Reliability analysis of available relations and suggestion of a new formula for effective moment of inertia in concrete beams with FRP bars reinforcement

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ABSTRACT: In recent years, strengthening and retrofitting of existing structures becomes an important field of engineering research and practice. One of the most common methods to achieve this purpose is to use Fiber Reinforced Polymers (FRPs) in order to strengthen concrete structural members. FRPs are used in different ways in concrete structures. One of this methods is to utilize FRP bars in flexural concrete members which reduce deflection of flexural members. Due to importance of the deflection in short term behavior of FRP reinforced concrete beam, it is needed to perform comprehensive investigations on this topic. According to the available relations for deflection of reinforced beams, the effective moment of inertia of the strengthened section is an important parameter. This effective moment of inertia is a function of different factors and there are a few relations available to calculate this parameter. Most of the available formulas are based on modifying Branson relation for effective moment of inertia in concrete members. The main goal of this study is to evaluate accuracy and reliability of the available relations for computation of effective moment of inertia in concrete beams with FRP bars. To this purpose, different reliability indexes are assessed in conjunction with Monte Carlo simulation technique. In addition, experimental results from different studies are used to evaluate accuracy of the available relations. Base on this analysis, the most accurate and reliable relations are identified. Furthermore, a new more reliable and accurate relation based on numerical methods will be proposed.

1 INTRODUCTION

Nowadays, using Fiber Reinforced Polymers (FRPs) to rehabilitate and strengthened structures becomes a common engineering practice due to positive characteristics of these materials such as high corrosion resistance, better fatigue behavior in comparison with metals, possibility of production in complex shapes and etc. One method to take advantage of FRP in concrete structures, is to use FRP bars in flexural members such as concrete beams. So, various research works are conducted on flexural behavior and load carrying capacity of the concrete beams reinforced by FRP bars. Results show that using FRP bars instead of common steel rebar would increase the ultimate load carrying capacity of the beams. Another important parameter of flexural behavior is the short term deflection under different loadings. The beam's moment of inertia is utilized in the proposed relations for computation of the short term deflection of general RC beams. ACI suggest that this moment of inertia must be equal to the effective moment of inertia of the beam after cracking according to the Branson relation. But various researchers stated that the mentioned relation is not appropriate for the concrete beams with FRP bars, and it will not provide accurate estimate of the beam deflection. Thus different relations are proposed for calculation of the effective moment of inertia in concrete beams reinforced by FRP bars. Even investigators modified Branson relation for this case. The main goal of this study is to evaluate accuracy and reliability of the available relations for this purpose and also proposed new more accurate and reliable equations.

2 AVAILABLE RELATIONS FOR EFFECTIVE MOMENT OF INERTIA

In most of the conducted studies on concrete beams reinforced by FRP bars, direct integration from the moment-curvature diagram is used to calculate deflection of the beam. This approach is very accurate and in many standards such as ACI440.2R-08, CNR-DT 2004 and FIB 2001, using this method is suggested. But because the direct integration method is very time consuming, it is not appropriate for practical engineering and design purposes. Thus, a simple but accurate method is needed to calculate short term deflection of the concrete beams reinforced by FRP bars. For this purpose it is possible to use existing relations for deflection of the common RC beams, but the effective moment of inertia must be modified.

All the existing modified relations are obtained based on the four-point bending experiments. Therefore, the maximum deflection of the experimental specimen, which occurs at mid-span, is derived from the following relation:

$$\delta_{\max} = \frac{PL_a}{48E_cI}(3L^2 - 4{L_a}^2) \tag{1}$$

In which, L is the beam length and P is the total load which is sum of the two *P/2* concentrated loads that are applied at the distance L_a from each support. E_c is the concrete modulus of elasticity and I is the beam's moment of inertia which must be replaced by the section effective moment of inertia after cracking, I_e.

According to ACI318-11, the effective moment of inertia can be computed by the Branson relation which is as follows (ACI, 2008):

$$I_e = (\frac{M_{cr}}{M_a})^3 I_g + \left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \leq I_g \tag{2}$$

In equation (2), M_a and I_g are the maximum applied bending moment and the gross moment of inertia, respectively. And M_{cr} are I_{cr} also the cracking bending moment and moment of inertia of the equivalent cracked section, respectively. When FRP bars used to reinforce the concrete beam, using the previous relation is not possible and it needs to be modified or replaced by another appropriate equation.

Various investigators proposed different relation to calculate effective moment of inertia for concrete beams strengthened by FRP bars. Benmokrane et al. (1996) modified the Branson relation for the concrete beams with GFRP bars and suggested the following equation:

$$I_e = (\frac{M_{cr}}{M_a})^3 \frac{I_g}{7} + 0.84\left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \leq I_g \tag{3}$$

Toutanji and Saffi (2000) incorporate the effect of the FRP modulus of elasticity in the relation and modified the Branson's relation in the next form:

$$I_e = (\frac{M_{cr}}{M_a})^m I_g + \left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \tag{4}$$

where m is computed from the succeeding equations:

$$m = \begin{cases} 6 - 1000\left(\frac{E_f}{E_s}\right)\rho_f & for \quad \left(\frac{E_f}{E_s}\right)\rho_f < 0.3\% \\ 3 & for \quad \left(\frac{E_f}{E_s}\right)\rho_f \geq 0.3\% \end{cases} \tag{5}$$

In the previous equation, $\frac{E_f}{E_s}$ is the ratio of the FRP modulus of elasticity to the steel modulus of elasticity, and ρ_f stand for ratio of the FRP bar area to the beam cross section area.

Alsayed et al. (2000) presented two different relations for calculation of the effective moment of inertia in concrete beams reinforced by FRP bars. These two relations are as follows:

$$I_e = (\frac{M_{cr}}{M_a})^{5.5} I_g + \left[1 - (\frac{M_{cr}}{M_a})^{5.5}\right] I_{cr} \tag{6}$$

$$I_e = \begin{cases} \left[1.4 - \frac{2}{15}\left(\frac{M_a}{M_{cr}}\right)\right] I_{cr} & for \ 1 < \frac{M_a}{M_{cr}} < 3 \\ I_{cr} & for \ \frac{M_a}{M_{cr}} > 3 \end{cases} \tag{7}$$

The Canadian standard ISIS (2001) suggest the following equation for computation of the effective moment of inertia in concrete beams with FRP bars:

$$I_e = \frac{I_g I_{cr}}{I_{cr} + \left[1 - 0.5(\frac{M_{cr}}{M_a})^2\right](I_g - I_{cr})} \tag{8}$$

In 2003, ACI440-1R proposed the next modified version of the Branson's relation to calculate the effective moment of inertia:

$$I_e = (\frac{M_{cr}}{M_a})^3 \beta_d I_g + \left[1 - (\frac{M_{cr}}{M_a})^3\right] I_{cr} \leq I_g \tag{9}$$

Where β_d is a function of the FRP and steel modulus of elasticity in the following form:

$$\beta_d = \alpha\left[\frac{E_f}{E_s} + 1\right] \tag{10}$$

The coefficient α for GFRP bars is equal to 0.5. Yost et al. (2003) proposed the next equation for computation of this coefficient:

$$\alpha = 0.064\left(\frac{\rho_f}{\rho_{fb}}\right) + 0.13 \tag{11}$$

Later in 2006, ACI440.1R (2006) proposed another relation for β_d:

$$\beta_d = 0.2\left[\frac{\rho_f}{\rho_{fb}}\right] \leq 1 \tag{12}$$

In the previous equation ρ_f and ρ_{fb} are the available ratio and the balance ratio of the FRP bars.

In another study, Al-Sunna et al. (2005) suggest the coming relation for the effective moment of inertia:

$$I_e = (\beta_d I_g - \alpha I_{cr})(\frac{M_{cr}}{M_a})^3 + \alpha I_{cr} \tag{13}$$

This equation is applicable for both steel and FRP bars. β_d is derived from equation (10) and α for steel, CFRP and GFRP bars is equal to 1, 0.85 and 0.9, respectively.

Bischoff et al. (2005) suggested the effective moment of inertia in the following form:

$$I_e = \frac{I_{cr}}{1-(\frac{I_{cr}}{I_g})(\frac{M_{cr}}{M_a})^2} \tag{14}$$

Later, Bischoff et al. (2011) revised their previously presented equation by introducing a modifying coefficient in the denominator:

$$I_e = \frac{I_{cr}}{1-\psi(\frac{I_{cr}}{I_g})(\frac{M_{cr}}{M_a})^2} \tag{15}$$

In equation (15), ψ is a function of the loading type and boundary conditions of the beam. In their study, Bischoff et al. (2011) computes this coefficient for different loading and boundary conditions.

Also, Rafi and Nadjai (2009) proposed another equation, which is again a modification of the Branson relation:

$$I_e = (\frac{M_{cr}}{M_a})^3 \beta_d I_g + \left[1-(\frac{M_{cr}}{M_a})^3\right]\frac{I_{cr}}{\gamma} \leq I_g \tag{16}$$

The coefficient γ in the previous equation is derived from the next relationship:

$$\gamma = \left(0.0017\frac{\rho_f}{\rho_{fb}} + 0.8541\right)(1+\frac{E_f}{2E_s}) \tag{17}$$

Finally, the latest proposed relations are presented by Mousavi and Esfahani (2012). They suggest three different relations for computing the effective moment of inertia. These relations are as follows:

$$I_e = 0.13(\frac{M_{cr}}{M_a})^{m_1} I_g + 0.89\left[1-(\frac{M_{cr}}{M_a})^{m_1}\right] I_{cr} \leq I_g \tag{18}$$

$$I_e = 0.15(\frac{M_{cr}}{M_a})^{m_2} I_g + 0.89\left[1-(\frac{M_{cr}}{M_a})^{m_2}\right] I_{cr} \leq I_g \tag{19}$$

$$I_e = 0.17(\frac{M_{cr}}{M_a})^{m_3} I_g + 0.96\left[1-(\frac{M_{cr}}{M_a})^{m_3}\right] I_{cr} \tag{20}$$

In these relations, m_1 , m_2 and m_3 are derived from the successive relationships:

$$m_1 = -0.24\left(\frac{\rho_f}{\rho_{fb}}\right) + 5.35\left(\frac{M_{cr}}{M_a}\right) + 2.28\left(\frac{E_f}{2E_s}\right) \tag{21}$$

$$m_2 = 0.66 - 0.3\left(\frac{\rho_f}{\rho_{fb}}\right) + 1.94\left(\frac{M_{cr}}{M_a}\right) + 4.64\left(\frac{E_f}{2E_s}\right) \tag{22}$$

$$m_3 = 1.69 - 0.51\left(\frac{\rho_f}{\rho_{fb}}\right) + 1.77\left(\frac{M_{cr}}{M_a}\right) + 6.67\left(\frac{E_f}{2E_s}\right) \tag{23}$$

3 EVALUATION OF THE EXISTING MODELS

In this section, the relations presented in the previous section will be evaluated by computation of Hasofer-Lind reliability index and the SRSS error for them according to the available experimental results. For this purpose, wide range of test data including 400 experimental data points are used. These data points were extracted from load-displacement relationship of thirty eight FRP-reinforced concrete beam specimens were tested by researchers. The results derived from experiments performed by Benmokrane et al. (1996), Toutanji (2000), Alsayed et al. (2000), Masmoudi et al. (1998), Alsayed (1998), Theriault et al. (1998), Abdalla (2002), El-Salakawy et al. (2002), Rafi et al.(2009), Tavares et al. (2008), Pawłowski et al(2015).

Taking advantage of these experimental results, the Hasofer-Lind reliability index, β, and SRSS error are computed for the available relations. The obtained results are presented in Table (1):

Table 1. Reliability index and SRSS error of the available relations

No.	Relation	Hosofer-Lind reliability index	SRSS error($\times 10^{-5}$)
1	ACI 440, 1st	11.1	2.98
2	ACI 440, 2nd	5.11	2.18
3	Alsayed, 1st	10.7	19.36
4	Alsayed,2nd	3.7	2.18
5	Benmokrane	4.9	2.14
6	Rafi	4.99	2.17
7	Yost	7.1	2.19
8	Bischoff	4.6	2.37
9	ISIS	4.86	38.5
10	Mosavi, 1st	4.82	56.9
11	Mosavi, 2nd	5.2	13.1
12	Mosavi, 3rd	5.14	12.1
13	Toutanji	9.87	6.53

A relation is more accurate if its SRSS error is lower. On the other hand, higher reliability index indicates that the relation is more reliable.

4 PROPOSITION OF THE NEW RELATION

Using the most accurate relations which are identified previously, and by taking advantage of the genetic algorithm for optimization, about 280000 data sets are produced in a wide range of the effective parameters. Then by using numerical nonlinear regression techniques, different equations with unknown coefficients and powers are proposed. In the proposition of theses relations, effects of parameters such as FRP bar modulus of elasticity, FRP bar ratio and balanced bar ratio are considered. The proposed relations are listed in Table (2):

Table 2. The proposed relations

Relation Name	Proposed relation for computation of the effective moment of inertia
TAG1	$I_e = I_g(\frac{M_{cr}}{M_a})^{\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_1}+\left(\frac{E_f}{E_s}\right)^{x_2}+(\frac{M_{cr}}{M_a})} + \left[I_{cr}(1-(\frac{M_{cr}}{M_a})\right]$
TAG2	$I_e = (I_g(\frac{M_{cr}}{M_a}))^{\left(\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_1}+\left(\frac{E_f}{E_s}\right)^{x_2}\right)} + \left[1-(\frac{M_{cr}}{M_a})^{x_3}\right]$
TAG3	$I_e = (I_g(\frac{M_{cr}}{M_a})^{x_1})^{\left(\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_2}+\left(\frac{E_f}{E_s}\right)^{x_3}\right)} + \left[I_{cr}(\frac{M_{cr}}{M_a})\right]$
TAG4	$I_e = \left(I_g(\frac{M_{cr}}{M_a})\right)^{\left(x_1+\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_2}\right)} + x_3\left[1-(\frac{M_{cr}}{M_a})\right]I_{cr}$
TAG5	$I_e = \left(x_1 I_g(\frac{M_{cr}}{M_a})\right)^{\left(x_2+\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_3}\right)} + x_4\left[1-(\frac{M_{cr}}{M_a})\right]I_{cr}$
TAG6	$I_e = \left(I_g(\frac{M_{cr}}{M_a})^{x_1}\right)^{\left(x_2+\left(\frac{\rho_f}{\rho_{fb}}\right)^{x_3}\right)} + x_4\left[1-(\frac{M_{cr}}{M_a})\right]I_{cr}$

The unknown coefficients and powers are derived by optimization technique such that to minimize error and maximize reliability index of the suggested relations. These values are presented in Table (3):

Table 3. Optimized coefficients and powers for the proposed relations

Relation Name	Optimum coefficients and powers
TAG1	$x_1 = -0.103$ $x_2 = -0.007$
TAG2	$x_1 = 0.092$ $x_2 = 0.253$ $x_3 = 1.035$
TAG3	$x_1 = 0.376$ $x_2 = 0.015$ $x_3 = 0.105$
TAG4	$x_1 = 1.47$ $x_2 = 15877.3$ $x_3 = 0.987$
TAG5	$x_1 = 0.01$ $x_2 = 2.422$ $x_3 = -0.007$ $x_4 = 1.033$
TAG6	$x_1 = 0.852$ $x_2 = 0.45$ $x_3 = -0.028$ $x_4 = 0.984$

Now, accuracy and reliability of the suggested relations are evaluated by using the extracted data points from the experimental studies. The obtained results, which are presented in Table (4) demonstrate acceptable accuracy of the proposed relations:

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Table 4. Reliability index and SRSS of the proposed relations

No.	Relation	Hosofer-Lind reliability index	SRSS error($\times 10^{-5}$)
1	TAG1	11.1	11.3
2	TAG2	10.8	2.16
3	TAG3	11.4	2.17
4	TAG4	10.4	2.15
5	TAG5	11.6	2.168
6	TAG6	10.2	2.15

According to the obtained results, it can be seen that the proposed relations are more accurate and also more reliable than the other existing equations for computation of the effective moment of inertia in concrete beams reinforced by FRP bars. In addition, it is reminded that in comparison with the other existing relations, a wider variation range of influential parameters are used in proposition of this new relations, therefore they are applicable for any kinds of FRP bars as well as concretes with different compressive strength.

5 CONCLUSION

The experiments on concrete beam reinforced by FRP bars show that using FRP bars instead steel bars leads to increase in the ultimate load carrying capacity of the flexural members. An important parameter in determination of flexural behavior of beams is the short term deflection under various loadings. To compute the short term deflection of FRP reinforced concrete beams, the effective moment of inertia of the section after cracking is needed. Different studies demonstrate that using Branson relation for this purpose is not appropriate. Therefore, various relations are proposed to compute this effective moment of inertia. In this study by taking advantage of nonlinear regression techniques and genetic algorithm optimization, six new relations are proposed for computation of the effective moment of inertia. Comparison of the result obtained by the proposed relations and the experimental outcomes shows that these new equations are more accurate and reliable than the other existing relations.

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