Exploring the solution space in error-domain model falsification using classification algorithms

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ABSTRACT: Most civil infrastructure possesses reserve capacity due to the use of justifiably conservative models in the design phase. This reserve capacity can be estimated by better understanding structural behaviour using model-based data interpretation. Error-domain model falsification is a multi-model methodology for data interpretation. This methodology explicitly accounts for model bias and is robust to unknown correlation values at measurement locations, thereby leading to robust identification and prediction. Most current implementations of the methodology employ grid sampling to explore the solution space for candidate models that reasonably explain the measurements.

The use of grid sampling is computationally expensive especially if large number of parameters are to be identified. Therefore, in this paper, a classification algorithm is used to identify the region of candidate model instances in the solution space. The application of classification algorithms is demonstrated using two examples to elucidate the reduction in computational cost. Using a classification algorithm to explore the solution space with fewer samples, complex model classes and detailed models can be employed for robust structural identification of full-scale structures and prediction of reserve capacities.

1 INTRODUCTION

Improving economic conditions and development in standard of living across the globe have increased the demand for civil infrastructure. Most present day infrastructure is aging and will need to be replaced according to new criteria and models used while designing these structures. However, most civil structures are designed using conservative design models and thus possess reserve capacity. Model-based data interpretation enables quantification of such reserve capacities for asset-management decision making.

Model-based data interpretation involves solving an inverse problem to determine the parameters of a structural model. In reality, this inverse problem is ill-defined, meaning that the uniqueness, stability and existence of a solution cannot be guaranteed. Therefore, robustness of the solution to uncertainty is an important consideration. Various methodologies are available for solving this inverse problem such as residual minimization (Alvin 1997), Bayesian model updating (Beck and Katafygiotis 1998) and error-domain model falsification (EDMF) (Goulet and Smith 2013).

In the context of civil infrastructure, the uncertainty at measurement locations is rarely Gaussian and modelling assumptions impart a high-degree of systematic bias. Also, the correlation between uncertainty at the measurement locations is usually not known and changes with the degree of systematic bias. Therefore, the assumption of Gaussian distributions for uncertainty and uncorrelated error is rarely satisfied (Tarantola 2005). Traditional Bayesian model updating and residual minimization utilise these assumptions for model updating and may thereby lead to biased updated probability distributions (Simoen et al. 2013).

Error-domain model falsification is a methodology for model-based data interpretation proposed by Goulet and Smith (2013) that is robust to the effects of systematic sources of uncertainty and correlation assumptions. In this methodology, combination of model parameters that constitute a model instance are falsified if the model response does not lie within threshold bounds. These bounds are determined using the uncertainty associated with the system. The dichotomy of a falsified and candidate model is utilized in this paper to train classification algorithms that can then be used to explore the parameter space to find further candidate model instances.

2 METHODOLOGY

In this section, the EDMF methodology is briefly explained. EDMF is a model-based data interpretation methodology, inspired from an assertion made by Karl Popper (Popper 1959) that models cannot be validated by data, they can only be falsified. In civil engineering, structures are designed using simple and conservative models. These models are biased and introduce modelling and systematic uncertainty along with parametric uncertainty. Most of these uncertainties can only be quantified using engineering knowledge, which is defined usually using bounds. Therefore, uncertainty is quantified using a uniform distribution rather than a zero-mean Gaussian distribution. Moreover, the correlation between uncertainty at various measurement locations on the structure is not known. Standard implementations of EDMF are robust to non-Gaussian sources of uncertainty and unknown correlation between measurement locations.

Consider a structure represented by a physics-based model, $g(\boldsymbol{\theta})$. Let the modelling and measurement uncertainty, at a measurement location i, be $U_{mod,i}$ and $U_{meas,i}$, respectively. The true response of the structure at a measurement location, Q_i, is given by,

$$Q_i = g_i(\theta^*) + U_{mod,i} = y_i + U_{meas,i} \tag{1}$$

where, $g_i(\theta^*)$ is the model response at measurement location i for the real values of the model parameters, θ^*, y_i is the measured response of the structure at measurement location i and $U_{mod,i}$ and $U_{meas,i}$ are the modelling and measurement uncertainty at the measurement location, respectively. Realigning Eq. (1), the following relationship between model response and measurement is obtained,

$$g_i(\theta^*) - y_i = U_{meas,i} - U_{mod,i} \tag{2}$$

where, the residual between model response and measurement at a sensor location is equal to the combined modelling and measurement uncertainty. Thresholds, $T_{high,i}$ and $T_{low,i}$, are established on this combined uncertainty based on a target reliability of identification using Eq. (3).

$$\phi^{1/m} = \int_{T_{low,i}}^{T_{high,i}} f_{Uc,i}(u_{c,i})\ du_{c,i} \tag{3}$$

In Eq. (3), $f_{Uc,i}$ is the combined uncertainty probability distribution function at measurement location i, ϕ is the target reliability of identification and the term $1/m$ is the Sidak correction (Šidák 1967) that accounts for a small number of measurements m. An initial grid of model instances is generated based on the prior distribution of model parameters. For every model instance, if the residual between model prediction and measurement does not lie within the thresholds then the model instance is falsified (Goulet et al. 2010, 2013b; Goulet and Smith 2013), as shown in Eq. (4).

$$\forall i \in \{1,..,m\} \qquad T_{low,i} \leq g_i(\theta) - y_i \leq T_{high,i} \tag{4}$$

The candidate model set comprises of the remaining model instances from the initial set, whose residuals for all measurement locations lies within the thresholds. These candidate models are then utilized for model prediction (Pasquier and Smith 2015), leak detection (Goulet et al. 2013a), fatigue life evaluation (Pasquier et al. 2014, 2016) and measurement system design (Goulet and Smith 2012a; b) among others. In total, the EDMF methodology has been developed and applied to fourteen full-scale systems since 1998 (Smith 2016).

The application of EDMF requires repeated evaluation of a structural model. Generally, full-scale civil infrastructure are modelled using the finite element method and these models are computationally expensive. In EDMF, a grid-based sampling is usually utilised to generate an initial population of model instances. For example, if two parameters are being identified with 10 instances of each, then, a total of 100 initial instances of model population have to be generated. However, if four parameters have to be identified, then 10000 initial model population instances have to be evaluated to retain the same grid density. In such a scenario, model updating can be carried out by developing surrogate models for structural response at measurement locations. However, surrogate models are not physics-based (unlike finite element models) limiting their applicability for estimation, for example strengthening requirements, through extrapolation. Therefore, in this paper, support vector machine (SVM) classifiers (Christianini and Shawe-Taylor 2000) is proposed as an effective strategy to determine candidate model instances.

In classification, an algorithm such as support vector machine is trained using a data set that includes parametric input and a categorical output. The trained classifier can then be used to predict the category or class for a given set of input parameters. There are various classification algorithms such as decision trees, naïve Bayes classification, k-nearest neighbour classification, SVM classification etc. A SVM classifier is a binary classifier, *i.e.*, it works only when the data is categorised into only two classes. In a simple SVM classifier, the dataset is used to train and find the best linear hyperplane that separates data points belonging to the two classes. However, the hyperplane separating the classes is not always linear. In such cases, a mathematical approach using kernels is utilised to develop a non-linear classifier. The kernel can be Gaussian, linear or of higher polynomial order. In this paper, model falsification results using a coarse grid sampling is used to train a SVM classifier with a quadratic kernel. As a coarse grid sampling is used, only few instances have to be evaluated with a physics-based model. Results for a finer grid sampling can then be predicted using this SVM classifier. This is a binary classifier that will predict based on the input parameter values if the model instance is a falsified or accepted candidate model instance. The applicability of using a SVM classifier is evaluated in the next section using two case studies.

3 EXAMPLES

3.1 Illustrative example

In this section, an illustrative analytical example is considered to demonstrate the use of SVM classifiers for determining the candidate model set. Figure 1 shows the beam considered in this example. The beam is pin supported at end A with rotational stiffness, K and roller supported at end B. In this example, 7 displacement measurements along the span of the beam are used to update parameter values of modulus of elasticity, E and rotational stiffness, K, when the beam is subjected to a point load at mid-span.

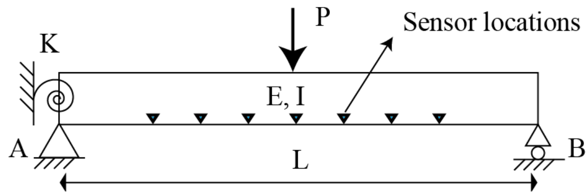

Figure 1 Illustrative beam example

The displacement measurements for the beam are simulated using Timoshenko beam theory, while the model for identification is developed using the Euler-Bernoulli beam theory. This bias in the model due to modelling assumptions introduces systematic uncertainty in the inverse problem. In Table 1, the real values of model parameters used to simulate the displacement measurements are presented. The uncertainty in parameter values is represented using uniform probability distributions whose bounds are provided in Table 1. The two parameters being identified are E and K, while the values for parameters I and L are assumed as shown in Table 1. This assumption contributes to modelling uncertainty associated with the identification problem.

Table 1 Model parameters and model class

	E (GPa)	**K log (Nmm/rad)**	**I $*10^8$ (mm^4)**	**L (mm)**
Real	70	9.8	7.01	3370
Prior	40-100	8-12	6.45-7.02	3350-3500
Model class	θ_1	θ_2	6.75	3400

The systematic and parametric uncertainty due to modelling assumptions are quantified as Gaussian random variables and combined at each measurement location. The mean value for modelling and systematic uncertainty at each measurement location is provided in Table 2. The coefficient of variation for uncertainty at each measurement location is assumed to be 10%. The measurements uncertainty is assumed to be zero-mean Gaussian with a standard deviation of 0.02mm.

Table 2 Identification uncertainty

Measurement location	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Mean of modelling uncertainty (%)	0.26	0.20	-0.08	-0.72	1.28	3.20	6.52
Mean of systematic uncertainty (%)	2.42	2.54	2.83	3.39	2.79	2.47	2.31

The thresholds for EDMF are determined using the distribution of combined uncertainty at each measurement location. For application of EDMF, a coarse two-dimensional grid containing 10 instances of each parameter is generated as shown in Figure 2. In the figure, the bubbles show initial model instances, while candidate model instances obtained using EDMF are shown using asterisks. Out of 100 initial samples generated, 36 candidate model instances were identified using EDMF.

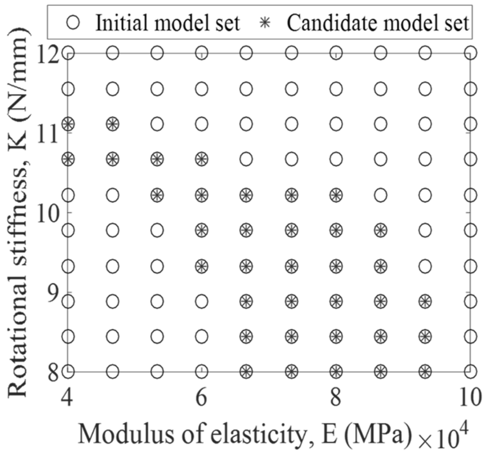

Figure 2 Sampling with a coarse grid

The falsified and candidate model instances shown in Figure 2 are used to train a SVM classifier to predict the candidate model instances for a finer grid density. In Figure 3, SVM classifier results are compared to candidate models obtained using a refined grid. The accuracy of prediction is good within the domain of the training data. However, at the boundaries of the candidate model region predicted using a finer grid, the SVM classifier prediction accuracy decreases. The classifier though provides a computationally efficient alternative to increase the density of candidate model population. 100 iterations of the analytical model of the beam were sufficient to train a SVM classifier and generate further candidate model instances. Without the use of a SVM classifier, to obtain results using a finer grid, 10000 iterations of the analytical model would be required. In the next example, an SVM classifier is used to predict candidate model instances of a highway bridge using in-service strain measurements.

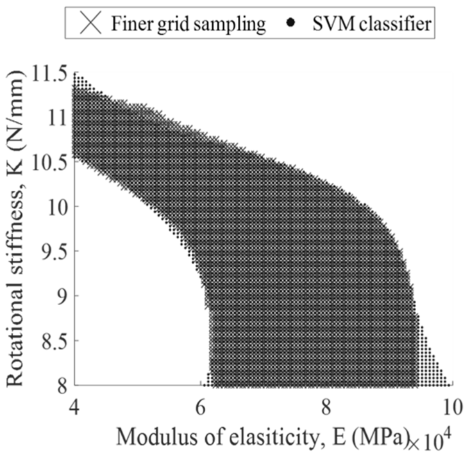

Figure 3. Comparison of candidate model set predicted using grid sampling and SVM classifier

3.2 *Venoge bridge*

Venoge bridge, shown in Figure 4, is a steel-concrete composite highway bridge of length 219.8m situated in Switzerland. Pai et al. (2017) have carried out model-based data interpretation for fatigue-life evaluation of the Venoge bridge using strain measurement data. The model parameters identified are the deck to girder connection stiffness along the bridge span (K_{deck}) and modulus of elasticity of concrete ($E_{concrete}$). Identification is carried out using in-service strain measurements from four sensors and traffic data from a weigh-in-motion station. The model response at sensor locations for various grid sampling instances has been obtained using a finite element model of the bridge developed in ANSYS (ANSYS 2012).

Figure 4 Venoge bridge in Switzerland

In this study, for the two parameters to be identified, an initial model set containing 400 instances were generated. Then, the combined uncertainty at each measurement location was quantified based on engineering heuristics to determine the thresholds for application of EDMF. Out of the 400 model instances considered, 192 model instances were identified as candidate model instances. A SVM classifier was trained using 33% of the dataset consisting of 400 analysed model instances. The trained classifier was then used to predict if the remaining model instances were falsified or candidate model instances as shown in Figure 5.

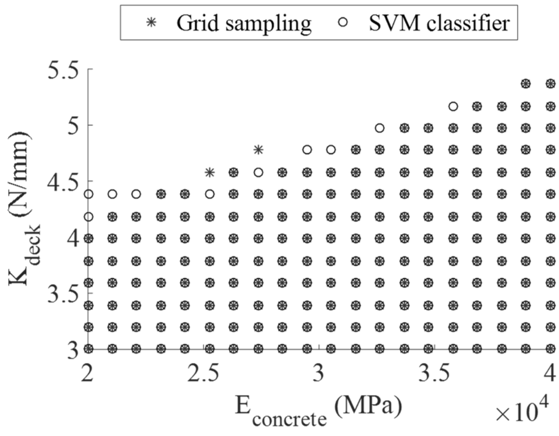

Figure 5. Comparison of candidate model set predicted using grid sampling and SVM classifier for the Venoge bridge

Figure 5 compares the candidate model set prediction using grid sampling and the trained SVM classifier. The comparison shows that the classifier performs well in predicting the candidate model set with few false classification instances. The classifier was trained using only one-third of the complete dataset. This implies only one-third of the model instances would have to be evaluated using a finite element model. The remaining model instances in the dataset could be classified as falsified or accepted using the SVM classifier without a finite element analysis. This would reduce the computation time associated with repeated evaluation of model instances as only one-third of the model instances have to be evaluated using a finite element model. As repeated evaluation of a finite element model is the most computationally expensive part of the analysis, the use of an SVM classifier reduces this cost significantly.

4 CONCLUSIONS

In this paper, the use of support vector machine classification in model-based data interpretation is presented. Through two examples, the applicability of this method and its advantage in reducing the computation time are demonstrated. In the illustrative example considered in this paper, the SVM classifier is able to provide reasonable results with only 1% of the computational time required when compared to finer grid sampling. For the Venoge-bridge case, only one-third of the dataset was considered for training the classifier. This classifier can then be used to explore the remaining parameter space without evaluation of a finite element model. However, classification accuracy decreases at the boundary delimitating falsified and candidate model instances. In future, false classification instances at the boundary could be reduced by evaluating these instances individually using the physics-based model. This will increase the computation time while the classification error is well controlled.

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