Duffing-like model for the hysteresis modelling of MR damper

Jia-Ying Tu[1], Ke, Jian-Cheng[2], and Yung-Bin Lin[3]

[1] Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu, Taiwan (R.O.C.)
[2] Department of Power Mechanical Engineering, National Tsing Hua University, Hsinchu, Taiwan (R.O.C.)
[3] National Center for Research on Earthquake Engineering, Tapei, Taiwan (R.O.C.)

ABSTRACT: Magnetorheological (MR) damper which is considered a smart damping device is extensively studied in the civil and earthquake engineering, because of the advantages of fast change in its damping properties. However, the nonlinear hysteresis dynamics of MR damper are difficult to model with high reliability, that renders the stability analysis and control compensation work difficult. Many existing hysteresis models in the literature include discontinuous and non-deterministic functions, such as absolute and sign functions. These non-ideal functions restrict further developments of analysis and control techniques. This paper introduces the continuous and deterministic Duffing-like equation to model the hysteresis curve. Because the discontinuous, piecewise, and singular functions are excluded, the optimal parameter identification, stability analysis, and semi-active control design can be developed in a more systematic manner; other hysteresis models cannot provide such strengths. Duffing-like model provides a foundation for better analysis of hysteresis dynamics of MR damper, and the parameter identification of a 400 N MR damper is presresnted in this article for illustration of the concept.

1 INSTRUCTIONS

In civil and earthquake engineering, magnetorheological (MR) damper is considered as a smart damping device and is extensively researched for vibration reduction of structural systems. Typically, the MR damper piston is wrapped in coils, and the MR damper cylinder is filled with silicon oil and magnetised iron particles. The coils are connected to a voltage-current converter. When the current is sent to the coils that generate magnetic flux to align the magnetized iron particles, the property of MR fluid becomes like a semi-solid. Therefore, the density, damping, and stiffness of the MR damper are changeable fast according to the input current; this advantage facilitates the use of MR dampers or MR fluid for vibration reduction in a variety of engineering application.

In order to achieve semi-active control of MR damper, the hysteresis dynamics in relation to the input current needs to be established in a mathematic model. In literature, the most common and well-known models for MR damper are Bingham and Bouc-Wen models. The equation of Bingham model is written as

$$y = y_m \operatorname{sgn}\left(\dot{u}\right) + c_{po}\dot{u} \tag{1}$$

where u and y are the input current and output force respectively, and the parameters are denoted as y_m and c_{po}. In addition, the Bouc-Wen model (Jr et al., 1997) is typically expressed by

$$y = c_0\dot{u} + k_0\left(u - u_0\right) + \alpha z \tag{2}$$

$$\dot{z} = -\gamma\left|\dot{u}\right|z\left|z\right|^{n-1} - \beta\dot{u}\left|z\right|^{n} + A\dot{u} \tag{3}$$

In equations (2) and (3), z is related to the nonlinear damping dynamics, and c_0, k_0, α, β, γ, n, and A are the parameters to be determined from experimental identification. As seen in equations (1)-(3), the conventional hysteresis models inevitably includes discontinuous functions, which are not appropriate for parameter analysis and control design. Furthermore, the parameters may have no physical meaning so that the relationship between the hysteresis input, output, and parameters could not be determined in an exact and systematic way.

Because of the disadvantages of the hysteresis models in literature, the authors study and modify Duffing equation to become a Duffing-like model for hysteresis modelling, which will be introduced in Section 2. The Duffing-like model is a second-order, continuous, ordinary differential equation, and the physical meaning of the parameters will be discussed. Parameter identification of a 400 N MR damper is presented in Section 3 for illustration of the concept. Finally, conclusion is drawn in Section 4.

2 INTRODUCTION TO DUFFING-LIKE MODEL AND PARAMETER TUNNING

By investigating the literature related to Duffing equation, civil engineering, and nonlinear hysteresis system, a relatively new method for modelling hysteresis is proposed in Tu et al. (2015), named Duffing-like equation and written as

$$\ddot{x} + \alpha\dot{x} + \beta x + \gamma\dot{x}^{n_1} + \delta x^{n_2} = c\dot{d} + kd \tag{4}$$

which represents a second-order continuous nonlinear ordinary differential equation. The right-hand side of the equation relates to the input dynamics, where c and k are the damping and stiffness coefficients, respectively, and d is the external input singal. Here, d can be any physical quantity, such as voltage, current, or displacement, depending on the application scenairo. Furthermore, the internal system dynamics are described in the left-hand side of equation (4), where α, β, γ, and δ correspond to the coefficients of linear damping, linear stiffness, nonlinear damping, and nonlinear stiffness, and x is the system state. The indices of n_1 and n_2 associated with $\dot{x}$ and x should be odd and determine the degree of nonlinearity of hysteresis. For the purpose of analysis and control design, equation (4) can be arranged to a state-space form as follows

$$\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{\dot{\mathbf{x}}} = \underbrace{\begin{bmatrix}0 & 1\\ -\beta & -\alpha\end{bmatrix}}_{\mathbf{A}}\underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}0\\ -\delta x_1^{n_2} - \gamma x_2^{n_1}\end{bmatrix}}_{\mathbf{F}(\mathbf{x})} + \underbrace{\begin{bmatrix}0 & 0\\ k & c\end{bmatrix}}_{\mathbf{B}}\underbrace{\begin{bmatrix}d\\ \dot{d}\end{bmatrix}}_{\mathbf{d}} \tag{5}$$

$$y = \underbrace{\begin{bmatrix}0 & A_y\end{bmatrix}}_{\mathbf{C}}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{6}$$

where the state variables are defined as $x_1 = x$ and $x_2 = \dot{x}$, and $\mathbf{x} = [x_1 \quad x_2]^{\mathrm{T}}$ and $\mathbf{d} = [d \quad \dot{d}]^{\mathrm{T}}$ denote the state and input vectors, respectively. In addition, y is the rate-dependent output of the hysteresis system, and **A**, **B**, and **C** denote the plant, input, and output matrices, respectively. An additional coefficient A_y is included in the output matrix to scale the output signal, and the nonlinear hysteresis dynamics are lumped into the **F**(**x**) matrix. The expression of equations (5) and (6) allows the stability analysis, parameter identification, and control design to be implemented in a more systematic way.

According to equations (4)-(6), an example of simulation result of the $\dot{d}$-y plot is depicted in Figure 1, and the curve characteristics are separated into linear and nonlinear regions. In general, the linear dynamics of a hysteresis system is determined by α, β, c, and k, and the nonlinear section is mostly governed by γ and n_1. The amplitude of the hysteresis output is adjusted by A_f. Because the parameters δ and n_2 have less effects on the hysteresis curve, their tuning performance is not discussed here for the sake of brevity.

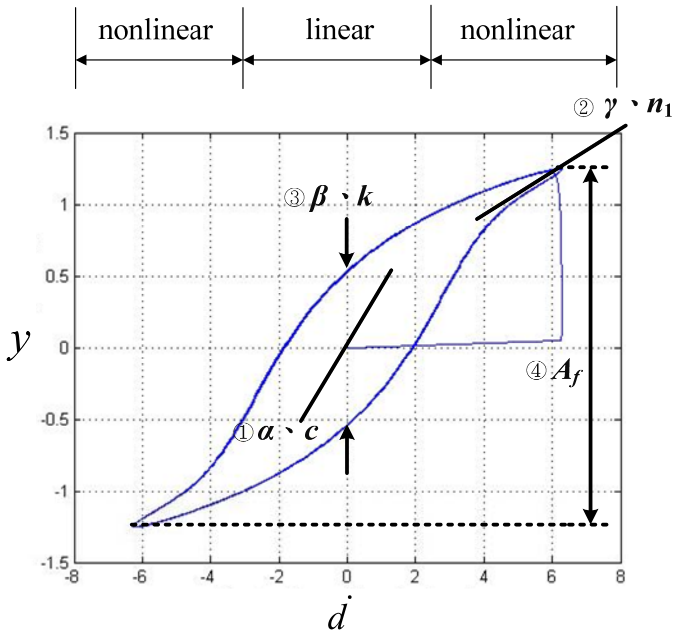

Figure 1. The hysteresis curve of Duffing-like model in relation to the parameters.

As shown in Figures 1 and 2, the simulation of parameter tuning indicates that the magnitude of y is decreased as α is increased. Thus, the linear area is enlarged and the nonlinear section gradually becomes unobvious. In contrast, as α is decreased, the magnitude of y is increased, and the distinction between linear and nonlinear sections becomes obvious. Therefore, α behaves like a linear damping coefficient which control the energy dissipation of the hysteresis system. In terms of β, the simulation work shows that the total area of hysteresis curve is strongly dependent on β; the area reduces as β increases, as seen in Figure 3. Given a large value of β, the hysteresis area is almost reduced to zero, and the forward and backward paths of the hysteresis curve are nearly overlapped. Nevertheless, the amplitude of y is not changeable by adjusting β. As a result, it is noted that β is related to the potential energy of the hysteresis system, and is denoted as the linear stiffness coefficient.

Furthermore, the variation of γ controls the output value of y; as γ increases, the amplitude of y decreases. In addition, as shown in Figure 1, the rate of the curve in the nonlinear part is determined by γ, and the transition point from linear to nonlinear part is also decided by γ. Therefore, it is noted that γ is associated with the damping dynamics in the nonlinear part, and is considered as the nonlinear damping coefficient.

On the other hand, the coefficients of c and k are associated with the damping and stiffness characteristics of the input dynamics. The simulation work show that the parameters c and α have similar but reverse effects on the linear dynamics of hysteresis system, and the parameters k and β have the same but reverse effects on the nonlinear dynamics. Therefore, parameter tuning related to the external input and internal dynamics needs to be taken into account simultaneously, in order to exactly reflect the external and internal effects on the hysteresis curve.

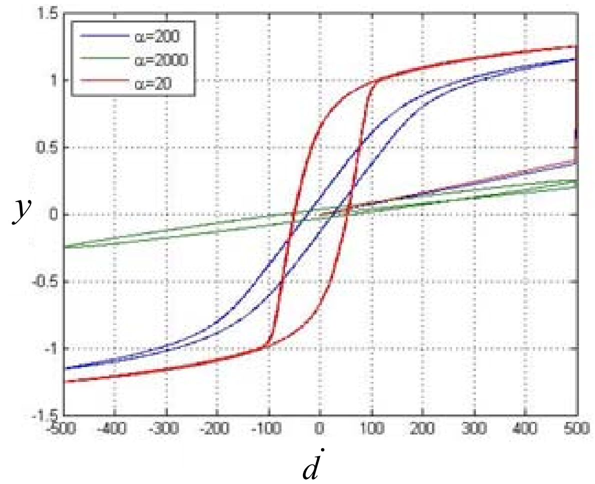

Figure 2. The effect of α on the hysteresis curve (Chien, 2016a; Chien et al. 2016b).

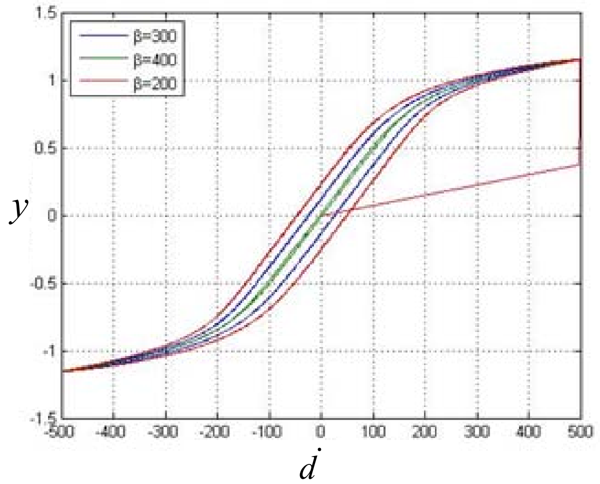

Figure 3. The effect of β on the hysteresis curve (Chien, 2016a; Chien et al. 2016b).

3 PARAMETER IDENTIFICATION OF MR DAMPER

Figure 4 displays the experimental rig of a 400 N MR damper, and the parameter of the MR damper is identified using the Duffing-like model in this section. As shown in Figure 4, the damper cylinder is fixed to the solid ground, and one end of the piston rod is connected to the electric actuator. The piston is wrapped by coils, and electric current is sent to the coils via a voltage-current converter to generate magnetic flux. The maximum voltage allowable for the MR damper is 3 V, and the corresponding maximum force is 400 N. In the present paper, the MR damper with 2 V excitations is considered. In addition, the piston rod is excited with a sinusoidal displacement input, where the magnitude is 1.5 cm and the frequency is 1.5 Hz. The resulting MR damper force is measured by a load cell.

Experimental data of the MR damper are illustrated using grey lines in Figure 5, and the identified results are plotted using black lines. In Figure 5, the input signal d is defined as the piston displacement exerted by the actuator, and thus $\dot{d}$ is the piston velocity, as depicted in the x axis. The output signal of hysteresis curve is the MR damper force, plotted in the y axis. As seen in Figure 5, the input-output relationship of the MR damper is similar to the pattern of Duffing-like model in Figure 1. The grey curves show that the maximum velocity and force recorded are around 14 cm/s and 230 N, respectively.

Table 1 lists the identified parameters using the Duffing-like model, and two sets of parameter tuning are considered. In Figure 5(a), the parameters are chosen to fit the linear part of the hysteresis curve; as the output force approaches zero, the identified black curves match the grey curves nearly. In addition, a different fitting strategy is considered in Figure 5(b), where the parameters are selected to fit the transition point between the linear and nonlinear sections. However, inevitable fitting errors are seen between the grey and black lines. In both Figures 5(a) and 5(b), the Duffing-like model can match the curves in nonlinear regions with reasonable accuracy, and one source of fitting errors is resulted from the asymmetric or defect design of the damper. Improvement on mechanical design and manufacturing of the MR damper would be considered in future work.

Table 1. The parameter tuning of Figure 5

	Parameters and values								
Notation	α	β	γ	δ	n_1	n_2	c	k	A_f
Figure 5(a)	36	170	500	0	11	0	16	30	250
Figure 5(b)	36	170	10	0	5	0	16	30	135

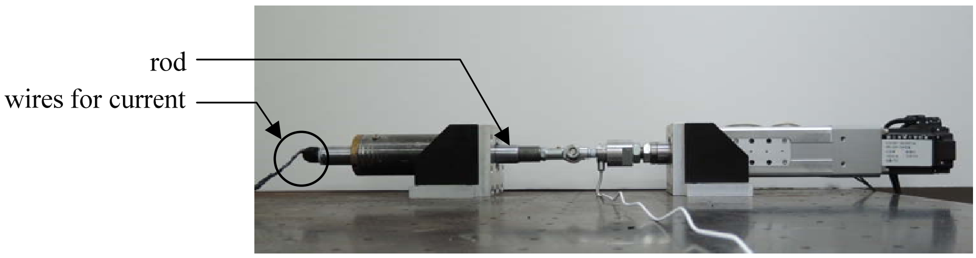

Figure 4. The experimental rig of the 400 N MR damper.

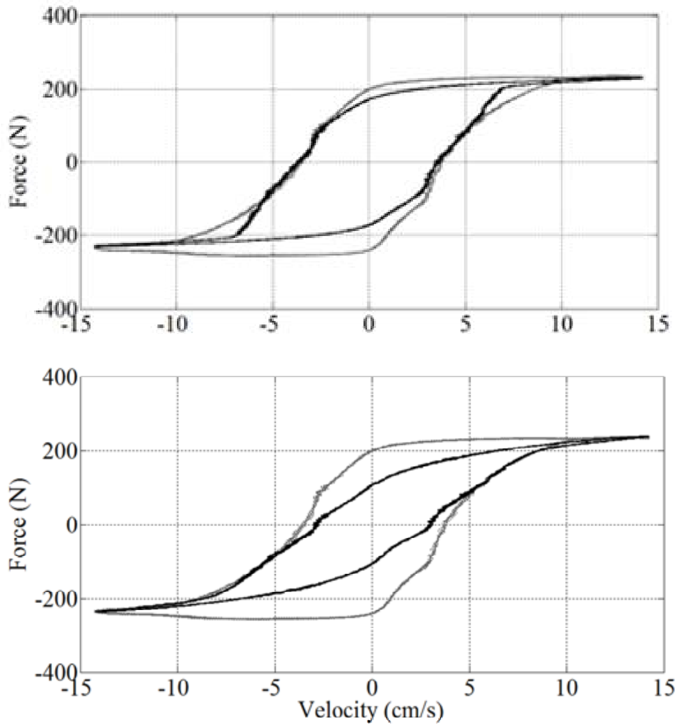

Figure 5. Comparison of parameter identification with sinusoidal excitations: (a) parameters selected to fit the linear part, and (b) parameters selected to fit the transition point.

4 CONCLUSIONS

Duffing-like model for identification of hysteresis curve has been proposed and is introduced in this paper. Parameter tuning in relation with the hysteresis characteristics is briefly discussed. Firstly, the hysteresis curve can be considered as a combination of linear and nonlinear dynamics, and the transition point from linear to nonlinear dynamics can be adjusted. Secondly, the Duffing-like model includes eight parameters, and the physical meaning and their effect on the hysteresis curve are discussed. In comparison with other hysteresis models in the engineering literature, the Duffing-like model gives the parameters exact physical meaning, facilitating the parameter identification work to be carried out in a more explicit and systematic manner. Experimental work of a 400 N MR damper is presented in this paper to demonstrate the effectiveness of Duffing-like model. The identification results show that the Duffing-like model can fit the hysteresis curve of the MR damper with reasonable accuracy. However, how to define an optimal or trade-off fitting strategy with sufficient accuracy requires further research.

ACKNOWLEDGEMENT

The authors gratefully acknowledge the support of the Taiwan Ministry of Science and Technology, under grant MOST 105-2625-M-007-001- ‘Development and validation of MR damper based on Duffing-like model for semi-active vibration reduction’, for the support in the pursuance of this work.

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