SMAR 2017 – Fourth Conference on Smart Monitoring,
Assessment and Rehabilitation of Civil Structures

# Identification of Structural Damage in Hybrid Bridge Truss Girders Using Relative Wavelet Entropy

Mohammad Moravvej[1] and Mamdouh El-Badry[1]

[1] University of Calgary, Calgary, Canada

ABSTRACT: Deterioration of bridge infrastructure has become a global concern. Recently, fiber-reinforced polymers (FRPs) have been used in concrete bridge construction to mitigate durability problems and to make infrastructure sustainable. However, even concrete bridges that include the corrosion-resistant FRP materials can still be subjected to various types of damage including cracking of concrete, local buckling of thin-walled FRP sections, and rupture of FRP reinforcement. Therefore, to ensure both safety and serviceability of such structures, monitoring their conditions during service is essential to identify damage as early as possible. An experimental implementation of a relative wavelet entropy (RWE)-based damage identification technique (DIT) in a hybrid FRP-concrete bridge truss girder is presented. The technique can detect and localize structural damage and estimate its severity in the truss girder. The girder specimen consisted of pretensioned top and bottom concrete chords connected by precast truss elements made of glass FRP tubes filled with concrete. The girder was tested under monotonic loading up to failure. The load-induced damage was identified in the various components of the girder, including the concrete chords, the truss connections, and the truss elements. The results were verified by visual inspection and by data obtained through instrumentation.

## 1 INTRODUCTION

Bridges are designed and built to be safe against failure, and to perform satisfactorily during their service life. Over the past few decades, these critical structures have been deteriorating at an alarming rate due to aging, inadequate maintenance, adverse environmental conditions, and constantly growing transportation demand. Corrosion of reinforcing steel is the major source of deterioration of concrete bridges (Qiao and Qu, 2007). Cracking of concrete reduces the structural stiffness and expedites corrosion of the steel reinforcement. Recently, fiber-reinforced polymers (FRPs) have been used in place of steel in reinforced concrete structures to mitigate the durability problems caused by corrosion of steel and to enhance their structural performance.

Nevertheless, performance of hybrid FRP-concrete bridges can be affected by various types of damage caused by excessive loading and extreme events. To ensure both safety and serviceability of bridge structures, damage identification techniques (DITs) are needed. A robust DIT can detect the presence of damage, determine its location, and quantify its severity (Carden et al., 2004). The outcome of a successful DIT can be used for prediction of the remaining service life of structures. Despite all the advanced developments in DITs, the following problems related to structural evaluation of bridges have been historically difficult to solve:

1. Most of the current DITs are designed for specific types of structure and are limited to identifying a particular type of damage (Carden et al., 2004).
2. Dynamic excitation tests are required for evaluation of the dynamic properties of bridges. However, it is not practical to interrupt the normal operations of the bridges to perform the tests (Farrar et al., 1999).
3. Even if dynamic tests could be performed, the varying operational and environmental conditions of bridges lead to changes in their measured dynamic responses. These changes may result in false positive damage indication (i.e. indication of damage while none actually exists) (Farrar and Worden, 2012).
4. In most *in-situ* cases, there are no data available from a reference (undamaged) state of bridges for comparison with the data measured from their current (damaged) state (Fan and Qiao, 2010). Therefore, development of DITs that allow structural health monitoring (SHM) systems to be implemented in bridge structures without a prior knowledge of the reference state is still needed (Lynch et al., 2016).
5. Due to inherent complexity of FRPs and their composite action with traditional construction materials, such as concrete, practical applications of current DITs in hybrid FRP-concrete structures are challenging (Cai et al., 2012).

To overcome the difficulties associated with the traditional techniques, a reference-free DIT has been developed (Moravvej et al., 2016). The technique can detect and localize structural damage, and estimate its severity. The technique uses only the data measured from current state of bridges and does not require interruption of their normal operations. The proposed technique combines discrete wavelet transforms (DWTs) and spectral entropy in a relative procedure to detect and quantify the damage-induced disturbances in the measured bridge acceleration signals similar to those shown in Figure 1.

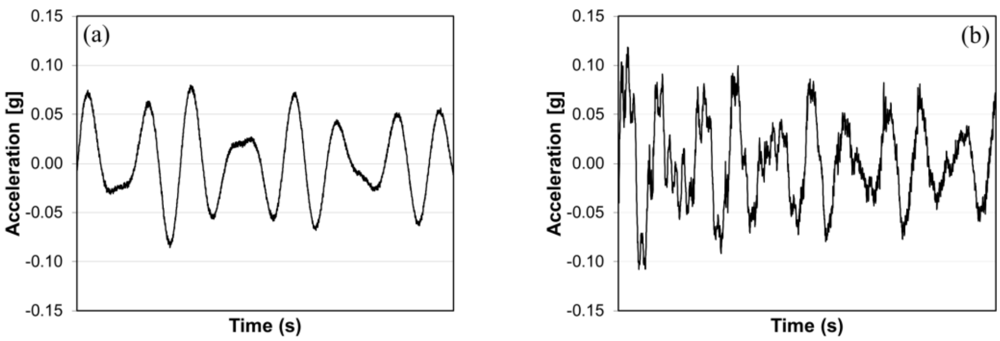


Figure 1. Acceleration signals at two locations on same structure: (a) slightly damaged; (b) severely damaged.

The efficiency of the technique was examined experimentally during testing a large-scale precast hybrid FRP-concrete bridge truss girder under static loading. The girder consisted of pretensioned top and bottom concrete chords connected by precast truss elements made of glass-FRP (GFRP) tubes filled with concrete (El-Badry, 2007). The concrete chords were reinforced with GFRP longitudinal bars and transverse stirrups. The truss elements were reinforced and connected to the chords by means of double-headed GFRP bars. The damage induced by the static loading in the girder elements, including the concrete chords, the truss connections, and the truss web elements, was identified using the proposed relative wavelet entropy (RWE)-based DIT.

## 2 BASES OF THE TECHNIQUE

### 2.1 *Wavelet transform (WT)*

In general, WT is a convertor of a signal into a different mathematical form to disclose the hidden characteristics of the signal and to focus on its specific properties that are of interest (Gao and Yan, 2011). Continuous wavelet transform (CWT) is defined as the product of a continuous signal, $f(t)$, and a basic wavelet function, $\psi(t)$. The result of this product is wavelet coefficients, defined by Equation (1), which shows how well the signal correlates with the basic wavelet function.

$$C(s,\tau)=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{1}$$

where $\psi^*(t)$ is the complex conjugate of the basic wavelet, shifted and scaled by factors $\tau$ and $s$, respectively. In practice, an acceleration signal is sampled at discrete time intervals. By adopting the values of $2^j k$ for the shifting factor and $2^j$ for the scaling factor ($j$ and $k$ are integers), the corresponding discrete wavelet coefficients, $C_j(k)$, can be obtained.

The discrete wavelet transform (DWT) works as a pair of filters, which decompose the acceleration signal into low- and high-frequency components and find corresponding wavelet coefficients for each component (Gao and Yan, 2011). The low-frequency component is filtered one more time. The process is repeated until the final level of decomposition, where the original acceleration signal is decomposed into $n$ groups of wavelet coefficients, from the highest frequency component ($j$ = 1) to the lowest frequency component ($j$ = $n$).

### 2.2 *Wavelet energy*

The wavelet coefficients can be used as a direct estimation of the wavelet energy. The energy of the signal at each scale, $E_j$, and the total wavelet energy, $E_{Total}$, are respectively defined as:

$$E_j=\sum_k |C_j(k)|^2 \tag{2}$$

$$E_{total}=\sum_j E_j=\sum_j\sum_k |C_j(k)|^2 \tag{3}$$

Consequently, the wavelet-energy ratio vector, $\{p\}$, which represents the wavelet energy distribution of the signal in the frequency domain, can be established by calculating the wavelet energy ratios for all the scales, as expressed in Equation (4). Structural damage changes the wavelet energy distribution of the acceleration signal (Lee et al., 2014). The wavelet-energy ratio vector is a suitable measure for characterizing this change to detect the structural damage.

$$\{p\}=\left\{\frac{E_1}{E_{total}} \quad \cdots \quad \frac{E_j}{E_{total}} \quad \cdots \quad \frac{E_n}{E_{total}}\right\} \tag{4}$$

### 2.3 *Relative wavelet entropy (RWE)*

In general, the entropy is a quantitative measure of the degree of disorder in a system (Shannon, 1948). The wavelet entropy thus quantifies the degree of disorder induced by structural damage in measured acceleration signals. Relative Wavelet Entropy (RWE), defined by Equation (5), describes the degree of dissimilarity between two sets of signals by comparing their wavelet-energy ratio vectors, $\{p\}$ and $\{q\}$ (Rosso et al., 2001). For identification of damage, these two sets of signals must be chosen in such a way that the degree of dissimilarity between them gives an estimate of severity of the targeted damage.

$$RWE\left(p \mid q\right) = \sum_j p_j \ln\left[\frac{p_j}{q_j}\right] \tag{5}$$

To apply the proposed RWE-based DIT, acceleration signals are measured during a series of dynamic excitations performed at multiple locations on the structural elements. The wavelet-energy ratio vector of the acceleration signal obtained at each measuring location is compared to the vectors obtained at all other locations through a RWE analysis. This process results in a RWE index, calculated for each location by Equation (5). The RWE indices identify the location and severity of the targeted damage. The RWE index for a location affected by a severe damage is higher than the indices corresponding to locations with slight or no damage.

## 3 EXPERIMENTAL IMPLEMENTATION

The proposed RWE-based DIT was applied on a precast hybrid FRP-concrete bridge truss girder tested under static loading. The truss girder consists of pretensioned top and bottom concrete chords connected by vertical and diagonal precast truss elements made of concrete-filled GFRP tubes. The truss elements are connected to the chords by means of double-headed GFRP bars. Under gravity loads, the vertical elements are mainly in compression while the diagonals are predominantly in tension. The GFRP tubes enhance the compressive strength of the verticals by confining the concrete core while the double-headed GFRP bars serve as corrosion-resistant internal reinforcement in the diagonals with excellent anchorage to the chords. The top and bottom chords are also reinforced with GFRP longitudinal bars and transverse stirrups (El-Badry, 2007). The truss girder specimen consisted of eight typical panels. The girder total length and overall depth were 9.82 m and 1.32 m, respectively. A vertical load was monotonically applied at the center of the top chord. The test setup and the load versus mid-span deflection diagram under the static loading are presented in Figure 2. More details of the experimental program and the test results can be found in El-Badry et al. (2017).

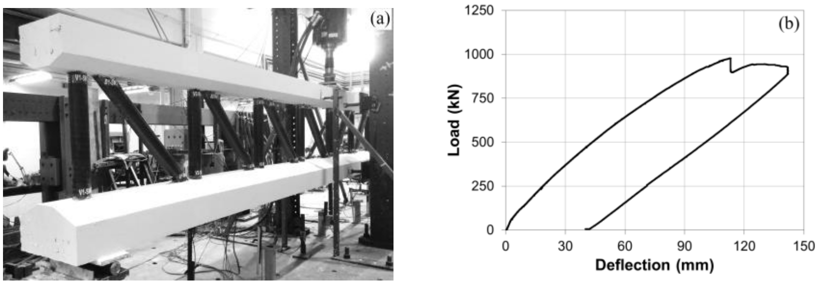


Figure 2. (a) Test setup and (b) load-deflection behaviour of the eight-panel truss girder.

The static loading test induced damage in the girder with different severities at different locations. In order to identify the test-induced damage using the RWE-based DIT, series of dynamic excitations were intermittently introduced to the girder after the static loading test by application of a vertical cyclic load at mid-span on the top chord. The cyclic load was applied at a low frequency (0.5 Hz) with amplitude of 87.5 kN (the maximum wheel load of CL-625 truck of CAN/CSA-S6-14), to simulate the ambient vibration of bridges due to traffic loads. To identify concrete cracking in the chords, acceleration signals were obtained during the excitations using accelerometers attached to the chords. To evaluate performance of the truss connections, one accelerometer was placed on the chord and another was attached to the diagonal element near the connection. To identify the damage induced in the vertical elements, accelerometers were attached to the outer surface of the tubes at three locations along their heights and a series of impact tests was conducted using a hammer. Figure 3 illustrates typical arrangement of accelerometers for damage identification in a panel of the girder. Measurement was performed by a maximum of four accelerometers. The location of the four accelerometers was successively changed along the eight panels of the girder to cover all the measuring locations including sixteen locations on the chords, sixteen truss connections, and three locations on each vertical element.

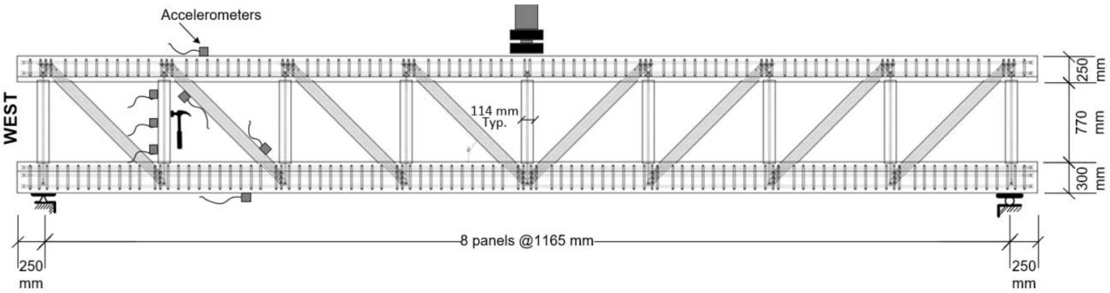


Figure 3. Dimensions and reinforcing details of the girder, and typical arrangement of accelerometers.

Wavelet analysis was performed on the acceleration signals using Haar wavelet as the basic wavelet function. Among several alternatives, Haar wavelet has been adopted because of its successful application in damage identification as reported in many research studies (Gao and Yan, 2011). In addition, as shown in Figure 4, Haar wavelet fits the pattern of damage-induced disturbances (i.e. the sudden transitions) in an acceleration signal obtained at a damaged location.

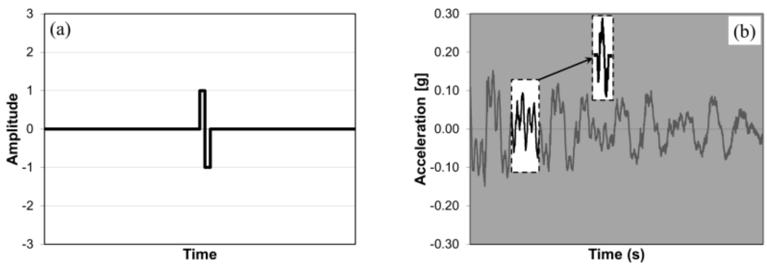


Figure 4. Similarity of (a) Haar wavelet and (b) damage-induced disturbances in a signal.

# 4 RESULTS AND DISCUSSION

## *4.1 Concrete chords*

To evaluate the condition of the chords after performing the static loading test, acceleration signals were obtained during the dynamic excitations at sixteen measuring locations along the eight panels of the girder. The wavelet analysis of the obtained signals results in wavelet-energy ratio vectors representing the wavelet energy distribution of the signals in the frequency domain. Comparing the wavelet-energy ratio vector of a measuring location to the wavelet-energy ratio vectors of the fifteen other locations through the RWE analysis results in a RWE index corresponding to that location. Repeating this process for all the measuring locations submits sixteen RWE indices, as listed in Table 1, representing the structural condition of the chords. When a location is severely affected by concrete cracking, its RWE index is higher than the indices corresponding to the locations with slight or no cracking. The table indicates that panels 4 and 5, particularly the top chord of Panel 5, have higher indices compared to other locations. Figure 5 illustrates visually inspected state of damage in the chords and corroborates the finding of the RWE-based DIT. The two panels located near the girder mid-span, particularly the top chord of Panel 5, are the most affected locations by the static load-induced damage. Other locations remained almost intact.

Table 1. RWE indices of the concrete chords at sixteen measuring locations

| | Panel 1 | Panel 2 | Panel 3 | Panel 4 | Panel 5 | Panel 6 | Panel 7 | Panel 8 |
|---|---|---|---|---|---|---|---|---|
| Top chord | 1.34 | 1.54 | 1.41 | 1.66 | 6.83 | 1.59 | 1.65 | 1.38 |
| Bottom chord | 1.60 | 1.35 | 1.35 | 3.01 | 2.58 | 1.58 | 1.65 | 1.21 |

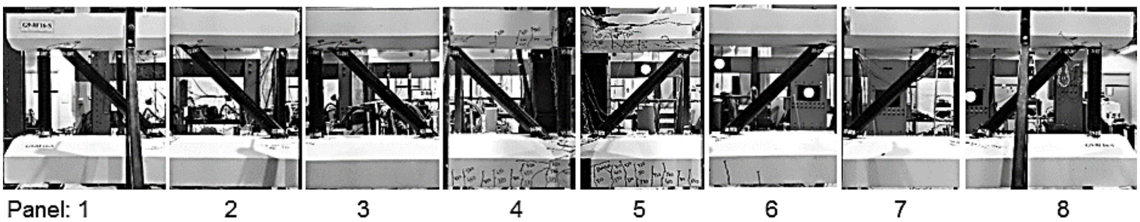


Figure 5. Structural condition of the concrete chords after performing the static loading test.

### *4.2 Truss connections*

To evaluate the structural condition of the truss connections after conducting the static loading test, the RWE analysis was performed on the acceleration signals obtained at the concrete chord and the diagonal element near each connection. The analysis results in a RWE index for each truss connection. The RWE indices, summarized in Table 2, identify the location and give an estimate of severity of the load-induced damage in the truss connections. The RWE indices indicate that the connections located near the two ends of the girder, particularly the bottom connection in Panel 7, are the most affected by the load-induced damage. The results were verified by visual inspection and by data obtained through instrumentation of the anticipated most stressed connections located in panels 2 and 7. Figure 6 compares the elongation of the diagonal truss elements and the average strain in the headed bars at the interface with the top and bottom chords during the static loading test. The actual state of damage in the four connections after the test can be seen in Figure 6-c.

Table 2. RWE indices of the sixteen truss connections

| | Panel 1 | Panel 2 | Panel 3 | Panel 4 | Panel 5 | Panel 6 | Panel 7 | Panel 8 |
|---|---|---|---|---|---|---|---|---|
| Top connection | 1.73 | 3.58 | 1.11 | 0.58 | 1.07 | 0.63 | 2.53 | 2.24 |
| Bottom connection | 6.46 | 9.28 | 0.01 | 0.80 | 0.75 | 0.06 | 26.10 | 9.99 |

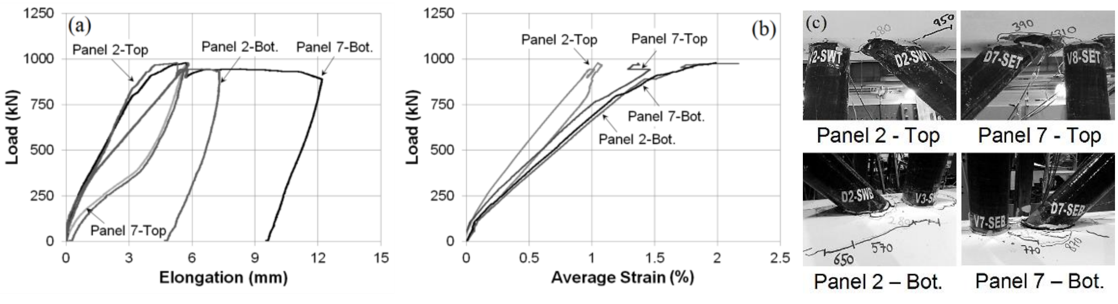


Figure 6. (a) elongation and (b) average strain in headed bars during the static loading test; and (c) state of damage after the test in the most stressed connections.

### *4.3 Vertical elements*

The analysis of the acceleration signals recorded by accelerometers during the impact tests on the vertical truss elements results in three wavelet-energy ratio vectors corresponding to the top, middle, and bottom of each element. Comparing the wavelet-energy ratio vector of a measuring location to the wavelet-energy ratio vectors of all other locations through the RWE analysis results in a RWE index for that location. Repeating this process for all the measuring locations results in RWE indices corresponding to all the locations on the vertical truss elements, as listed in Table 3. It can be concluded from the table that vertical element 8 has the highest RWE index followed by vertical element 5. This finding agrees with the static load test results shown in Figure 7-a, which indicates that among the anticipated most stressed vertical elements, Vertical 8 has the largest shortening, followed by Vertical 5. In addition, Figures 7-b and 7-c illustrate that the strain in the FRP tube at the bottom of Vertical 8 is significantly higher than at the top. This also agrees with the results of RWE-based DIT in Vertical 8, as presented in Table 3.

Table 3. RWE indices of the vertical elements

| | Vertical 1 | Ver. 2 | Ver. 3 | Ver. 4 | Ver. 5 | Ver. 6 | Ver. 7 | Ver. 8 | Ver. 9 |
|---|---|---|---|---|---|---|---|---|---|
| Top | 6.75 | 5.68 | 4.83 | 6.75 | 10.20 | 6.33 | 5.81 | 17.63 | 7.12 |
| Middle | 5.56 | 6.45 | 4.86 | 4.78 | 9.35 | 6.04 | 6.07 | 36.55 | 5.42 |
| Bottom | 4.97 | 4.98 | 5.01 | 5.84 | 7.36 | 5.64 | 8.31 | 65.77 | 5.01 |
| Sum = | 17.28 | 17.11 | 14.70 | 17.37 | 26.91 | 18.01 | 20.19 | 119.95 | 17.55 |

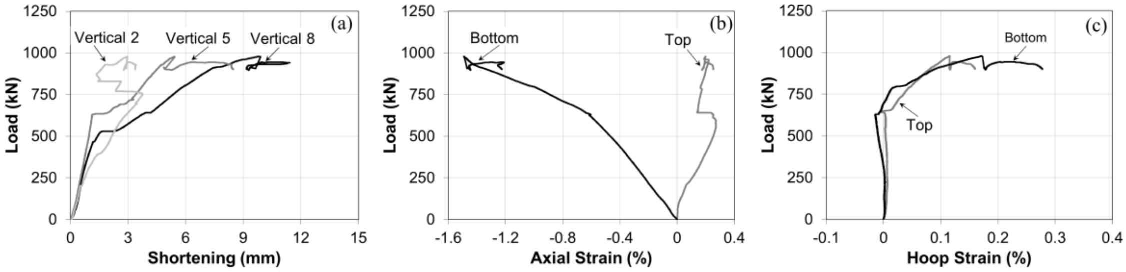


Figure 7. (a) shortening in the most stressed vertical elements; (b) axial strain and (c) hoop strain in the FRP tube of Vertical 8 during the static loading test.

## 5 CONCLUSIONS

Utilization of FRPs in bridge construction has become an encouraging solution for the durability problems and for enhancing the structural performance of concrete bridges. Therefore, structural evaluation of hybrid FRP-concrete bridges through damage identification is of great importance. The efficiency of a relative wavelet entropy-based damage identification technique (RWE-based DIT) was experimentally investigated in a precast hybrid FRP-concrete bridge truss girder tested under static loading. The main conclusions drawn from this study are:

1. The structural condition of the truss girder specimen was successfully evaluated by identifying the load-induced damage in the concrete chords, truss connections, and vertical elements of the girder. The results were verified by strain gauge and displacement transducers data, and by visual inspection of the actual damage.

2. The proposed technique successfully identified the load-induced damage in the girder using the acceleration signals obtained under simulated ambient vibration of bridges. This indicates the technique's practicality in damage identification in *in-situ* cases where the normal operation of bridges cannot be interrupted for performing dynamic excitations.
3. The location and severity of damage were successfully identified using only the acceleration signals obtained from the damaged state (i.e. after performing the static test) of the girder. This makes the technique applicable to *in-situ* cases where the data from a reference (e.g., undamaged) state of bridges are not available.

Although the RWE-based DIT has shown great potential for identification of damage, future work is needed for quantification of damage by adopting the technique in a Finite Element (FE) model updating procedure, in which the RWE index can work as a reference-free damage index.

## 6 ACKNOWLEDGEMENTS

The financial support received from the Natural Sciences and Engineering Research Council of Canada (NSERC) is gratefully acknowledged.

## 7 REFERENCES

Cai, J., Qiu, L., Yuan, S., Shi, L., Liu, P., and Liang, D. 2012. *Composites and Their Applications, Chapter 3: Structural Health Monitoring for Composite Materials*, INTECH Science, Technology and Medicine open access publisher, 24 pp.

Carden, E.P. and Fanning, P. 2004. Vibration Based Condition Monitoring: A Review. *Structural Health Monitoring*, 3(4): 355–377.

CSA-S6-14 2014, *Canadian Highway Bridge Design Code*, Canadian Standards Association, Ontario.

El-Badry, M. 2007. An Innovative Hybrid FRP-Concrete System for Short and Medium-Span Bridges. *Proceedings of the COBRAE Conference on Benefits of Composites in Civil Engineering*. Stuttgart, Germany, March 2007, 16 pp.

El-Badry, M., Moravvej, M., and Joulani, P. 2017. Performance of a Hybrid FRP-Reinforced Bridge Truss Girder System – Experimental Assessment. *4th Conference on Smart Monitoring, Assessment and Rehabilitation of Civil Structures*, SMAR 2017. Zurich, Switzerland. September 2017, 8 pp.

Fan, W. and Qiao, P. 2010. Vibration-Based Damage Identification Methods: A Review and Comparative Study. *Structural Health Monitoring*, 10(1): 83–111.

Farrar, C.R., Duffey, T.A., Cornwell, P.J., and Doebling, S.W. 1999. Excitation Methods for Bridge Structures. *17th International Modal Analysis Conference*. Kissimmee, Florida, USA. 1999, 5 pp.

Farrar, C.R. and Worden, K. 2012. *Structural Health Monitoring: A Machine Learning Perspective*. USA: John Wiley and Sons Ltd.

Gao, R. and Yan, R. 2011. *Wavelets: Theory and Applications for Manufacturing*. USA: Springer Science and Business Media.

Lee, S.G., Yun, G.J., and Shang, S. 2014. Reference-Free Damage Detection for Truss Bridge Structures by Continuous Relative Wavelet Entropy Method. *Structural Health* Monitoring, 13(3): 307–320.

Lynch, J., Farrar, C.R., and Michaels, J. 2016. Structural Health Monitoring: Technological Advances to Practical Implementations. *IEEE*, 104(8): 1508–1512.

Moravvej, M., El-Badry, M., and Joulani, P. 2016. Smart Structural Health Monitoring System for Damage Identification in Bridges Using Relative Wavelet Entropy. *Proceedings of the International Conference on Smart Infrastructure and Construction*, ICSIC 2016. Cambridge, UK. June 2016; 6 pp.

Qiao, G. and Qu, J. 2007. Corrosion Monitoring of Reinforcing Steel in Cement Mortar by EIS and ENA. *Journal of Electrochimica Acta*, 52: 8008–8019.

Rosso, O.A., Blanco, S., Yordanova, J. et al. 2001. Wavelet Entropy: A New Tool for Analysis of Short Duration Brain Electrical Signals. *Journal of Neuroscience* Methods, 105(1): 65–75.

Shannon, C.E. 1948. A Mathematical Theory of Communication. *Bell System Technical Journal*, 27(1): 379–443.