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Damage detection method based on state representation methodology (SRM)

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ABSTRACT: This paper introduces the details of a newly proposed "State Representation Methodology (SRM)" and its application to bridge condition assessment (damage detection) based on the bridge monitoring data. The SRM is a novel tool that can provide some ideas and algorithms for data mining in the bridge monitoring system. The state of a system such as a bridge structure can be obtained by a state variable that is calculated from a State Representation Equation (SRE). A Kernel function method which plays an important role in the Support Vector Machines (SVM) is applied to get solutions of the SRE. In the computation of the SRE, it needs to be changed into a Large-Scale Linear Constraint Problem (LSLCP). A new compatible algorithm is therefore proposed as solving technique of the LSLCP. Before applying the SRM, it is necessary that the system features need to extract from the complex responses observed data in the system. In this paper, with the aid of the frequency slice wavelet transform (FSWT) which is a time-frequency analysis tool, it is possible to clearly reveal a change of the characteristics (feature extract) in vibration signal from the bridge monitoring.

1 INTRODUCTION

Bridge monitoring system via information technology is capable of providing more accurate knowledge of bridge performance characteristics than traditional methods. Then, structural health monitoring (SHM) is becoming an important socio-economic concern for infrastructure maintenance all over the world because existing infrastructures are deteriorating with time and continuously accumulate damage throughout their service life due to material deterioration, natural hazards, such as earthquakes, storms, fires, long-term fatigue and corrosion, etc. (Miyamoto 2009, Brühwiler et al. 2013). In the SHM, it will be able to collect a huge amount of reliable and objective data every day and also every year. Then, an important issue is how to detect damages from the collected data (big data). This is a big challenge for analysis to identify damage information from a specific structure. Furthermore, there are not only many kinds of sensor data, but also many undetermined factors in the system, such as dynamic effects, severe noise interference, etc.

In this paper, a new idea called "State Representation Methodology (SRM)" is introduced which is about a non-parametric description system state theory, for describing and assessing the structural condition (state) and its change. In SRM, the system state is described as a state variable that can be expressed by a "State Representation Equation (SRE)". Kernel function method which plays an important role in supporting vector machines (SVM), is used to resolve the SRE. Then, the solution of SRE is modified into a large-scale linear constraint problem (LSLCP). A compatible gradient algorithm is proposed to solve the LSLCP. "Frequency Slice Wavelet Transform (FSWT)" which is a time-frequency analysis tool was also developed successfully (Yan et al. 2009). Finally, an experimental result shows that the proposed method is simple to assess the state change by using a laboratory bridge monitoring system.

2 CONCEPT OF STATE REPRESENTATION METHODOLOGY(SRM)

The state of a system is interpreted as the overall response to its internal and external factors, which essentially depends on the response of its structure or structural property and natural environment. The quantitative of the system state is the description for system responding to incentive factors. If the response satisfies our expectation, the system state is considered as normal state, otherwise as abnormal state. In usual circumstances or under normal use conditions, the system is in a stable state, which means that its state is constant, or generally fluctuates in the vicinity of a steady-state. Therefore, usually it is assumed that it is a steady random variable which often follows the normal distribution.

2.1 System state representation

Basic vibration equation as a parameterized example is shown as follows:

$$M\delta'' + C\delta' + K\delta = 0 \tag{1}$$

Although it is easy to determine the parameters, *M*, *C* and *K* for Single Degree of Freedom (S-DOF) system, in general, it is not easy to determine the parameters for Multi-DOF system, because *M*, *C* and *K* are correlated within the system characteristics. From the vibration equation for S-DOF system, the vibration frequency ω can be written as:

$$\omega^2 = \frac{K}{M} - \frac{C^2}{4M^2} \tag{2}$$

It can also be written as:

$$M = \frac{K \pm \sqrt{K^2 - (\omega C)^2}}{2\omega^2} \quad \text{or} \quad M = f(K, \omega, C) \tag{3}$$

On the other hand, for Multi-DOF system, it can be written as:

$$\sum_i f(K_i, \omega_i, C_i) = M_{total} = \text{CONSTANT} \quad \text{or} \quad \sum_i \frac{1}{M_{total}} f(K_i, \omega_i, C_i) = 1; \text{CONSTANT} \tag{4}$$

At this state, let's apply the Taylor formula to expand the function, $f(K_i, \omega_i, C_i)$ to obtain, then:

$$\sum_i A_i(K_i, C_i)\omega_i + o(\omega_1, \omega_2, ...\omega_m) = 1 \tag{5}$$

This equation can be written more generally as:

$$\sum_i \alpha_i \omega_i + o(\omega_1, \omega_2, ...\omega_m) = 1 \tag{5'}$$

If a state of the vibration system is noted by the variable ζ then:

$$\zeta = \sum_i \alpha_i \omega_i + o(\omega_1, \omega_2, ...\omega_m) = 1 \tag{6}$$

If the system is not changed, ζ is a constant value, and it is expressed as an implicit function of the system feature parameters. In an actual existing bridge system, the parameters *M, K* and *C* are not accurately known. However, it is relatively easy to know the response parameters $(\omega_1, \omega_2, ...\omega_n)$ from sensors. Then, the question is how to establish the function including the state variable ζ with $(\omega_1, \omega_2, ...\omega_n)$? These functions are called "State Representation Equation (SRE)" or "State Representation Function (SRF)" of the system.

2.2 State Representation Equation

There are a lot of responses in a complex structural system such as a bridge, that are implied in test data or experimental data. It is impossible to obtain all of the system features from data as obtained by an infinite number of sensors. Therefore, it is assumed that the system state of a complex system is a function of the system state space. A projector or a part of the system state can be revealed by means of limited observation, which will be able to realize in an actual application. Naturally, at the current time, all of the system responses are considered as the system state space or feature space, denoted as H^{∞}, i.e.

$$(h_1(t), h_2(t), h_3(t),\ldots\ldots h_n(t)\ldots\ldots) \in H^{\infty} \tag{7}$$

where each $h_i(t)$ is a projector of system feature, called a system sub-response function. For example, in an experiment, if the input is an excitation signal $r_i(t)$ to the system which has many responses, the output signal, $s_i(t)$ as a test result can be obtained by the sub-response function $h_i(t)$ using the following methods in the time domain:

$$s_i(t) = \int_{-\infty}^{\infty} r_i(\tau) h_i(t-\tau) d\tau, i = 1,2,3\ldots\ldots\infty \tag{8}$$

In the frequency domain, it becomes as:

$$S_i(\omega) = R_i(\omega) H_i(\omega), \quad i = 1,2,3\ldots\ldots\infty \tag{9}$$

Then, the sub-responses are:

$$H_i(\omega) = \frac{S_i(\omega)}{R_i(\omega)}, \quad i = 1,2,3\ldots\ldots\infty \tag{10}$$

Usually $h_i(t)$ is independent of $r_i(t), i = 1,2,3,\ldots \infty$.

Therefore in SRM, for an objective system *S*, a nonobjective condition state variable, ζ is defined as $\zeta \in [0,1]$ or $\zeta \in (0,1)$ which is a function of its state space:

$$\zeta = f(h_1(t), h_2(t), h_3(t),\ldots\ldots h_n(t)\ldots\ldots) \tag{11}$$

This equation can be changed into the frequency domain:

$$\zeta = f(H_1(\omega), H_2(\omega),\ldots\ldots H_n(\omega)\ldots\ldots) \tag{12}$$

Here, the main object in SRM is focused on the relationship between the system responses and its system state (feature), ζ in the system feature space H^{∞}. And, the SRM describes the system state and some system assessment methods when the system conditions have been changed.

In the SRM, if one regards the current system state as normal state, this means that the state subjects to the expectation, just as it is safety or reliability etc., which can be viewed as state (feature) $\zeta = 1$, i.e.:

$$\zeta = f(\cdot) \equiv 1, \quad \text{if the system is always in normal state} \tag{13}$$

With the passage of time, the system has some gradual deterioration in the structures such as bridge, it will depart out of the normal state, in general sense, called Deterioration State and it could be assumed as:

$$\zeta = f(\cdot) \le 1, \quad \text{for always} \tag{14}$$

In a complex system, it is very difficult to test the exact condition state by sensors. Therefore, the lowest level of state herein noted c_0 was taken into consideration, at the same time, by testing its part of responses to the environment, c_0 can then be estimated and represented using the following inequality:

$$\zeta = f(\cdot) \ge c_0 \tag{15}$$

Figure 1 shows a conceptual procedure of the feature extracting by the above mentioned SRM, where x is the system feature vector, ζ is the state variable, and λ is the system structure alias parameter.

3 DAMAGE DETECTION METHOD BASED ON SRM

3.1 Basic idea of state representation

3.1.1 Extraction of a finite dimension feature sub space to approximate the state function, *f*

Because the real representation function, *f* is unknown, some related properties to the relationships between their features and the condition state may be correlated to be what kinds of features. In fact, it is possible to be approximated by some mathematical expressions. The successful use of SRM is dependent upon the experimenter's ability to develop a suitable approximation for $f(\cdot)$.

Note: H^n is a finite dimension sub space such that $H^n \subset H^\infty$ and state variable, ζ can be limited on $H^n \subset H^\infty$ i.e. $\zeta = f(h),\ h \in H^n \subset H^\infty$.

3.1.2 Standardization of feature vector

Let's

$$h \in H^n \subset H^\infty, \quad h := \frac{h}{\|h\|} \tag{16}$$

It is necessary to standardize the unit representation of feature vector, *h*.

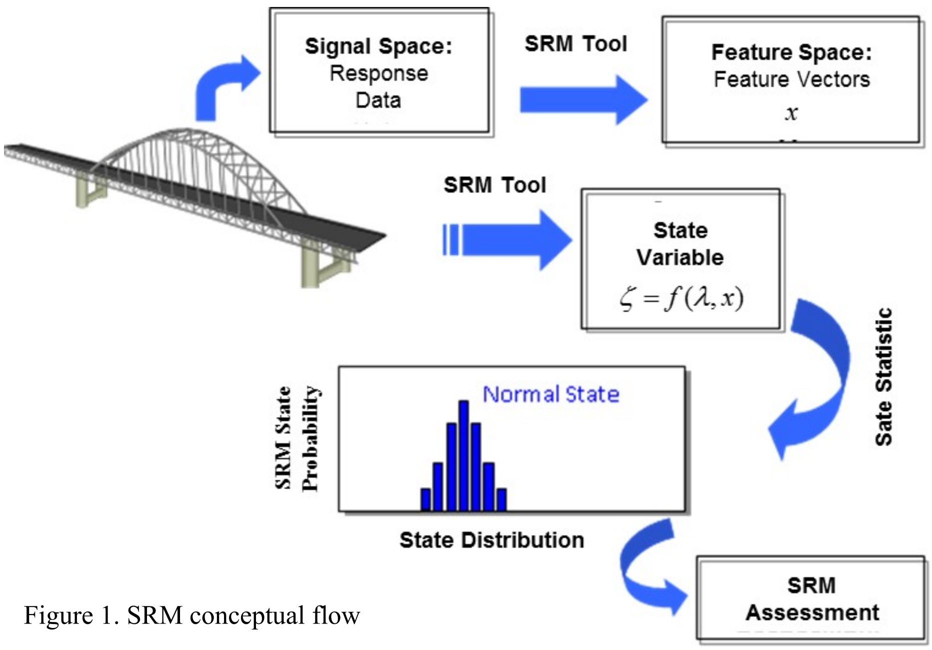


Figure 1. SRM conceptual flow

3.1.3 Find system state support vector

Let's assume system feature vector, $x := (x(1), x(2),...x(n)) \in H^n$, if constant vector $w \in R^n$ called as system state support vector or feature director exists, then it needs to satisfy the following relation (equation):

$$1 =< w, x >= \sum_{k=1}^{n} w(k)x(k) \tag{17}$$

This is the first-order model of state, ζ and recorded as:

$$\zeta = f(w, x) = \sum_{k=1}^{n} w(k)x(k) \tag{18}$$

If the system has only one response vector signal $h \in H^n$, the solution using the Least Squares Estimators method should be: $w = h$. But if it is a multiple response vector, then let

$$w_i = h_i, i = 1,2....m \tag{19}$$

Now let every response vector $h \in H^n$ be as a projector of the current system state along with its feature directors $w_i, i = 1,2....m$ (Cristianini et al. 2000), and it has a weight: $\lambda_i \geq 0$ and

$$\lambda_i \in (0,1), \sum_{i=1}^{m} \lambda_i = 1, \quad w = \sum_{i=1}^{m} \lambda_i w_i \tag{20}$$

Then,

$$1 =< w, x >=< \sum_{i=1}^{m} \lambda_i h_i, x >= \sum_{i=1}^{m} \lambda_i < h_i, x > \tag{21}$$

In generally, the state function included parameter vector, α is written as:

$$f(\alpha, x) = \sum_{i=1}^{m} \alpha_i < h_i, x > \tag{22}$$

The function, f is a linear operator, the vector, α herein called a system state parameter.

3.1.4 Modify the product relation about feature vectors by means of Kernel Function Method

As representation of the system state with one-order linear model is limited, it is a very useful idea to use the Kernel function method to solve the high order nonlinear representation of the system state.

A Kernel Function Map $x \to \phi(x)$ is defined herein as a nonlinear function that changes from the current variable space into another parameter space, which can practically be understood as a transformation for the sensor's function. Then redefine $< s, x >, s \in H^n, x \in H^n$ as,

$$< s, x >=< \phi(s), \phi(x) >= k(s, x), s \in H^n, x \in H^n \tag{23}$$

Here,
$k(s, x) =< \phi(s), \phi(x) >$ is a Kernel Function. There are many choices for Kernel Function, $k(\cdot,\cdot)$.

In the present paper, the Kernel Functions recommended in (Cristianini et al. 2000) are used:

$$k(s, x) = \exp(-\frac{d(s, x)}{\sigma^2}) \quad \text{, where, } \sigma \text{ is the SRM scale.} \tag{24}$$

$$\text{Here, } d(s, x) = \sum_{i=1}^{n} \frac{(s_i - x_i)^2}{s_i + x_i} \quad \text{or} \quad d(s, x) = (\sum_{i=1}^{n} |s_i - x_i|^p)^{1/p} \tag{25}$$

$$k(s, x) = (s \cdot x)^d, \quad (\|s\| = 1, \|x\| = 1) \tag{26}$$

Let's define $\Omega = \sum_{k=1}^{m} \lambda_k k(h_k,.)$ as an operator, called system state support operator or system state representation operator, which is a highly nonlinear operator. It is important to find the support operator to represent and assess the system state.

3.1.5 Computing support vector

Take a Train Set of system feature vectors recorded as *TS*.

$$\text{Define the Gram matrix:} \quad G = \begin{pmatrix} k(x_1,x_1), k(x_1,x_2),....k(x_1,x_m) \\ k(x_2,x_1), k(x_2,x_2),....k(x_2,x_m) \\ .... \\ k(x_m,x_1), k(x_m,x_2),....k(x_m,x_m) \end{pmatrix} \tag{27}$$

$e = (1,1,...1)^T$.

$$\begin{aligned} &\min \quad \left\| (I - \frac{1}{n} ee^T) G\lambda \right\|^2 \\ &\text{Subject to} \quad \sum_i \lambda_i = 1, 0 \le \lambda_i \le 1, \mathrm{i} = 1,2,3,....\mathrm{m} \end{aligned} \tag{28}$$

It follows that:

$$\zeta = f(\lambda, x) = \sum_i^m \lambda_i k(x_i, x) \tag{29}$$

3.2 SRM algorithm

SRM algorithm for computing of state variable, $\zeta = f(\lambda, x)$ or $\zeta = f(\alpha, x)$ by using Eq. (28) with a Kernel function such as Eqs. (24), (26), etc. is shown for linear programming or quadratic convex programming which are similar with SVM methods, as follows as an example:

STEP 1: Input: $(x_1, x_2, ...x_m), x_i \in H^n$, matrix, *A* by Eq. (28), iterate number *N*, and error *ε*. **STEP 2**: Set initial point, $x = e / n$, $D_0 = diag(\lambda_{0i})$. **STEP 3**: $k = 0$. **STEP 4**: while, $k < N$ do. **STEP 5**: Set $D_k = diag(\lambda_{ki})$. **STEP 6**: Compute search direction, p_k, such as, set $p_k = D_k A^T A D_k e$. **STEP 7**: Compute optimal search step *t* to satisfy $\min_{tp<e} \| AD_k(e - tp_k) \|^2$, and $\lambda_{k+1} = \lambda_k - tD_k p_k$ set. **STEP 8**: if $|\lambda_{k+1} - \lambda_k| \le \varepsilon$, break and go to STEP 11. **STEP 9**: set $k = k+1$; go to STEP 4. **STEP 10**: end while. **STEP 11**: stop.

4 FREQUENCY SLICE WAVELET TRANSFORMATION(FSWT) TOOL

4.1 Instruction of FSWT

In the following, suppose $\hat{p}(\omega)$ is the Fourier Transformation of the function $p(t)$. For any $f(t) \in L^2(R)$, the Frequency Slice Wavelet Transform (FSWT) is defined directly in the frequency domain as (Yan et al. 2010):

$$W_f(t, \omega, \kappa) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{f}(u) \hat{p}^*(\kappa \frac{u - \omega}{\omega}) e^{iut} du \tag{30}$$

where, the scale κ is a constant or a function of ω, *t* and *u*, and the star '*' means the conjugate of a function. Here, $\hat{p}(\omega)$ is called a Frequency Slice Function (FSF). More details of FSWT can be found in (Yan et al. 2009) and (Yan et al. 2010).

4.2 FSWT envelope analysis

It is stated that if $t < 0, p(t) = 0$, $p(t)$ is a single side function (SSF) in the time domain.
Next, an analytic function is assumed as:

$$f(t) = \begin{cases} Ae^{-\alpha t}e^{i\beta t} & t \geq t_0 \\ 0 & t < t_0 \end{cases} \tag{31}$$

and also $p(t) = e^{-\frac{1}{2}t^2}$ is a SSF. Moreover, if $t \geq t_0$, then

$$|W(t,\omega,\sigma)| = e^{-\alpha t}|W(0,\omega,\sigma)| \tag{32}$$

It is obvious that according to the above equation, $|W(t,\omega,\sigma)|$ can be viewed as the envelope of free decay response (FDR), *f*(*t*) by using Eq. (32). Then, the damping parameter, *α* can be obtained.

4.3 Feature extract algorithm

In a real bridge system, the system response may be a stochastic state. The structural system performs some response when it is excited by environmental conditions such as moving cars, wind, earthquake, etc. Therefore, some trigger conditions always exist, and the time of a trigger condition is called the Time Trigger Lines (TTLs). Figure 2 shows a FSWT image with a feature extracting grid, as an example to explain the feature extract algorithm. The steps for the feature extract algorithm are as follows:

STEP 1: Compute the FSWT expression for each sensor response, and denote it, $W_i(t,\omega,\alpha)$, $i = 1,2,...N_s$, where N_s is number of sensors.

STEP 2: Take the maxima of $|W_i(t,\omega,\kappa)|$, i.e. $|W|_M = \max_{t\geq 0,\omega\geq 0,i}|W_i(t,\omega,\kappa)|$ and record $(t_M, \omega_M, |W|_M)$ as a trigger condition.

STEP 3: Define the Frequency feature Sampling Line (FSL) and Time feature Sampling Line (TSL), and, record M_f, the number of all FSL and M_t, the number of all TSL. Call small block as Feature Block (FB), as shown in Figure 2.

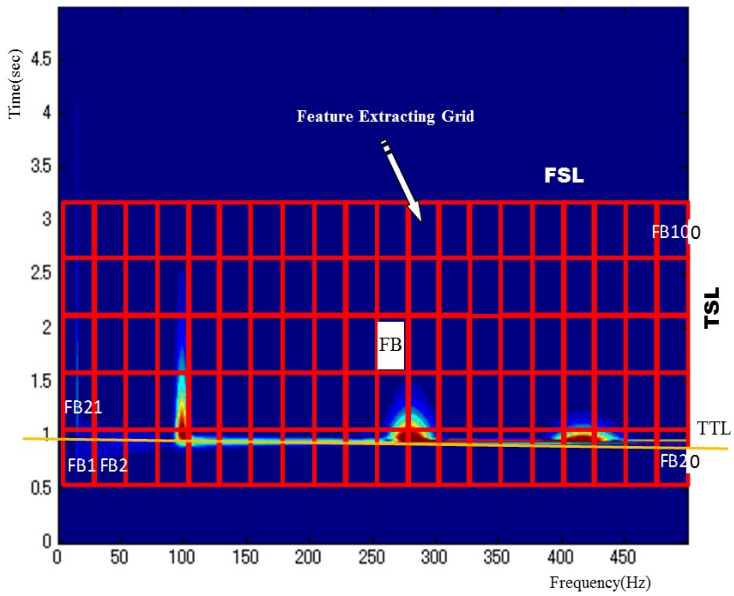


Figure 2. Example of feature extracting grid

STEP 4: Compute the modal damping feature at each feature block as,

$$\alpha_{ipq} = MDF\left(\left|W_i(t,\omega,\kappa)\right|\right)\Big|_{(t,\omega)\in \mathrm{FB_{pq}}}, \quad i = 1,2,\ldots N_s,\ p = 1,2\ldots M_f,\ q = 1,2\ldots M_t \tag{33}$$

STEP 5: Define: $V_i = (W_{M1}, \alpha_{ipg})\ i = 1,2,\ldots N_s,\ p = 1,2\ldots M_f,\ q = 1,2\ldots M_t$, where,

$$W_{ipq} = E\left(\left|W_i(t,\omega,\kappa)\right|\right)\Big|_{(t,\omega)\in \mathrm{FB_{pq}}} \tag{34}$$

where, $E(\cdot)$ means the average value, and call V_i : the Time-Frequency Feature (TFF) vectors of the system.

5 EXPERIMENTAL STUDY

5.1 Application to laboratory bridge monitoring system

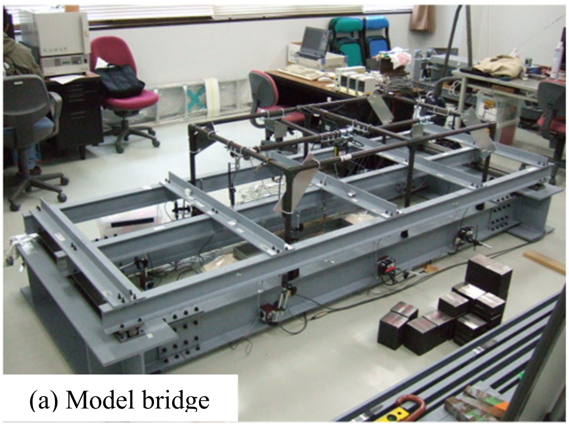
Figure 3 shows a laboratory bridge monitoring system: (a) the model bridge prepared for this study (a simply supported girder bridge model with three main girders (Miyamoto et al. 2012), and (b) acceleration sensor locations. Total 9 sensors which are piezo-type, ARF-10A (flat frequency response: 0-50Hz) acceleration sensors were arranged to the lower flange of the girders. Type DC-104R data logger was used to acquire and record the impact hammer vibration response data. The system sampling frequency was 1,000Hz and sampling time was set to 5.0 sec.

Single impact force as vibration test was applied to the cross points between the main girders and the cross beams (see Figure 3(b)). To enhance the accuracy of the transfer function, impact forces were applied 10 times at each point. Two kinds of test states were carried out for verification of the featuring (damage detection) method, that is normal (no-damaged) state phase and changed (damaged) state phase by additional weights on the model bridge, as a specific example.

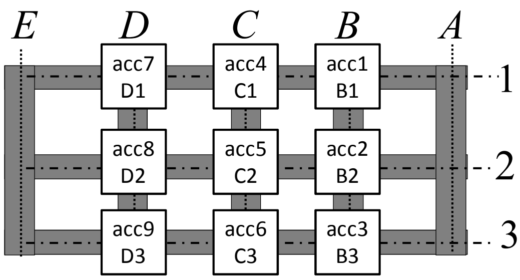
5.2 Feature extraction (Damage detection) based on SRM

Figure 4(a) shows an example of an acceleration response of one main girder as obtained from the laboratory monitoring data. Based on the time domain data, Figures 4(b)-(g) show the FSWT results as calculated by Eq. (30) in section *4.1* (in case of

$\hat{p}(\omega) = e^{-\frac{1}{2}\omega^2}$: Frequency Slice Function (FSF)).



(a) Model bridge



(b) Sensor Locations

Figure 3. Laboratory bridge monitoring system

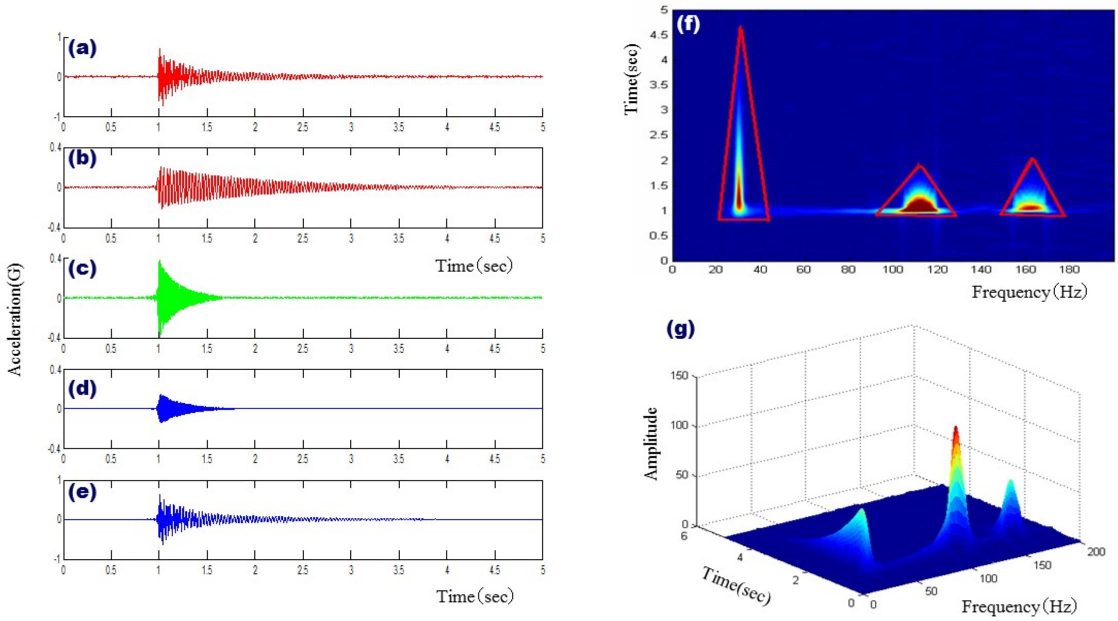


Figure 4. (a) is an original signal; there are three segmented signals (b), (c) and (d), and the left bottom (e) is their synthesized signal.; The right top (f) and (g) are **a**mplitude 2D and 3D map of FSWT coefficients for original signal, respectively

Based on the SRM algorithm given in Chapter 3, it will be able to get the distribution of the state variable, ζ that is the SRM state probability distribution by using future extracting grid as shown in Figure 5. In the experiment, there are 1,200 features for each sensor, and 9 impact points × 11 sensors × 10 times =990 groups of acceleration sensor data. Therefore, there are 990 × 1,200 = 1.1million feature numbers.

Figure 6 shows the results in terms of SRM state probability distributions with two observing SRM scales when weights (2.5kgf/weight) were added. It is shown clearly that increasing the additional weights, the state of system tends to move far away from the normal state, and the state changes of the model bridge due to additional loads (weights) can be clearly distinguished by the SRM state probability distribution. Then, based on these distributions, it will be possible to evaluate the difference between the current (damaged) state and previous (normal) state.

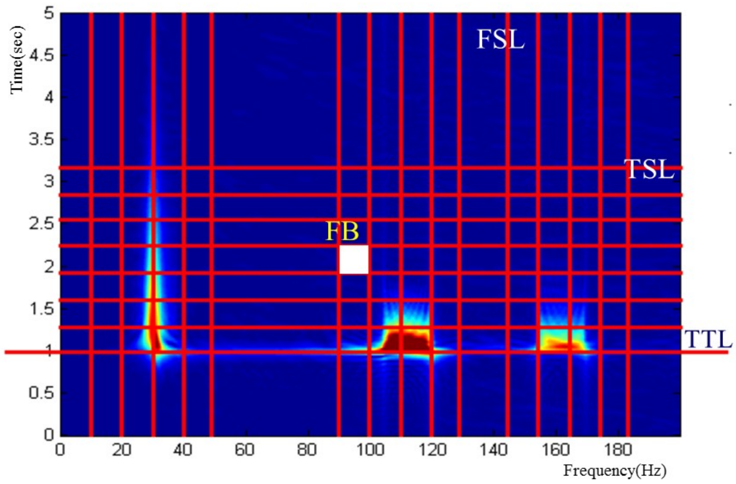


Figure 5. Feature extracting grid

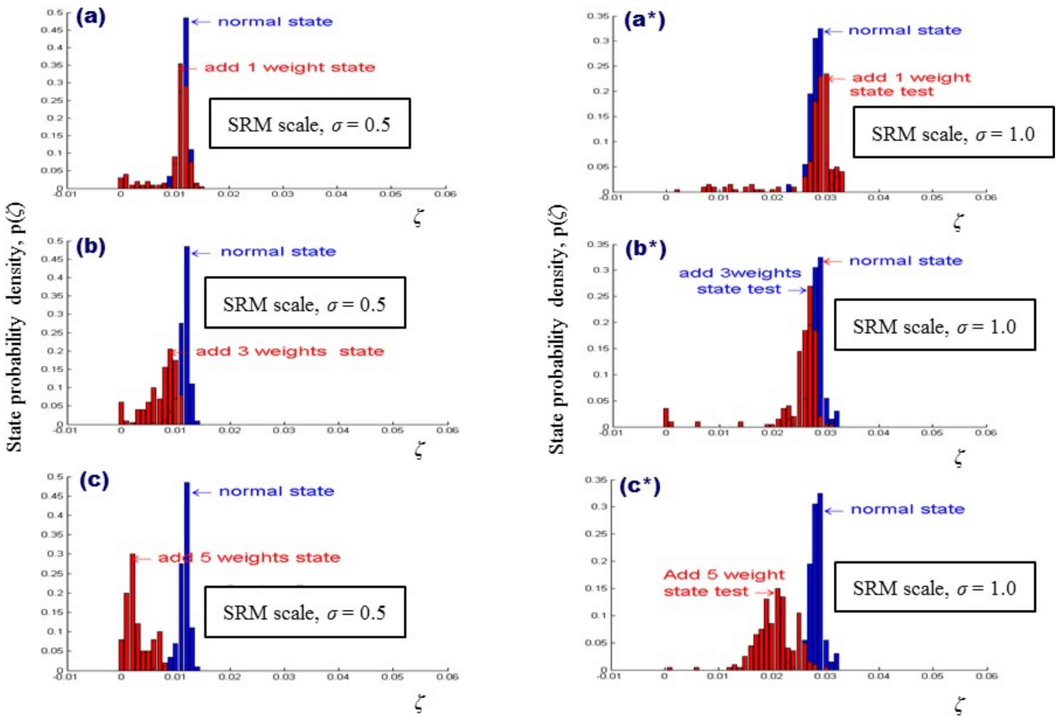


Figure 6. Comparison of SRM state probability distributions under different additional weights

6 CONCLUSIONS

This paper introduces the details of a newly proposed "State Representation Methodology (SRM)" and its application to damage detection based on laboratory bridge monitoring data. The main conclusions in this study are summarized as follows:

1. SRM is a new method for a non-parametric description of system state that described by state variables.
2. The state variables are calculated by the state representation equation (SRE) for steady expression of the system state.
3. SRM is an useful method for bridge condition assessment (damage detection) based on Structural Health Monitoring(SHM).

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