A Three-dimensional Structural Health Monitoring System Using Multi-Scale Sample Entropy

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ABSTRACT: A three-dimensional structural health monitoring (SHM) system based on multi-scale entropy (MSE) and multi-scale cross-sample entropy (MSCE) is proposed in this article. By measuring the ambient vibration signal on the roof of a structure, the damage condition can be rapidly screened by the MSE analysis. The vertical damage location can then be evaluated by analyzing the individual signals of different floors with the vertical MSCE analysis. Results are illustrated by the vertical damage index. With the progressive vertical analyses, the damaged floor and damage locations can be diagnosed accurately and efficiently. In order to demonstrate the performance of the proposed SHM algorithm, numerical simulation is conducted on a three-dimensional seven-story steel structure. Based on the results, the damaged condition and elevation can be detected reliably. Additionally, the damage location can be quantified by the damage index efficiently. The average accuracy rates of 91% can be achieved by the proposed damage index method for the vertical damage locations, respectively. As simply a reference measurement of the current stage can initially launch the SHM system, the proposed SHM system can be implemented widely and practically.

Keywords: three-dimensional, structural health monitoring, vertical, planar, cross-sample entropy, multi-scale

1 INTRODUCTION

Structural health monitoring (SHM) is emerging as a popular research area in civil engineering. SHM techniques can be employed for the periodical inspection of aging structures and for post-disaster damage detection and reinforcement. Newly built structures require extended service and periodic damage assessment, thus highlighting the importance of structural damage detection. The concept of entropy was first introduced by German physicist Clausius in 1865 to evaluate the uncertainty of events in a thermodynamic system. In 1948, Shannon entropy was proposed and formally introduced into the field of information (Shannon 1948).

In 1991, an analytical method called “approximate entropy” (ApEn) was developed by Pincus (1991). ApEn, an improvement on traditional methods of entropy analysis, can be used to statistically determine regularity in real-world time series.

In 2000, Richman and Moorman (2000) proposed a modification of ApEn called “sample entropy” (SampEn). The advantage of SampEn is that the entropy value obtained is not affected by the length of the time series. Moreover, greater relative consistency can be achieved under different parameters such as the threshold, sample length, and signal length. In 2002, multiscale entropy (MSE) analysis was proposed and subsequently validated through clinical experiments (Costa et al. 2002, 2005).

Cross-sample entropy (Cross-SampEn) was developed to evaluate the degree of synchronicity or similarity between a pair of cardiovascular time series (Richman and Moorman 2000). In 2013, Fabris et al. (2013) utilized Cross-SampEn to identify healthy patients and those with throat or vocal disorders by quantifying the degree of asynchrony between time series. In 2015, Lin and Liang (2015) proposed an SHM system based on multiscale cross-sample entropy (MSCE), which was subsequently verified numerically and experimentally. The results demonstrated that high Cross-SampEn values can be observed for damaged floors. Following MSCE analysis, specific locations were determined.

According to the results of previous studies, an SHM system was developed in the present study; this system employs the MSE and MSCE methods to analyze the dynamic response signals of a numerically simulated high-rise structure. Furthermore, vertical analyses were conducted to diagnose the damage of the structure. The remainder of this paper is organized as follows: The proposed MSE and MSCE methods are described in Section 2. In Section 3, a numerical evaluation conducted on a seven-story steel structure is presented. Based on the numerical evaluation results, the performance of the vertical MSCE and damage index (DI) analyses are described in Section 4. Finally, Section 5 provides a discussion and conclusions.

2 THE PROPOSED SHM ALGORITHM

2.1 Cross-Sample Entropy

SampEn is a statistical method for analyzing time series. The complexity of a system can be quantified by calculating the entropy value of a measured time series. As an extension of ApEn, results are not affected by the time series length or calculation parameters in SampEn. Cross-SampEn is utilized to evaluate the degree of asynchrony or dissimilarity between two time series derived from the same system.

Let $\{X_i\} = \{x_1, \ldots, x_I, \ldots, x_N\}$ and $\{Y_j\} = \{y_1, \ldots, y_j, \ldots, y_N\}$ represent two individual time series of length N. The signals are segmented into the following templates of length m: $u_m(I) = \{x_i, x_{i+1}, \ldots, x_{i+m-1}\}$, $1 \le I \le N - m + 1$ and $v_m(j) = \{y_j, y_{j+1}, \ldots, y_{j+m-1}\}$, $1 \le j \le N - m + 1$. The template space T_x is presented as:

$$T_x = \begin{bmatrix} x_1 & x_2 & \cdots & x_m \\ x_2 & x_3 & \cdots & x_{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N-m+1} & x_{N-m+2} & \cdots & x_N \end{bmatrix} \tag{1}$$

Similarly, the template space T_y is expressed as:

$$T_y = \begin{bmatrix} y_1 & y_2 & \cdots & y_m \\ y_2 & y_3 & \cdots & y_{m-1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-m+1} & y_{N-m+2} & \cdots & y_N \end{bmatrix} \tag{2}$$

The degree of similarity between templates $u_m(i)$ and $v_m(j)$ is defined as $n_i^m(r)$ and is calculated under the criterion of:

$$d[u_m(i), v_m(j)] \le r,\ 1 \le j \le N - m \tag{3}$$

The similarity probability of the templates can be evaluated as follows:

$$U_i^m(r)(v\|u) = \frac{n^m(r)}{(N - m)} \tag{4}$$

The average similarity probability of length m can be calculated using the following equation:

$$U^m(r)(v\|u) = \frac{1}{(N - m)} \sum_{i=1}^{N-m} U_i^m(r)(v\|u) \tag{5}$$

Where $U^m(r)(v\|u)$ is the degree of dissimilarity between the two time series when m points are segmented.

New template spaces T_x and T_y are created by assembling templates with length m + 1, and the average similarity probability $U^{m+1}(r)(v\|u)$ is used to derive the Cross-SampEn values as:

$$CS_E(m, r, N) = -\ln\left\{\frac{U^{m+1}(r)(v\|u)}{U^m(r)(v\|u)}\right\} \tag{6}$$

2.2 *Multi-Scale Entropy*

MSE analysis is defined as the process of converting an original signal into signals at different time scales through coarse-graining. After completion of the process, the entropy values for each time scale are calculated using SampEn. Thus, compared with the results obtained using traditional entropy measures, healthy and pathological signals can be distinguished. The procedure is described as follows: A time series $x_1, x_2, \ldots, x_N$ of length N is segmented into multiple time series with a length of τ points, where τ is the scale factor. Subsequently, each set of data values is averaged, and a new time series $\{y_j^{(\tau)}\}$ is constructed. Each element is calculated according to the following equation:

$$y_j^{(\tau)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{j\tau} x_i, \ 1 \le j \le N/\tau \tag{7}$$

SampEn is calculated for each coarse-grained time series $\{y_j^{(\tau)}\}$. The S_E values for each time scale is the MSE of the time series. Finally, the S_E values are plotted as a function of the scale factor $(f(\tau) = S_E)$.

2.3 *Vertical Damage Index*

The MSE and MSCE methods are integrated to achieve structural health diagnosis along with the development of a set of vertical damage indices. These indices provide a means of determining the damaged floor and damage direction in a structure.

In the vertical analysis, two groups of curves representing the condition of the structure (healthy or damaged) are analyzed. For a structure with N stories, the MSCE plot of each floor is expressed as the cross-sample curves of each adjacent floor to the Nth story, where each curve depicts the Cross-SampEn at different scales.

H and D represent the MSCE curves for the healthy and damaged conditions of the structure, respectively. The subscripts depict the analyzed floor; for example, H_1 is the MSCE between the ground and first floor of the healthy structure. After MSCE analysis, the resulting curve illustrates the single-axis vertical characteristics of the first floor, expressed as $H_1 = \{CS_{E\,H_1}^{1}, CS_{E\,H_1}^{2}, CS_{E\,H_1}^{3}, \cdots, CS_{E\,H_1}^{\tau}\}$, where CS_E is the Cross-SampEn value, the superscript τ is the scale factor and the subscript number is the analyzed floor. Thus, the MSCE of each floor can be expressed as follows:

$$D_F = \{CS_{E\,D_F}^{1}, CS_{E\,D_F}^{2}, CS_{E\,D_F}^{3}, \cdots, CS_{E\,D_F}^{\tau}\} \tag{8}$$

Subsequently, the following formula can be used to calculate the DI:

$$DI_F = \sum_{q=1}^{\tau}\left(CS_{E\,D_F}^{q} - CS_{E\,H_F}^{q}\right) \tag{9}$$

Where F is the number of the floor to be evaluated for damage.

The DI for a single floor is evaluated by calculating the difference between the MSCE values of the damaged and healthy structures. For a specific floor, a positive DI value indicates the existence of damage on the floor, whereas a negative value indicates a lack of damage.

3 SHM DATABASE

In this study, the simulated response signals of a seven-story benchmark structure located at the National Center for Research on Earthquake Engineering (NCREE) were used as the SHM database.

3.1 Preliminary Experiment

For the benchmark structure, the height of each story was 1.18 m, and the length and width of each floor were 1.32 and 0.92 m, respectively. The cross section of the columns was set as plate-type of 20 x 75 mm and the beam size was set as 100 x 70 mm. Detachable braces with a cross section defined as L-shaped steel angles measuring 65 x 65 x 6 mm were installed on each face of every floor. An additional mass of 500 kg was mounted on each story. To record the response of the structure under ambient vibrations, biaxial velocity sensors were installed in the center of the floors.

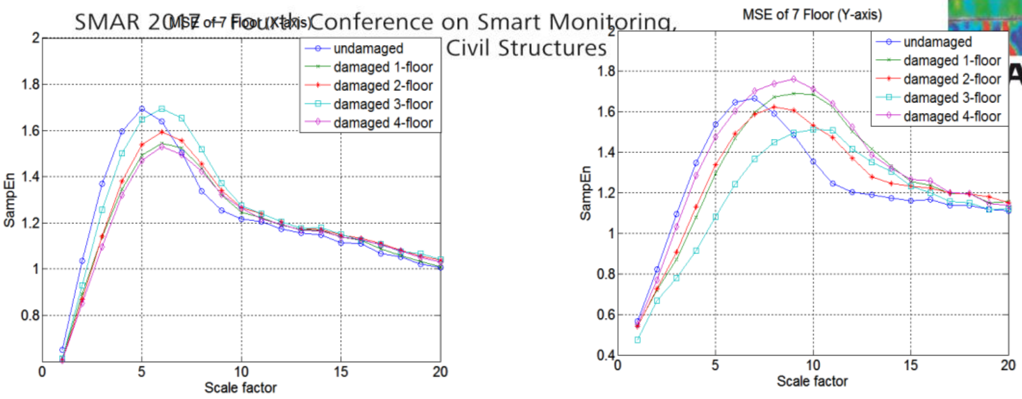
Table 1. displays the fundamental vibration frequencies of both the experimental and simulated specimens, indicating that they are consistent with each other. The numerical model was employed to verify the proposed SHM system, as well as to enhance the reliability of the proposed method and its feasibility in engineering practice.

Table 1. Modal comparison of experimental specimen and numerical simulation.

	Experimental Specimen	Numerical Simulation
Mode 1 (long axis)	4.18 Hz	4.15 Hz
Mode 2 (long axis)	17.8 Hz	17.24 Hz
Mode 1 (short axis)	3.13 Hz	3.12 Hz
Mode 2 (short axis)	13.06 Hz	13.06 Hz

3.2 Damage Database

In the simulation of the seven-story steel structure, the two sides in the long axis direction of the structure were defined as E and W, and those in the short axis direction were defined as N and S. The four sides of each floor were fitted with a single diagonal brace to support the long and short



(a) X (long) direction (b) Y (short) direction

Figure 1. MSE diagrams for seventh floor response.

axis directions. The damaged condition of the floor was denoted by the removal of a brace.

4 PERFORMANCE EVALUATION

The MSE and MSCE methods were used to diagnose the damage condition of a 3D biaxial structure. After a series of optimization searches, the template length m, threshold r, and signal length N were determined as 3, 0.15 * standard deviation (SD), and 60,000, respectively. Through the analysis of the biaxial velocity response signals of each floor, the damaged story was detected.

4.1 Damage Conditions

For each damage condition, a signal from the roof was selected for assessment. The various damage conditions are outlined as follows: healthy, single-story damage, two-story damage, three-story damage, and multistory damage.

Figure 1.(a)-(b) show the MSE diagrams derived for the long (strong) axis and short (weak) axis signals, respectively, on the seventh floor. Regarding the trends of the curves for the long (X) axis signal in Figure 1.(a), as the scale rises from 1 to 4, all the curves are shown to increase, but the trend of the curve derived for the signal from the healthy structure remains slightly higher than those of the curves derived for the other damage conditions. Moreover, the gaps between all curves are demonstrated to gradually increase. When the scale reaches 5, the curve for the healthy condition starts to plummet below the others, whereas all the curves for the damage conditions continue to rise. However, the gap between the curves for the damage and healthy conditions is maintained. Under scales ranging from 5 to 20, the curves for the damage conditions are all above the curve for the healthy condition. These results thus indicate damage in the long (strong) direction of the structure.

Regarding the trend of the curves for the short (Y) axis signal in Figure 1.(b), all the curves are clearly separated at a scale of 4. When a scale of 7 is reached, the curve for the healthy condition is shown to fall, whereas the curves for the other damage conditions continue to rise until a scale of 9 is reached. Furthermore, the gaps between all the curves are maintained to at least 0.1. Finally, as the scale increases, the curves gradually converge. All the curves in the short axis MSE graph are shown to have larger gaps than those in the long axis graph, because when the structure sustained damage on the short axis, its dynamic response was more severe and the signal produced was more complex. Based on these results, the possible damage on the long and short axes of the structure can be determined.

4.2 *Vertical Damage Locations*

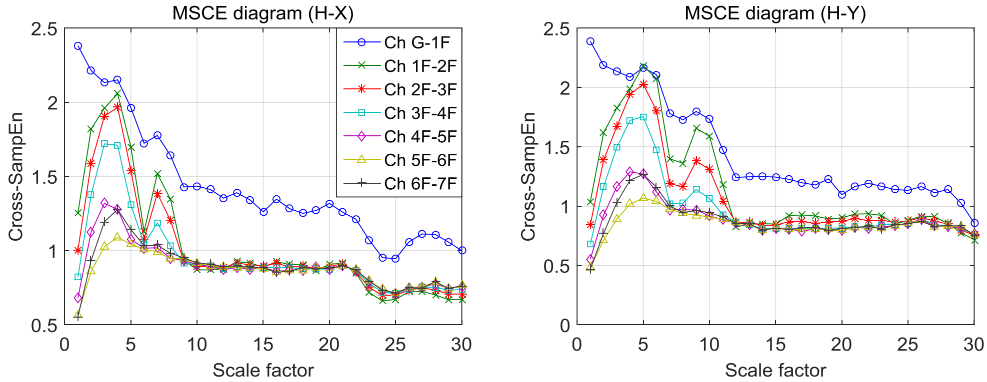
The velocity signals for the X (long) and Y (short) directions were extracted from the center of each floor for every damage condition. The signals of two adjacent floors under identical damage conditions were processed through Cross-SampEn to evaluate the dissimilarity between floors. Furthermore, the differences between the obtained MSCE curves for healthy and damaged structures were calculated to obtain the DI values of the adjacent floors. The damaged floor and damage direction could be obtained accordingly.

Figure 2. (a)-(b) present the MSCE diagrams derived for the healthy condition. Vertical MSCE analysis was performed after acquiring the time history in two directions for every floor. In these figures, G-1F denotes the curve for the first floor, 1F-2F denotes the curve for the second floor, 2F-3F denotes the curve for the third floor, and so forth.

4.2.1 Single-story damage: Case V1-2W

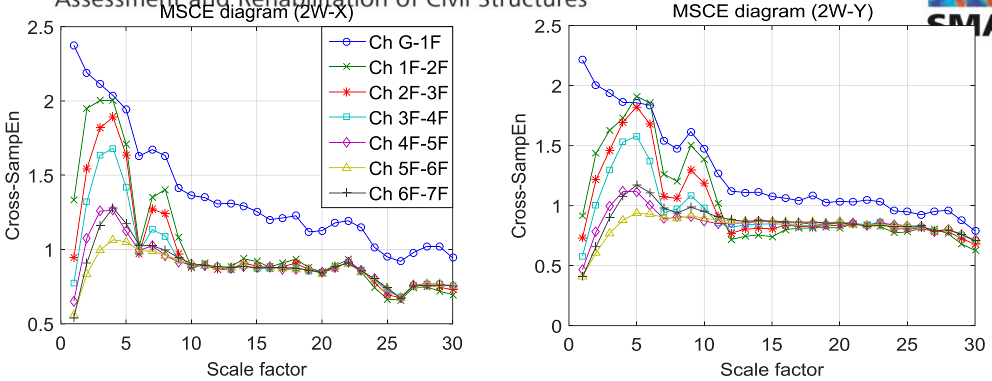
The MSCE diagrams for Case V1 are presented in Figure 3. , indicating that compared with the curves for the healthy condition, the change in trends of the curves for each floor in the Y-direction is non-significant. Therefore, the structure did not sustain any damage in the Y-direction. By contrast, the curve derived for the second floor in the X-direction increases significantly at scale factors ranging between 5 and 20, thus signifying an anomaly on this floor. Moreover, this curve exhibits more complexity than the other curves. The second floor was thereby determined to have been damaged in the X-direction.

The DI for this case is presented in Figure 4. , revealing that the value of the second floor in the X-direction is positive and that those of all the other floors are negative. In the Y-direction, all the DI values are negative, which is consistent with the results obtained from the MSCE curves.



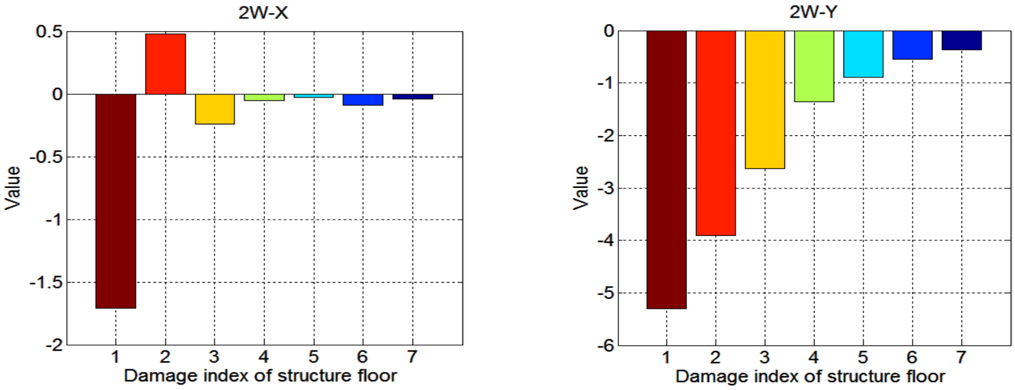
(a) X-direction (b) Y-direction

Figure 2. MSCE diagrams for the healthy condition.



(a) X-direction (b) Y-direction

Figure 3. MSCE diagrams for damage on the W side of the second floor.



(a) X-direction (b) Y-direction

Figure 4. DI diagrams for damage on the W side of the second floor.

4.3 Accuracy of vertical DI analysis

The complete results of vertical DI analysis are summarized in Table 2. In the short (Y) direction, all the damaged floors were successfully identified for every analyzed case, whereas in the long (X) direction, one case was misidentified, lowering the accuracy of the analysis. The overall recognition accuracy in both directions was 91%.

5 CONCLUSIONS

In this study, a series of analysis was conducted to highlight and extend the application of MSCE analysis for 3D SHM. A health diagnosis algorithm is proposed, and the reliability and feasibility of the MSCE-based system were verified by analyzing the ambient vibration response signals of a structure. Through the assessment of the complexity of the signals, the damage severity of the structure can be distinguished, indicating that the MSCE-based method is an effective replacement for the more complicated forced vibration response method. The results have demonstrated that for the 12 verical cases with simple and complex damage conditions, the proposed method can be successfully applied for biaxial vertical and planar damage diagnoses. The identification accuracy levels of the vertical damage detection were 91%, thereby further validating the potential of the proposed method for practical application.

Table 2. Identification accuracy of vertical DI analysis.

Case Number	**Damaged Floor and Direction**	**DI (Y-axis)**	**DI (X-axis)**	**DI (Both axes)**
0	H	✓	✓	✓
V1	5N	✓	✓	✓
V2	2W	✓	✓	✓
V3	6NES	✓	✓	✓
V4	2S7E	✓	✓	✓
V5	3NS4WE	✓	✓	✓
V6	5NE6SW	✓	✓	✓
V7	1NS4E7NS	✓	✓	✓
V8	1NES5W7W	✓	✓	✓
V9	2SW4WE6S	✓	✓	✓
V10	3N4W5S6E	✓	✓	✓
V11	1NE3NS5SW7NE	✓	×(6F&7F)	×(6F&7F)
V12	2N4W5N6SE7NS	✓	✓	✓
	Accuracy (%)	100%	91%	91%

✓ Indicates that damage on all floors was successfully detected; × indicates that damage on some floors was not successfully detected.

6 REFERENCES

Costa, M., Goldberger, A.L. and Peng, C.K. (2002), "Multiscale entropy analysis of complex physiologic time series", Phys. Rev. Lett., 89(6), 068102.

Costa, M., Goldberger, A.L. and Peng, C.K. (2005), "Multiscale entropy analysis of biological signals", Phys. Rev. E., 71(2), 021906.

Fabris, C., De Colle, W. and Sparacino, G. (2013), "Voice disorders assessed by (cross-) sample entropy of electroglottogram and microphone signals", *Biomed. Signal Proces.*, **8**(6), 920-926.

Lin, T.K. and Liang, J.C. (2015), "Application of multi-scale (cross-) sample entropy for structural health monitoring", *Smart Mater. Struct.*, **24**(8), 085003.

Pincus S.M. (1991), "Approximate entropy as a measure of system complexity", Proc. Natl. Acad. Sci. U.S.A., 88(6), 2297-301.

Richman, J.S. and Moorman, J.R. (2000), "Physiological time-series analysis using approximate entropy and sample entropy", *Am. J. Physiol. Heart Circ. Physiol.*, **278**(6), H2039-49.

Shannon, C.E. (1948), "A mathematical theory of communication", *Bell Syst. Tech. J.*, **27**, 379-423.