fatigue. The analysis scheme is validated using the results of previously conducted experimental tests on columns with a rectangular and circular cross-section. A satisfactory agreement between the analytically obtained and experimentally recorded global hysteretic response has been obtained. The analyses have also been capable of capturing the impact of rebar rupture on the strength of the columns. The importance of accounting for the effect of rebar buckling and strain penetration has been demonstrated through parametric analyses. The numerical efficiency of the proposed scheme (compared to more refined analysis methods, such as finite element models) renders it appropriate for the systematic performance assessment of RC components and systems.

ACKNOWLEDGEMENTS

The work presented in this paper was conducted during the visit of the first author as a post-doctoral scholar to the Virginia Polytechnic Institute and State University. This visit was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) 2219 Program.

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