

Design oriented Hoek-Brown strength criterion and strain models in frp confined columns

Z.Canan Girgin¹, Konuralp Girgin²

¹Yildiz Technical University Architecture Faculty Structural Systems Div., Istanbul, Turkey

²Istanbul Technical University Civil Engineering Faculty Structural Systems Div., Istanbul, Turkey

ABSTRACT: In this study, the Hoek-Brown strength criterion from rock mechanics, which was previously extended to FRP confined concrete from 7 MPa to 108 MPa, is revisited to propose design oriented models. Ultimate strengths and strains from low strength to ultra-high strength concrete (UHSC) in the wide range of unconfined strength (7 MPa to 190 MPa) are assessed via this modified strength criterion and other proposed strain models in axially loaded circular short columns. The assessment performance of the proposed strain models and the some models in the current literature are compared. Strength and strain predictions through new data for ultra-high performance concrete (UHPC) are included in this paper as well.

1 INTRODUCTION

FRP composites are used in the construction sector over two decades due to their properties such as high strength-to-weight ratio, high tensile strength and modulus, corrosion resistance and durability. FRP confinement through glass-fiber tube, frequently carbon, sometimes aramid, recently basalt sheets and some recycled materials were applied in the experiments so far (Samaan et. al., 1998; Rousakis, 2001; Dai et.al., 2011).

The experimental studies are often in the range $f_{co}=20-50$ MPa of unconfined compressive strength (Karbhari and Gao, 1997; Song et.al. 2013). Low strength data under 20 MPa (Ilki et.al., 2004; Pon et.al., 1998; Karbhari and Gao, 1997), and UHSC or UHPC data over 100 MPa (Miyachi, 1999; Berthet et.al. 2005; Cui and Sheikh, 2010; Zohrevand and Mirmiran, 2011; Guler et.al. 2013) are limited compared with the other range. High compressive strength levels in the construction sector enable to reduce member sizes. FRP wrapping prevents the inherent brittle behavior due to reducing confinement effectiveness of steel in UHSC or UHPC. UHPC has different stress-strain characteristics due to steel fiber content to prevent the fragile behavior.

The Hoek-Brown criterion (Hoek et.al. 1995) provides an advantage over Mohr-Coulomb criterion for the tensile stress predictions in the compression-tension region. The Hoek-Brown criterion was first extended to FRP-confined concrete by Girgin (2009) for the compressive strength range from 7 MPa to 108 MPa. This study focuses to propose to revisit this modified criterion and to develop new models covering stresses and strains. Database was updated (7 MPa to 190 MPa) with new data covering UHSC and UHPC from the current literature. Stresses and strains agree with experimental results.

2 STRENGTH AND STRAIN MODELS FOR FRP CONFINEMENT

2.1 Confinement with FRP

Under triaxial compressive stresses, the columns are subjected to major compressive stresses σ_1 uniformly applied along the axial axis of the column and minor principal stresses (lateral confining pressure) σ_3 (f_l) (Fig.1) enhancing the unconfined compressive strength of concrete (σ_c, f_{co}). Passive confinement pressure σ_3 (f_l) can be presently provided by FRP confinement (sheets and tubes) instead of steel confinement as well. f_l (σ_3) is expressed in terms of hoop tensile strength of FRP (σ_{frp} or f_{frp})

$$f_l = \sigma_3 = \frac{2t \sigma_{frp}}{D} \quad (1)$$

or in terms of FRP ultimate strain (ε_f)

$$f_l = \frac{2t E_f \varepsilon_f}{D} = \frac{1}{2} \rho_f E_f \varepsilon_f, \quad \rho_f = \frac{4t}{D} \quad (2)$$

where D, E_f, ρ_f, t denote the diameter of concrete core, Young's modulus, volumetric ratio and thickness of FRP, respectively.

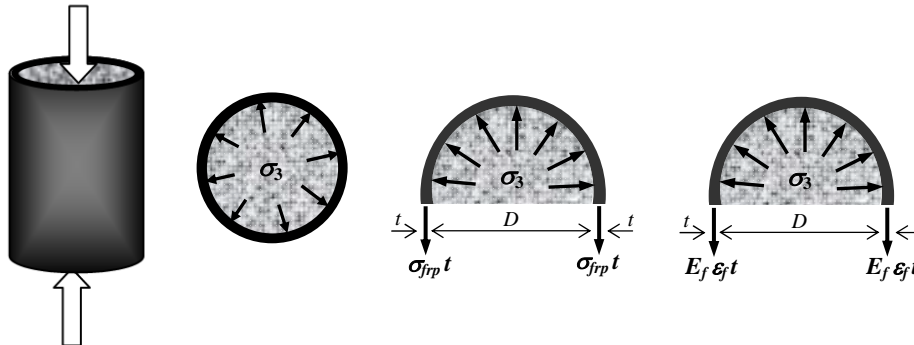


Figure 1 Development of confining pressure in FRP confined concrete

In the literature, several strength models are available, however the models predicting both stress and strain are more limited. Specific design oriented models assessing ultimate strengths and strains in circular short columns are given in Table 1.

Table 1. Strength and strain models in circular sections

Source	Strength model	Strain model
Fardis and Khalili (1981) -strength model based on Richart et.al.(1929) (f_{co} =20-50 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 4.1 \frac{f_l}{f_{co}}$	$\varepsilon_{cu} = \varepsilon_{co} + 0.0005 \frac{E_f}{f_{co}}$
Mander et al.(1988) ^a Saadatmanesh et.al.(1994)	$\frac{f_{cc}}{f_{co}} = 2.254 \sqrt{1 + 7.94 \frac{f_l}{f_{co}}} - 2 \frac{f_l}{f_{co}} - 1.254$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 5 \left(\frac{f_{cc}}{f_{co}} - 1 \right)$
ACI 440 (2008)		$\varepsilon_{cu} = \frac{1.71(5f_{cc} - 4f_{co})}{E_c}$
Karbhari and Gao (1997) (f_{co} = 38 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 2.1 \left(\frac{f_l}{f_{co}} \right)^{0.87}$ Model II	$\varepsilon_{cu} = \varepsilon_{co} + 0.01 \frac{f_l}{f_{co}}$
Kono et.al. (1998) (f_{co} =32-35 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 0.0572 f_l$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 0.28 f_l$

Saafi et. al. (1999) (f_{co} = 38 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 2.2 \left(\frac{f_l}{f_{co}} \right)^{0.84}$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + (537\varepsilon_f + 2.6) \left(\frac{f_{cc}}{f_{co}} - 1 \right)$
Spoelstra and Monti (1999) (f_{co} = 30-50 MPa)	$\frac{f_{cc}}{f_{co}} = 0.2 + 3 \left(\frac{f_l}{f_{co}} \right)^{0.5}$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 1.25 \frac{E_{co}}{f_{co}} \varepsilon_f \left(\frac{f_l}{f_{co}} \right)^{0.5}$
Xiao and Wu (2000) (f_{co} = 34-55 MPa, CFRP)	$\frac{f_{cc}}{f_{co}} = 1.1 + [4.1 - 0.75(f_{co}^2 / E_l)] \frac{f_l}{f_{co}}$	$\varepsilon_{cu} = \frac{\varepsilon_f - 0.0005}{7(f_{co} / E_l)^{0.8}}$
Toutanji (1999) -modified- (f_{co} = 31 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 2.3 \left(\frac{f_l}{f_{co}} \right)^{0.85}$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + (310.57\varepsilon_f + 1.9) \left(\frac{f_{cc}}{f_{co}} - 1 \right)$
Lam and Teng (2003) (f_{co} = 27-55 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 3.3 \frac{f_{l,a}}{f_{co}} \left(\frac{f_{l,a}}{f_{co}} \geq 0.07 \right)$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.75 + 12 \frac{f_{l,a}}{f_{co}} \left(\frac{\varepsilon_{h,u}}{\varepsilon_{co}} \right)^{0.45}$
Teng et.al. (2009) (f_{co} = 38-46 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + 3.5(\rho_k - 0.01)\rho_e \quad (\rho_k \geq 0.01)$ $\rho_k = \frac{2E_f t \varepsilon_{co}}{D f_{co}}, \rho_e = \frac{0.586 \varepsilon_f}{\varepsilon_{co}}$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.75 + 6.5 \rho_k^{0.8} \rho_e^{1.45}$
Benzaid et. al. (2010) (f_{co} = 29-62 MPa, CFRP)	$\frac{f_{cc}}{f_{co}} = 1 + 1.6 \frac{f_l}{f_{co}}, \frac{f_{cc}}{f_{co}} = 1 + 2.2 \frac{f_{l,a}}{f_{co}}$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 5.5 \frac{f_l}{f_{co}}, \frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 7.6 \frac{f_{l,a}}{f_{co}}$
Rousakis et.al. (2012) (f_{co} = 9-170 MPa)	$\frac{f_{cc}}{f_{co}} = 1 + \left(\frac{\rho_f E_f}{f_{co}} \right) \left(\frac{\alpha E_f 10^{-6}}{E_{f\mu}} + \beta \right)^a$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1 + 24.8 \frac{4E_f t}{D f_{co}} \left(\frac{-0.45E_f 10^{-6}}{E_{f\mu}} + 0.0223 \right) \left(\frac{40E_f t}{E_{f\mu} D} \right)^{-0.16}$
Ozbakkaloglu and Lim (2013)	$\frac{f_{cc}}{f_{co}} = 1 + 3.64 \frac{f_{l,a}}{f_{co}} \quad (CFRP)$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 17.41 \frac{f_{l,a}}{f_{co}} \quad (CFRP)$
	$\frac{f_{cc}}{f_{co}} = 1 + 2.64 \frac{f_{l,a}}{f_{co}} \quad (GFRP)$	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 2 + 24.47 \frac{f_{l,a}}{f_{co}} \quad (GFRP)$

^a $\rho_f = 4t/d$, $E_{f\mu} = 10$ MPa (for units compliance); $\alpha = -0.336$, $\beta = 0.0223$ for FRP sheets ; $\alpha = -0.23$, $\beta = 0.0195$ for FRP tube
 $Y = f_{cc}/f_{co} - 1$, $X = \rho_f E_f / f_{co}$, $Y = AX$, carbon: $A = 0.0151$ ($E_f = 234$ GPa), 0.0093 ($E_f = 377$ GPa), 0.0021 ($E_f = 640$ GPa), glass: $A = 0.0187$ ($E_f = 80.1$ GPa)

2.2 Strength Models

Earlier models for FRP confinement (Table 1) were proposed based on steel-confined concrete (Fardis and Khalili, 1981; Mander et.al. 1988; Saadatmanesh et.al. 1994). Most models for steel and FRP confined concrete are based on the Mohr-Coulomb criterion (Richart et.al. 1929; Saafi et.al. 1999; Spoelstra and Monti, 1999; Benzaid et.al. 2010).

Hoek-Brown and Johnston failure criteria from rock mechanics were firstly extended and modified for FRP confined concrete to precisely predict ultimate strength (Table 2) by including common FRP confinement types (CFRP, GFRP, AFRP sheets; and GFRP tube) in a wide range (7 to 108 MPa). The Integral Absolute Error (IAE) in two modified criteria is under 6% in the each specific range of overall averaged data ($n=135$). According to the verification results, these criteria can be successfully used for the reliable ultimate strength assessments. In the previous studies (Girgin 2009, 2014), the prediction performances of the strength models (Table 2 and other models) in the current literature were investigated based on each strength range. For this reason, herein, the assessment performances they will not be mentioned again. In passing, the performance assessment based on strength range may be more meaningful compared with investigating one error ratio (IAE, AAE etc.) through overall data in the current literature.

Recent HSC (40-100 MPa) and UHSC (100-150 MPa) data added to database were used to reverify m coefficient. The expressions of m coefficient (Table 2) were not changed, $m=0.1$ is valid from $f_{co}=82$ MPa to 114 MPa according to the comparisons in this study (Fig.2a). Meanwhile comparable UHSC data is available up to 113.6 MPa.

Table 2. Modified strength models from rock mechanics

	Failure criterion for rock	Modified form for FRP confinement of concrete
Girgin (2009)	Hoek-Brown (Hoek et.al. 1995)	$f_{cc} = f_l + (s f_{co}^2 + m f_{co} f_l)^{1/2}$
	$\sigma_1 = \sigma_3 + \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s \right)^{0.5}$ $\sigma_c \geq 20 \text{ MPa}$	$s=1$ for intact rock or undamaged concrete $m=2.9$ ($f_{co}=7 \text{ MPa}$ to 18 MPa) $m=6.34-0.076 f_{co}$ ($f_{co}=20 \text{ MPa}$ to 82 MPa) $m=0.1$ ($f_{co}=82 \text{ MPa}$ to 108 MPa)
Girgin (2014)	Johnston (Johnston, 1985)	$\frac{f_{cc}}{f_{co}} = \left(1 + \frac{M}{B} \cdot \frac{f_l}{f_{co}} \right)^B$
	$\frac{\sigma_1}{\sigma_c} = \left(1 + \frac{M}{B} \cdot \frac{\sigma_3}{\sigma_c} \right)^B$	$B = 1 - 0.0172 (\log f_{co})^2$, f_{co} in kPa $M = 0.0035 f_{co}^2 - 0.056 f_{co} + 2.83$ ($f_{co}=7 \text{ MPa}$ to 25 MPa) $M = 0.0003 f_{co}^2 - 0.076 f_{co} + 5.46$ ($f_{co}=25 \text{ MPa}$ to 108 MPa)

As for UHPC, a practical relationship is proposed by covering UHSC data as well as UHPC data (Miyachi et.al. 1999; Cui and Sheikh, 2010; Berthet et.al. 2005; Zohrevand and Mirmiran, 2011; Ozbakkaloglu and Vincent, 2013; Guler et.al.,2013) from $f_{co} = 108 \text{ MPa}$ to 188 MPa for $f_l/f_{co} = 0.07$ to 1.6 (Fig. 2b) and all FRP types mentioned above with high accuracy

$$f_{cc} = 163 \frac{f_l}{f_{co}} + 105 \quad (n=25, R=0.975, \text{IAE}=5.9\%, \text{AAE}=6.4\%) \quad (3)$$

in which IAE (Integral Absolute Error) and AAE (Average Absolute Error) are

$$\text{IAE} = \frac{\sum |o_i - p_i|}{\sum o_i}, \quad \text{AAE} = \frac{\sum |(o_i - p_i) / o_i|}{n}, \quad (o_i = \text{observed}, p_i = \text{predicted})$$

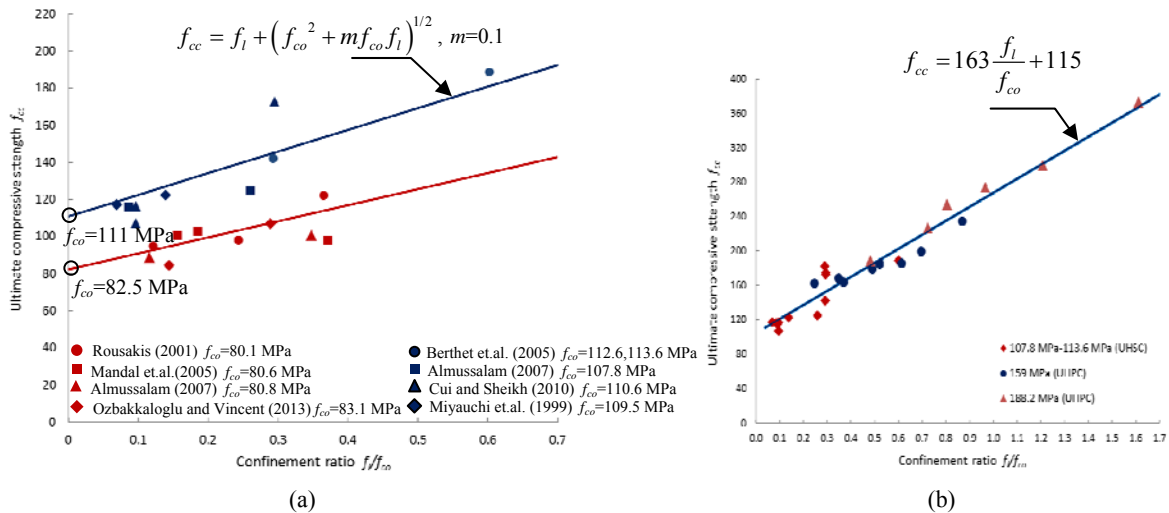


Figure 2 (a) Verification of modified Hoek-Brown modified criterion for very high compressive strength levels (b) Proposed linear relationship for UHSC and UHPC

2.3 Strain Models

The ultimate strain of confined concrete ε_{cu} is generally defined in terms of ε_{co} and as a function of f_l , f_l/f_{co} , f_{cc}/f_{co} , ε_f and some additional parameters such as elastic modulus of concrete (E_c), confinement modulus (E_l) or volumetric ratio (ρ_f). Meanwhile, there is a reduction factor (k) (Spoelstra and Monti, 1999; Pessiki et. al., 2001) between ε_f and the hoop rupture strain $\varepsilon_{h,u}$

$$\varepsilon_{h,u} = k\varepsilon_f \quad (4)$$

CFRP confined circular specimens cover the widest test data in the literature. For this reason, the database was mainly compiled with the experimental results of CFRP ($n=177$) wrapped circular specimens to enhance reliability. More limited data for GFRP wrapped specimens ($n=62$) were also included to the database. The available ultimate strain data are in the range $f_{co}=18$ MPa to 188 MPa. The following relationship in Eurocode 2 (2004) was used for the prediction of ε_{co}

$$\varepsilon_{co} = \frac{0.7}{1000} f_{co}^{0.31} \quad (5)$$

The assessment performances of some analytical models (Table 1) were compared with the experimental results in Fig. 3. It is noted that Ozbakkaloglu and Lim (2013)'s model is based on actual hoop rupture strain values ($\varepsilon_{h,u}$) and confining pressure ($f_{l,a}$) while other models used FRP ultimate strains (ε_f) and theoretical pressures (f_l). The assessment performance of any model in Table 1 was investigated based on the database not the range of that model. As almost all the models are deviated at or over about $f_{co}=100$ MPa, the experimental data in the model comparisons was limited with upper 110 MPa strength level. Only the proposed models in this study take all database into consideration.

Generally, a higher scattering in ultimate strains (ε_{cu}) in the database is under consideration compared with those of ultimate strengths (f_{cc}). In this study, to compare the actual models and to develop practical relationships, some test results significantly deviated from the general trend were discarded from the analyses. The main target is to propose practical models (Table 3) through main parameters (f_l , f_l/f_{co} , f_{cc}/f_{co}) predicting the ultimate strains with high accuracy by using currently available fiber or jacket data. It is noted there are discontinuities in the strength ranges of limited GFRP wrapped specimens.

Table 3 The proposed strain models developed in this study

Parameter	Relationship	Data range	IAE %	AAE %	Sheet type
f_l	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 4.3 * 10^{-5} f_l^3 - 0.008 f_l^2 + 0.57 f_l + 1$	15 MPa \leq f_{co} < 50 MPa	15.8	15.9	CFRP
			16.4	17.6	GFRP
f_l/f_{co}	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = -4.24 \left(\frac{f_l}{f_{co}}\right)^2 + 15.42 \left(\frac{f_l}{f_{co}}\right) + 2.23$	15 MP a \leq f_{co} \leq 50 MPa 0.01 \leq f_l/f_{co} \leq 8.1	See Fig. 3		
	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = -2.62 \left(\frac{f_l}{f_{co}}\right)^2 + 10.94 \left(\frac{f_l}{f_{co}}\right) + 1.0$	50 MPa < f_{co} \leq 109 MPa (CFRP) 0.07 \leq f_l/f_{co} \leq 1.14			
	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = -3.42 \left(\frac{f_l}{f_{co}}\right)^2 + 12.7 \left(\frac{f_l}{f_{co}}\right) + 0.27$	80 MPa < f_{co} \leq 159 MPa* (GFRP) 0.10 \leq f_l/f_{co} \leq 0.61			
	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 0.57 \left(\frac{f_l}{f_{co}}\right) + 1$	109 MPa < f_{co} \leq 170 MPa* (CFRP) 0.07 \leq f_l/f_{co} \leq 8.14			
f_{cc}/f_{co}	$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 4.46 \left(\frac{f_{cc}}{f_{co}}\right) - 2.14$	15 MPa < f_{co} \leq 103 MPa (CFRP) 1.02 \leq f_{cc}/f_{co} \leq 4.24	15.4	14.2	CFRP

* UHPC data was also included.

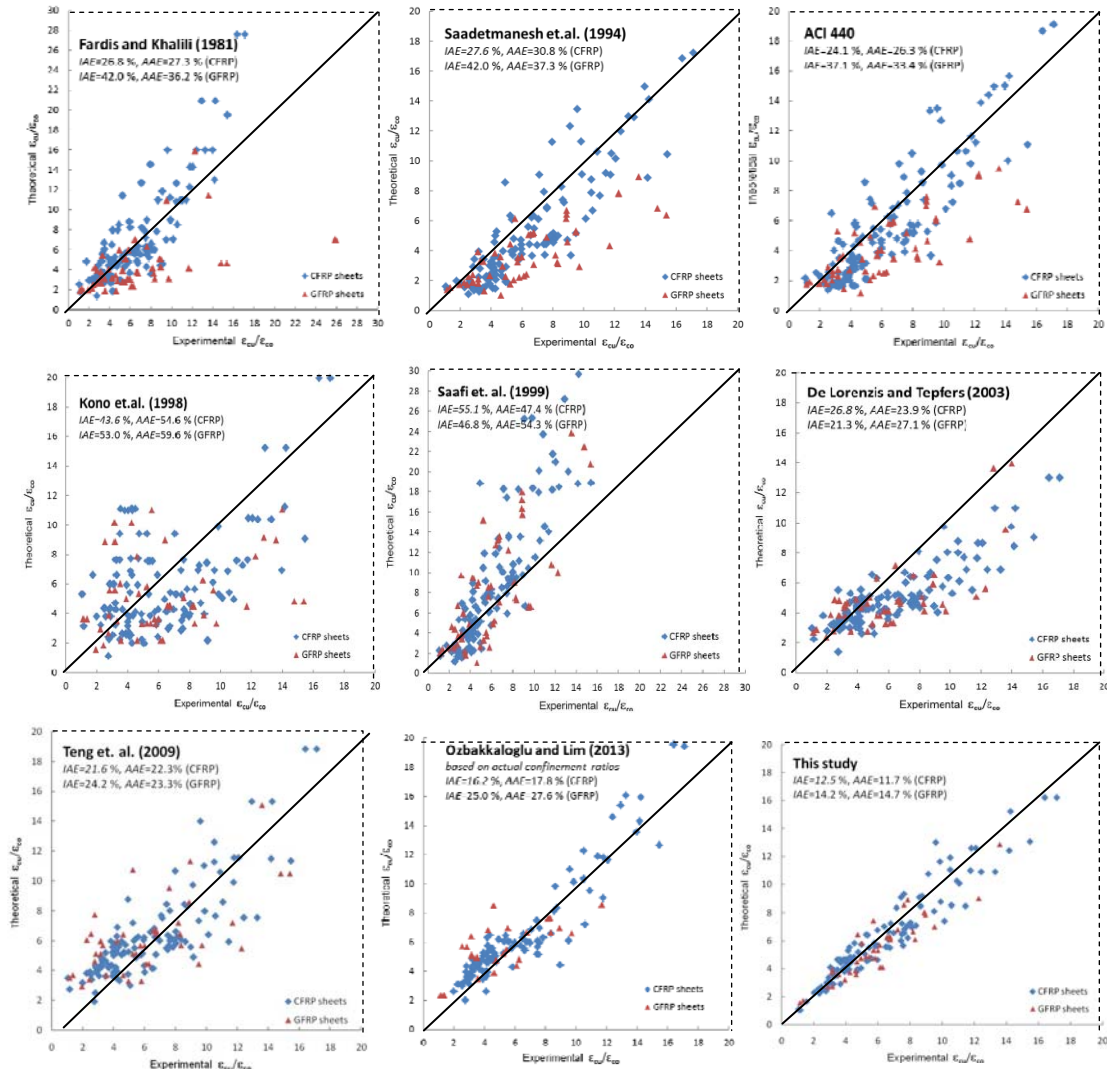


Figure 3 Assessment performance of models for the short columns confined with CFRP and GFRP wraps in the current literature and proposed model

It is seen from Fig. 3 that the developed strain model in this study may predict the ultimate strains of CFRP and GFRP confined specimens.

3. RESULTS

In this study, the modified Hoek-Brown criterion is revisited and generalized with other stress and strain models developed in this study to propose a design oriented model for FRP confined circular short columns. Ultimate strength and strain estimations are included into this paper. A database of very wide-range of unconfined strength (7 to 190 MPa) are used in the analyses. The successful estimations for ultra-high strength (UHSC) and ultra-high performance concrete (UHPC) are considered in this paper as well.

References

- ACI 440.2R-08, American Concrete Institute (ACI) Committee, 2008, Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures, Farmington Hills.
- Almussalam, TH. 2007, Behavior of normal and high-strength concrete cylinders confined with E-Glass/epoxy composite laminates, *Composites, Part B*, 38: 629-639.
- Benzaid, R., Mesbah, H. and Chikh, N., 2010, FRP-confined concrete cylinders: axial compression experiments and strength model, *J Reinforced Plastics and Composites*, 29(16):2469–88.
- Berthet, JF, Ferrier, E, and Hamelin, P. 2005, Compressive behavior of concrete externally confined by composite jackets. Part A: Experimental Study, *Construction and Building Materials*, 19(3):223-232.
- Campione, G., and Miraglia N. 2003, Strength and strain capacities of concrete compression members reinforced with FRP, *Cement & Concrete Composites*, 25: 31–41.
- Chikh N., Benzaid R., and Mesbah H. 2012, An experimental investigation of circular RC columns with various slenderness confined with CFRP Sheets, *Arab J Science Engineering*, 37:315–323.
- Chu, G.D. 1998. The Technology and application of composites in the reinforcement of structures in civil engineering. *Technical Conference on Reinforcement of RC Structure*, Industry Technology Research Institute, Taipei, Taiwan, 45 p.
- Dai, JG, Bai, YL, and Teng, JG. 2011, Behaviour and modeling of concrete confined with FRP composites of large deformability. *ASCE Journal of Composite Construction*, 15:963–973.
- Cui, C. and Sheikh, A. 2010, Experimental study of normal- and high-strength concrete confined with fiber-reinforced polymers, *ASCE Journal of Composite Construction*, 14(5):553–61.
- Eurocode 2: Design of Concrete Structures : Part 1–1 : General Rules and Rules for Buildings; British Standards Institution: Brussels, Belgium, 2004.
- Fardis MN., and Khalili H. 1981, Concrete encased in fiberglass-reinforced plastic, *ACI Structural Journal*, 78:440-445.
- Girgin, ZC. 2009, A modified failure criterion to predict ultimate strength of circular columns confined by different materials. *ACI Structural Journal*, 106(6):800-809.
- Girgin, ZC. 2014, Modified Johnston failure criterion from rock mechanics to predict the ultimate strength of fiber reinforced polymer (FRP) confined columns. *Polymers*, 6:59-75.
- Guler S., Copur A., and Aydogan M., 2013, Nonlinear finite element modeling of FRP-wrapped UHPC columns, *Computers and Concrete*, 12(4): 413-429.
- Hoek, E, Kaiser, PK., and Bawden, WF. 1995. *Support of Underground Excavations in Hard Rock*, A.A. Balkema, Rotterdam.
- Howie, I and Karbhari, VM. 1994, Effect of materials architecture on strengthening efficiency of composite wraps for deteriorating columns in the North-East. *Proc. of ASCE 3rd Materials Engineering Conf. Infrastructure: New Materials and Methods of Repair*, San Diego, 199-206.
- Ilki, A, and Kumbasar, N. 2003, Compressive behaviour of carbon fibre composite jacketed concrete with circular and non-circular cross-sections. *Journal of Earthquake Engineering*, 7(3):381-406.
- Ilki, A., Kumbasar, N. and Koc, V. 2004, Low strength concrete members externally confined with FRP sheets, *Structural Engineering and Mechanics*, 18(2):167-194.
- Johnston, IW. 1985, Strength of intact geomechanical materials. *ASCE Journal of Geotechnical Engineering Division*, 111(6):730-748.
- Karabinis, AI, and Rousakis, TC. 2002, Concrete confined by FRP material: A plasticity approach. *Engineering Structures*, 24(7):923-932.
- Karbhari, VM, and Gao, Y. 1997, Composite jacketed concrete under uniaxial compression—verification of simple design equations, *ASCE Journal of Materials in Civil Engineering*, 9(4):185–193.
- Kono S., Inazumi M., and Kaku T., 1998, Evaluation of confining effects of CFRP sheets on reinforced concrete members, *In: Proc 2nd int conf on composites in infrastructures*.
- Lam, L, and Teng, JG. 2004, Ultimate condition of FRP-confined concrete. *ASCE Journal of Composites for Construction*, 8(6):539–548.
- Lam, L., and Teng, JG. 2003, Design-oriented stress-strain model for FRP-confined concrete, *Construction and Building Materials*, 17(6-7):471–489.
- Lin, HL. and Liao, CI. 2004, Compressive strength of reinforced concrete-column confined by composite materials. *Composite Structures*, 65:239–250.

- Mandal S., Hoskin A., and Fam A., 2005, An experimental investigation of circular RC columns with various slenderness confined with CFRP sheets, *ACI Structural Journal*, 102(3):383-392.
- Mander, JB., Priestley JN., and Park, R. 1988, Theoretical stress-strain model for confined concrete, *Journal of Structural Engineering*, ASCE, 114(8): 1804-1826.
- Miyauchi, K., Nishibayashi, S., and Inoue, S. 1997, Estimation of strengthening effects with carbon fiber sheet for concrete column. *Proc., FRPRCS-3*, Sapporo, Japan, 1: 217-224.
- Ozbakkaloglu T., and Vincent T., 2013, Axial compressive behavior of circular high-strength concrete-filled FRP tubes, *Journal of Composites for Construction ASCE*, 18(2):1-11.
- Pessiki, S, Harries, KA., Kestner, JT., Sause, R. and Ricles, JM. 2001, Axial behaviour reinforced concrete columns confined with FRP jackets, *Journal of Composites for Construction*, ASCE, 5(4):237-245.
- Pon, TH, Li, YF, Shih, BJ, Han, MS et. al. 1998, Experiments of scale effects on the strength of FRP reinforced concrete. [in Chinese] *In: Proc. of the 4th National Conference on Structural Engineering*, Taipei, Taiwan, 2133–2140.
- Richart, E, Brandtzaeg, A, and Brown, RL. 1929, Failure of plain and spirally reinforced concrete in compression. *Bulletin 190*, University of Illinois, Champaign, Illinois.
- Rochette, P, and Labossière, P. 2000, Axial testing of rectangular column models confined with composites. *ASCE Journal of Composites for Construction*, 4(3):129–136.
- Rousakis, T., 2001, Experimental investigation of concrete cylinders confined by carbon FRP sheets, under monotonic and cyclic axial compressive load, *Research Report No. 44*, Division of Building Technology, Chalmers University of Technology, Göteborg, Sweden: 87 pp.
- Rousakis, TC., Rakitzis, TD., Karabinis, AI. 2012, Design-oriented strength model for FRP-confined concrete members. *ASCE Journal of Composites Construction*, 16:615-625.
- Saadatmanesh, H.; Ehsani, MR. and Li, MW. 1994, Seismic retrofit of circular bridge columns for enhanced flexural performance, *ACI Structural Journal*, 91(4):434-447.
- Saafi, M., Toutanji, HA., and Li, Z. 1999, Behaviour of concrete columns confined with fiber reinforced polymer tubes. *ACI Material Journal*, 96(4):500–509.
- Samaan, M, Mirmiran, A, and Shahawy, M. 1998, Modeling of concrete confined by fiber composites. *ASCE Journal of Structural Engineering*, 124(9):1025–1031.
- Song, X., Gu, X., Li, Y., Chen, T., and Zhang W., 2013, Mechanical behavior of FRP-strengthened concrete columns subjected to concentric and eccentric compression loading. *Journal of Composites for Construction ASCE*, 17(3): 336-346.
- Spoelstra, MR, and Monti, G. 1999, FRP-confined concrete model. *ASCE Journal of Composites for Construction*, 3(3):143–150.
- Teng, J., Jiang, T., Lam, L., and Luo, Y. 2009, Refinement of a design-oriented stress–strain model for FRP-confined concrete, *ASCE Journal of Composite Construction*, 13(4):269–278.
- Vincent, T. and Ozbakkaloglu, T., 2013, Influence of concrete strength and confinement method on axial compressive behavior of FRP confined high- and ultra high-strength concrete. *Composites, Part B*, 50: 413–428.
- Thériault, M, Neale, KW, and Claude, S. 2004, Fiber-reinforced polymer-confined circular concrete columns: Investigation of size and slenderness effects. *ASCE Journal of Composites for Construction*, 8(4):323-331.
- Toutanji, HA. 1999, Stress–Strain Characteristics of concrete columns externally confined with advanced fibre composite sheets. *ACI Material Journal*, 96(3):397–402.
- Triantafillou, TC, Papanicolaou, CG, Zissimopoulos, P, and Laourdekis, T. 2006, Concrete confinement with textile-reinforced mortar jackets. *ACI Structural Journal*, 103(1):28-37.
- Xiao, Y, and Wu, H. 2000, Compressive behavior of concrete confined by carbon fiber composite jackets. *ASCE Journal of Materials in Civil Engineering*, 12(2):139–146.
- Watanabe, K, Nakamura, H, Honda, T et. al. 1997, Confinement effect of FRP sheet on strength and ductility of concrete cylinders under uniaxial compression. *Proc. of 3rd Int. Symposium on Non-Metallic (FRP) Reinforcement for Concrete Structures*. Sapporo, Japan: 233–240.
- Zohrevand P., and Mirmiran A., 2011, Behavior of ultrahigh-performance concrete confined by fiber-reinforced polymers, *Journal of Materials in Civil Engineering ASCE*, 23(12): 1727-1734.