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# Modelling the cyclic response of FRP strips glued to a brittle substrate

Enzo Martinelli[1], and Antonio Caggiano[2]

[1] DICiv - University of Salerno, Fisciano (SA), Italy
[2] CONICET & University of Buenos Aires, Buenos Aires, Argentina

ABSTRACT: The present paper is intended at formulating a theoretical contribution to simulate the bond behaviour of FRP strips glued to a brittle substrate and subjected to low-cycle fatigue loads, such as the ones applied to the anchorage zones of FRP strips employed in seismic retrofitting of concrete or masonry members. The constitutive proposal is formulated within the general framework of the Fracture Mechanics and is based on two alternative expressions for modelling the softening response of the bond-slip relationship. Closed-form expressions, obtained for determining the key damage-related quantities, are derived for the present formulation. The comparison between some experimental results, available in the scientific literature, and the numerical simulations performed by means of the present model are reported for investigating the formulations soundness and capabilities.

## 1 INTRODUCTION

Fiber-Reinforced Polymer (FRP) materials are more and more employed as technically convenient solutions in strengthening deficient structural members as part of retrofitting interventions of existing structures. The most common applications of FRP deals with confining concrete columns (Pan et al., 2007), strengthening wooden floor beams (Corradi et al., 2006) or steel profiles in bending (Zhao and Zhang, 2007) and masonry panels in shear (Marcari et al., 2007). However, the adhesive interface between the FRP strips or fabrics and the existing material substrate often controls the resulting performance of the aforementioned members strengthened by Externally-Bonded (EB) FRP strips: this is particularly the case of Reinforced Concrete (RC) members (Buyukozturk and Hearing, 1998). Therefore, a big deal of research was carried out in the last decades for investigating the bond behaviour of FRP strips glued to concrete (Kang et al., 2012). Particularly, the FRP-to-concrete fracture and debonding process were thoroughly investigated via both experimental (see, for instance, Chajes et al., 1996; Czaderski et al., 2012) and theoretical (Ferracuti et al., 2006, Cornetti & Carpinteri, 2011; Martinelli et al., 2011; Caggiano et al., 2012; Caggiano & Martinelli, 2013) studies. However, these studies were generally restricted at the case of monotonic loading processes.

As a matter of fact, FRP strips are widely used for enhancing the structural performance of RC beams under cyclic actions possibly induced by either traffic loads or earthquakes. Despite the significant differences between the two aforementioned load cases, the state of knowledge about the actual behaviour of both the adhesive FRP-to-concrete interface and the performance of EB-FRP strengthened RC members under cyclic actions is not as developed and as their behaviour under monotonic load cases. In fact, few studies are nowadays available on this topic.

Particularly, Nigro et al. (2011) reported the results of low-cycle fatigue tests carried out by assuming a single shear test set-up, whereas the results of high-cycle fatigue tests were recently documented by Carloni et al. (2012). Regarding theoretical modelling, Ko & Sato (2007) proposed an empirical bond-slip model intended for simulating the behaviour observed in a series of monotonic and cyclic tests carried out on Aramid (A), Carbon (C) and Polyacetal (P) FRP strips glued to concrete blocks and tested in double shear configuration. The model was based on assuming a Popovics-like law and involved seven mechanical parameters, which should be calibrated experimentally as a result of the empirical nature of the model under consideration.

This paper moves from a study recently proposed by the same authors (Martinelli and Caggiano, 2014) and is intended at further enhancing a theoretical model capable of simulating the progressive cracking processes developing at the FRP-to-concrete interface as a result of cyclic actions applied to the composite member. Particularly, it presents a theoretical model formulated within the general framework of Fracture Mechanics (FM) to describe the post-elastic behaviour of the aforementioned adhesive interface. Mode II fracture mode is assumed in this formulation and a three-parameter double-exponential bond-slip law is assumed in this study as a generalisation of an existing proposal already available in the scientific literature (Dai et al, 2005). The fully nonlinear shape of the bond-slip law assumed in this study results in the capability of simulating the debonding damage accumulation observed experimentally also under cyclic actions characterised by low-to-medium force amplitude with respect to the failure load determined under static and monotonic processes (Yun et al., 2008).

The paper is organised as follows. Firstly, Section 2 outlines the key theoretical foundations of the proposed model and proposes some closed-form expressions of the fracture work that can be derived by the analytical expression of the assumed bond-slip law. Then, Section 3 proposes some comparisons between the model simulations and a series of monotonic and cyclic test results available in the scientific literature. Finally, concluding remarks as well as future developments of the present work are highlighted in Section 4.

## 2 THE THEORETICAL MODEL

A simplified theoretical model is proposed to model the cyclic response of FRP strips glued to brittle substrates, made of materials such as concrete or masonry. Particularly, the present proposal is based upon the following key assumptions:

- the crack develops at the FRP-to-concrete interface in (pure shear) "mode II";
- a three-parameter-double-exponential analytical law is assumed for expressing the bond-slip relationship characterising the FRP-to-concrete interface;
- stiffness degradation in the unloading stages depends upon the actual value of the "fracture work" spent in each interface point;
- small displacements are assumed at the interface while axial strains possibly developing in the concrete substrate are neglected.

The four assumptions listed above lead to define the general governing equations for the mechanical behaviour of FRP strips glued to a brittle substrate. They are derived by writing the classical “equilibrium”, “compatibility” and “(generalised) stress–strain” relationships, in both monotonic and cyclic response.

## 2.1 Main assumptions

The proposed model is intended at modelling the FRP strip glued to a brittle support and schematically depicted in Fig. 1.

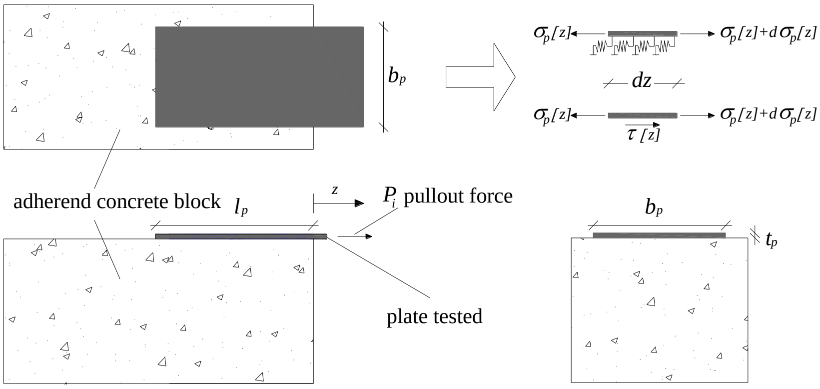


Figure 1. Single-lap shear test of a FRP-to-concrete bonded joint.

The assumptions of uniform width and thickness, $b_p$ and $t_p$ respectively, and a unique bond-slip relationship throughout the adhesive interface, lead to the following equilibrium condition:

$$\frac{d\sigma_p(z)}{dz} = -\frac{\tau(z)}{t_p} \tag{1}$$

being $\tau(z)$ the interface bond stress and $\sigma_p(z)$ the axial stress in the FRP cross section. The bond-slip equations for the adhesive behaviour can be expressed by means of the following double exponential law, which can be intended as a generalisation of the one proposed by Dai et al. (2005):

$$\tau(z) = \frac{\alpha\beta}{\beta-\alpha} \cdot G_F \cdot \left[ e^{-\alpha s(z)} - e^{-\beta s(z)} \right] \tag{2}$$

where $s(z)$ *is* the shear slip at the considered $z$-abscissa, $\alpha$ and $\beta$ are two conceptually independent constants (with $\beta>\alpha$) and $G_F$ is the fracture energy (in Mode II) of the FRP-to-concrete interface. As a matter of fact, the following identity can be easily checked:

$$G_F \overset{\text{def}}{=} \int_0^\infty \tau(s) \cdot ds = \frac{\alpha\beta}{\beta-\alpha} \cdot G_F \cdot \int_0^\infty \left( e^{-\alpha s} - e^{-\beta s} \right) \cdot ds = G_F \tag{3}$$

Moreover, it is worth mentioning that a three-parameter bond-slip law (2) was actually adopted in this study (as an extension of the two-parameter one proposed by Dai et al., 2005) because it allows for establishing the following correspondence between the three independent parameters (i.e., the initial elastic stiffness $k_E$, the maximum bond stress $\tau_{max}$ and the fracture energy $G_F$) needed for identifying the widely-adopted bi-linear bond-slip relationship:

$$k_E = \left. \frac{d\tau}{ds} \right|_{s=0} = \alpha\beta G_F \tag{3}$$

$$\tau_{\max} = \left[ \left( \frac{\alpha}{\beta} \right)^{\frac{\beta}{\beta-\alpha}} \right] \cdot \beta \cdot G_F \tag{4}$$

being $G_F$ explicitly involved in the expression (2).

The linear elastic behaviour of the FRP strip can be easily represented by the following relationship:

$$\sigma_p[z] = E_p \varepsilon_p \tag{5}$$

where $E_p$ is the Young modulus of the composite, whereas the strain field can be calculated by means of the following compatibility condition:

$$\varepsilon_p = \frac{ds[z]}{dz}. \tag{6}$$

Finally, the following differential equation can be obtained by introducing eqs. (5) and (6) into the equilibrium condition (eq. 1):

$$\frac{d^2 s[z]}{dz^2} + \frac{\tau[z]}{E_p t_p} = 0. \tag{7}$$

### 2.2 *Fracture-based damage modelling*

The unloading/reloading stiffness is modelled within the framework of the theory of Fracture Mechanics by considering, for each point of the adhesive interface, the fracture work and the corresponding fracture energy in "mode II" $G_F$. The fracture work, $w_{sl}$, developed during the sliding fracture process, controls the evolution of damage. Particularly, the variable $w_{sl}(s)$ represents the "inelastic portion" of the enclosed area of the $\tau$-$s$ curve in the range *(0-s)* (Fig. 2) and can be expressed in closed-form as follows:

$$w_{sl} = \int_0^s |\tau(s)| ds - \frac{\tau^2(s)}{2k_E} = \frac{\alpha\beta}{\beta-\alpha} \cdot G_F \cdot \left[ \frac{1-e^{-\alpha s}}{\alpha} - \frac{1-e^{-\beta s}}{\beta} \right] - \frac{\tau^2(s)}{2k_E}. \tag{8}$$

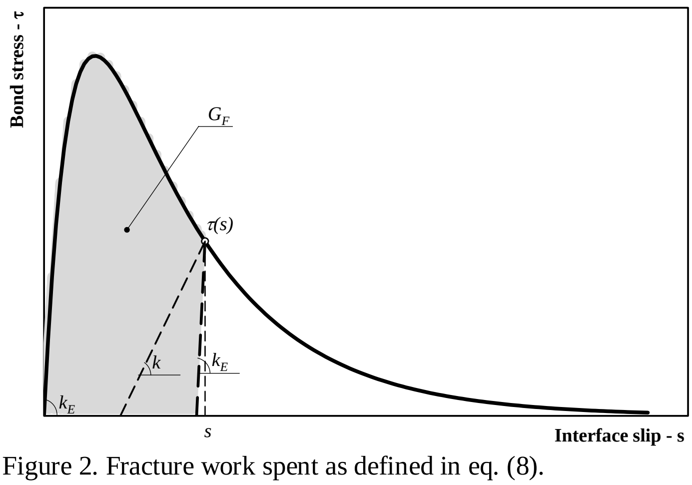


Figure 2. Fracture work spent as defined in eq. (8).

Finally, the damage parameter $d$ can be defined in each point of the adhesive interface:

$$d = \xi^{\alpha_d}, \text{ with } \xi = \frac{w_{sl}}{G_F}, \tag{9}$$

where $\alpha_d$ controls the shape of the damage curve and the loading/unloading stiffness $k$ is related to the elastic one through the following relationship:

$$k = k_E \left(1 - d\right). \tag{10}$$

### 2.3 Outline of the numerical procedure

A Finite Difference (FD) procedure is developed for integrating equation (7) under monotonic and cyclic actions. Particularly, a Central-Difference (CD) expression is assumed to express the second derivative of eq. (7) in the internal nodes of the FD mesh represented in Fig. 3:

$$\Delta s_i{}^j = \frac{\Delta s_{i+1}{}^j + \Delta s_{i-1}{}^j}{2 + \dfrac{k_{T,i}}{E_p t_p} \cdot \Delta z^2} \qquad \text{for } i=0,..n-1, \tag{10}$$

where $j$ is the current analysis step, $i$ the node number and $k_T$ the corresponding tangential stiffness of the local bond-slip law depending on $s_i^{j-1}$. Since the analyses are intended to proceed in displacement control, the following boundary conditions are applied at the unloaded and loaded end of the FRP strip, respectively:

$$\Delta s_{-1}{}^j = \Delta s_1{}^j \tag{11}$$

$$\Delta s_n{}^j = \Delta s_c{}^j \tag{12}$$

where eq. (11) corresponds to the condition of zero stress (and strain) at the unloaded end, and eq. (12) to the imposed slip increment at the loaded end (i.e., node $n$).

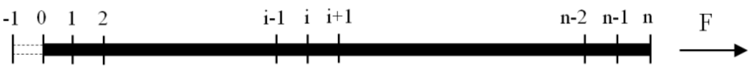


Figure 3. Finite difference discretisation of the FRP-to-concrete interface.

The set of ($n+2$) simultaneous equations (10)-(12) can be solved in terms of slip increment vector $\boldsymbol{\Delta s}^j$ and, in principle, the final solution in the *j-th* analysis step can be obtained iteratively to take into account the possible interface nonlinearity. Particularly, the trial solution at the *k-th* iteration of the *j-th* incremental analysis step can be obtained in terms of both interface slip and bond stress vectors (which collect the $n+2$ components of both quantities):

$$\boldsymbol{s}^j\Big|_k = \boldsymbol{s}^{j-1} + \boldsymbol{\Delta s}^j\Big|_k \tag{13}$$

$$\boldsymbol{\tau}^j\Big|_k = \boldsymbol{\tau}^{j-1} + \boldsymbol{\Delta \tau}^j\Big|_k = \boldsymbol{\tau}^{j-1} - \boldsymbol{k}_T \cdot \boldsymbol{\Delta s}^j\Big|_k \tag{14}$$

where $\boldsymbol{s}^{j-1}$ and $\boldsymbol{\tau}^{j-1}$ are slip and bond stress vectors, at the convergence of the *j-th* incremental analysis step, and $\boldsymbol{k}_T$ a vector collecting the tangential stiffnesses $k_{T,i}$ at the various nodes of the FD discretisation. If the node *i-th* ended up the *(j-1)-th* analysis step in the elastic stage, the following condition should be met by the trial solution (14) for the same node to remain in elastic stage:

$$\left|\tau_i{}^j\big|_k\right| \le \left|\tau\left(s_{cr,i}\right)\right| \tag{15}$$

where $\tau$ is the bond-slip law expressed by either of equations (2) and $s_{cr,i}$ a state variable which represents the total slip developed in the node $i$ during the fracture process and, in monotonic conditions, could be simply expressed as $s_{cr,i}=s_i-s_{el}$. If eq. (15) is satisfied in all nodes at the first iteration ($k=1$), then they hold their elastic status and the force $\Delta F^j$ increment, corresponding to the imposed slip increment $\Delta s_c^j$, can be derived by equilibrium:

$$\Delta F^j \overset{[k=1]}{=} \Delta F^j\big|_k = \left[\sum_{i=1}^{n} \frac{\tau_i{}^j\big|_k + \tau_{i-1}{}^j\big|_k}{2}\right] \cdot b_p \cdot \Delta z\,. \tag{16}$$

If this is not the case, the slip increment $\Delta s_i^j|_k$ should be subdivided in an elastic part $\Delta s_i^j|_{k,el}$, corresponding to the achievement of the equality in equation (15) and the cracking part $\Delta s_i^j|_{k,cr}= \Delta s_i^j|_k- \Delta s_i^j|_{k,el}$. Then, an iterative search of the equilibrium for the *j-th* can be carried out by employing eqs. (10)-(12) as an elastic predictor and the equality in eq. (15) to obtain the corrector. Once convergence is achieved (i.e. in terms of unbalanced forces at the *k-th* iteration of the *j-th* increment), the vector $\boldsymbol{s}_{cr}$, collecting the state variable $s_{cr,i}$, can be updated as follows:

$$\boldsymbol{s}_{cr}{}^j = \boldsymbol{s}_{cr}{}^{j-1} + \boldsymbol{\Delta s}_{cr}{}^{j-1}\big|_k \tag{17}$$

and the corresponding force determined through eq. (16). Then, in the following incremental analysis steps, the same node $i$ will keep the cracking status if no sign change occurs between the increment slip at the previous step *(j-1)-th* and the one obtained by solving eqs. (10)-(12):

$$\Delta s_i{}^{j-1} \cdot \Delta s_i{}^j\big|_k > 0\,. \tag{18}$$

If this is the case for all the nodes, the corresponding force can be determined through eq. (16) and the status variable updated via eq. (17). Otherwise, an unloading stage starts in the nodes where the inequality (18) is not satisfied and an iterative predictor-corrector search leads to the new system status. The incremental analysis proceeds up to the achievement of a given failure condition which could be formulated in terms of maximum slip occurring at the unloaded end.

It is worth mentioning that the "analysis of accuracy and stability" of the FD algorithm, employed for the integration of the constitutive relations, is out of the scopes of this paper and will be carried out in future works also by comparing the numerical results obtainable with this model against numerical simulations based on discontinuous-based finite element integrations. Moreover, It is worth highlighting that the current version of the formulation is limited to quasi-static actions; The extension to dynamic ones is currently under development.

## 3 EXPERIMENTAL COMPARISONS

The formulation presented in Section 2 needs to be validated in its soundness and capability to simulate the FRP-to-concrete pull-out behaviour under both monotonic and cyclic conditions. Experimental data characterising both of the above mentioned experimental situations, on three types of FRP sheets are available in Ko & Sato (2007). The results of some tests carried out on a single ply of A-FRP strips are considered to achieve a preliminary validation of the proposal as already proposed in a previous work by the authors (Martinelli & Caggiano, 2014).

Three equal specimens were tested under monotonic and cyclic actions. They are characterised by an A-FRP strip with relative axial stiffness $E_pt_p = 10.4\ kN/mm$ and width $b_p = 50\ mm$. Then, the values of the bond-slip material parameters are identified for the two (alternative) softening laws (namely, the exponential and linear one). Particularly, $k_E = 52.22\ MPa/mm$, $\tau_{max} = 2.256\ MPa$ and $G_F = 0.958\ N/mm$ are assumed in the following numerical simulations for the mechanical quantities which are relevant for both the bond-slip relationships, according to the average values identified by the cited authors for the specimens A11, A12 and A13, tested under monotonic actions. Consequently, the values of the coefficients $\alpha$ and $\beta$ involved in the mathematical expression of the assumed bond-slip law (eq. 2) can be determined by solving eqs. 3-4 and depending on the aforementioned numerical values of $k_E$, $\tau_{max}$ and $G_F$: the values $\alpha$=3.658 and $\beta$=14.575 are determined and assumed in the following numerical applications. Moreover, the unit value is considered for the damage parameter $\alpha_d$.

Fig. 4 compares the results (in terms of force-slip relationship) obtained in the cyclic test labelled as "A14" by Ko & Sato (2007) with the corresponding numerical simulations obtained by assuming the bond-slip law described in eq. (2). The agreement between experimental and numerical results is rather satisfactory, especially if it is kept in mind that no fine tuning of the relevant mechanical parameters was performed in this paper, but they were simply assumed in accordance to the values identified by Ko & Sato (2007) on monotonic tests.

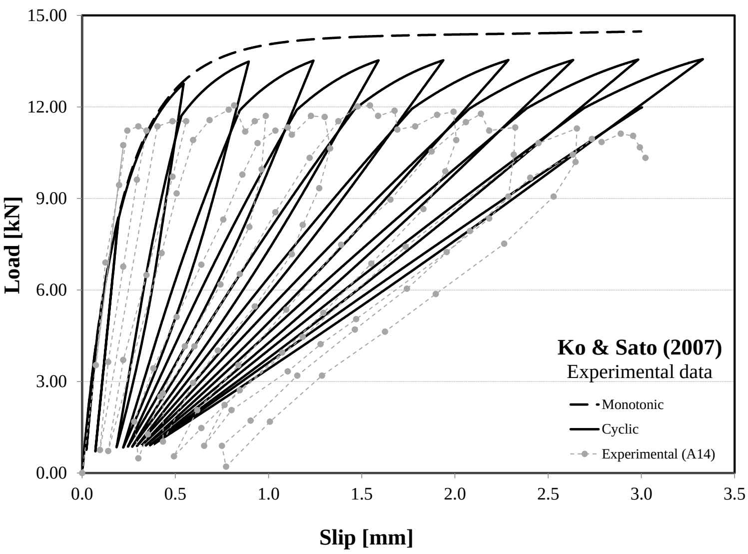


Figure 4.Load-slip response under monotonic and cyclic actions of FRP strips glued to concrete (Ko & Sato, 2007).

## 4 CONCLUDING REMARKS

This paper proposed the further development of a previous model already presented by the authors for simulating the cyclic behaviour of FRP-to-concrete interface. Particularly, the proposed model has been formulated within the framework of Fracture Mechanics and assumed two alternative expressions for the softening branch of the bond-slip relationship. The closed-form expressions obtained for determining the key damage-related quantities are among the novel and most attractive features of the present formulation. The comparison between some experimental results available in the literature and the numerical simulations performed by means of the present model highlighted the predictive potential of the latter. As a matter of principle, the proposed double exponential formulation of the bond-slip law should be more capable of other available laws (generally assuming an initial linear-elastic branch) for simulating both the low-cycle fatigue phenomenon (as demonstrated in this paper) and the high-cycle elastic fatigue. The overall validation of the proposed formulation will be proposed in the following developments of the present proposal that will also be aimed at covering the viscoelastic/viscoplastic behaviour of the adhesive interface, taking into account the actual rate of application of the dynamic actions and, hence, simulating the resulting strain-rate effect of the cyclic response.

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