

## Statistical evaluation of maximum displacement demands of SDOF systems using code-compatible ground motion records sets

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**ABSTRACT:** Due to rapidly increasing computational power and the evolution of engineering software, nonlinear dynamic analysis is becoming a common practice for seismic design and performance evaluation of structures. It is important to select appropriate ground motions since dynamic analysis results are depend on the used ground motions. According to current consensus between modern seismic codes, acceptable ground motions should be compatible with code-prescribed design spectrum. The aim of this study is to evaluate the statistical distribution of maximum displacement demands of single degree of freedom (SDOF) systems using Turkish Earthquake Code (TEC) compatible ground motion record sets. Three different ground motion sets obtained for soil class Z3 defined in TEC. Elastic-perfectly plastic (EPP) hysteresis behavior is considered for SDOF systems. 15 SDOF systems with various natural period and strength ratio are used for analysis. Results show that although each of the three ground motion sets are code-compatible, different displacement demands for SDOF systems may be obtained.

### 1 INTRODUCTION

Performance-based design methods have been commonly used for seismic design, evaluation and rehabilitation of structures. Using these methods, engineers can estimate better the maximum inelastic deformation demands under an earthquake ground motion. Some studies for the evaluation and rehabilitation of existing structures have proposed simplified analysis methods in which single degree of freedom (SDOF) systems are used to estimate maximum inelastic displacement demands on structures. For example, Ruiz-Garcia and Miranda (2010) proposed a probabilistic approach to estimate maximum displacement of the SDOF systems for performance based design. In FEMA-356 (2000), four analytical methods are presented to assess the seismic demands of buildings. Hatzigeorgiou and Beskos (2009) suggest a new empirical equation for prediction of nonlinear displacement demand under the repeated earthquake ground motions.

During the last decades, linear and nonlinear dynamic analysis have been made feasible for structures with lots of degree of freedom, thanks to rapidly increasing computational power and the evolution of engineering software (Katsanos et al. 2010). The selection of earthquake ground motions is an important step for nonlinear dynamic analysis because it strongly affects the result of building's response. Generally, three different types of ground motions are used for dynamic analyses (Bommer and Acevedo 2004). These are artificial, simulated, and real ground motions. In many of current seismic codes, the regional hazard characteristics are described

through design spectrum. According to these codes, acceptable ground motions should be compatible with code-prescribed design spectrum (Eurocode-8 2003, TEC 2007 etc.).

The purpose of this paper is to evaluate the distribution of displacement demands of SDOF systems using different code-compatible ground motion record sets. For this purpose, three different code-compatible ground motion sets used considering soil class Z3 defined in TEC. Each set has 10 different ground motion records. Dynamic analysis of 15 SDOF systems, which have various period and strength ratio, are performed using Prism 1.0.2 (Jeong et al. 2010) and maximum displacement demands are obtained. The mean, standard deviation and coefficient of variation of the displacement demands are calculated for each set and compared. In addition, the mean values of displacement demands obtained using different ground motion sets are compared.

## 2 STRUCTURAL MODEL

### 2.1 Single Degree of Freedom Systems

Degree of freedom is the number of parameters necessary for determining the position in the case of the structure of the vibration. A single degree of freedom system (SDOF) requires only one parameter for describe its position at any instant of time. Basic components of a SDOF system are stiffness ( $k$ ), mass ( $m$ ), damping coefficient ( $c$ ) and external forces (Figure 1). Herein damping mechanism represents the energy consumption and ground motions ( $x_g$ ) represent the external forces. Determine the dynamic behavior of the SDOF systems need to solve equations of motions (Eq. 1).

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{x}_g(t) \quad (1)$$

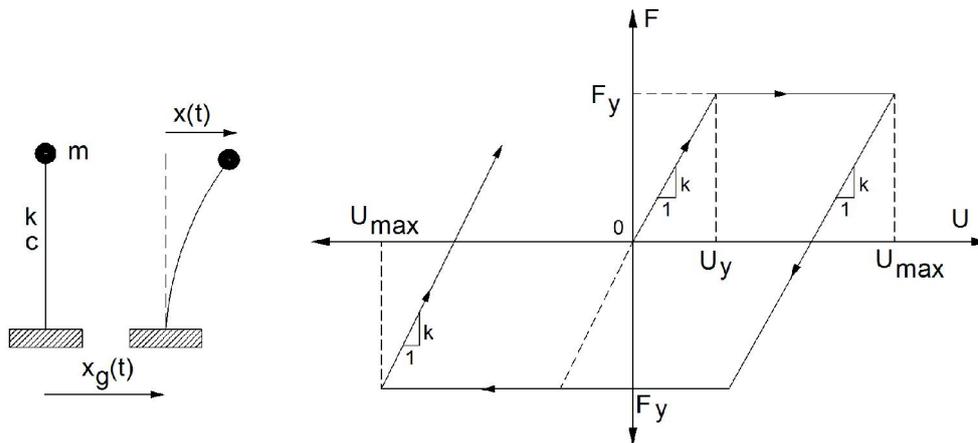


Figure 1. Representation of a SDOF system and EPP hysteresis model.

In this study, nonlinear behaviors of the SDOF systems are considered using the elastic perfectly plastic (EPP) hysteresis model (Figure 1). This is generally used as a reference model (FEMA-356 2000). The force-displacement relationship of the EPP hysteresis model is totally specified through three parameters:  $k$  is initial stiffness,  $u_y$  is yield displacement and  $F_y$  is yield strength. As can be seen in Figure 1, force-displacement relationship is linear for lower displacements than  $u_y$ . If the displacements exceed  $u_y$ , plastic deformations start. During unloading, elastic deformations returned but plastic deformations are not. Stiffness does not change during loading or reloading.

15 different SDOF systems with various periods ( $T$ ) between 0.20s and 0.40s (0.20s, 0.25s, 0.30s, 0.35s, 0.40s) and strength ratios ( $F_y/W$ ) between 0.10 and 0.30 (0.10, 0.20, 0.30) are considered for nonlinear dynamic analysis. Damping ratio of SDOF systems are assumed as 5% of critical damping.

### 3 GROUND MOTION RECORD SETS

#### 3.1 Design spectrum in TEC

In TEC, elastic design spectrum with 5% of damping is defined by Eq.2 for different type of soil classes.

$$S(T) = \begin{cases} 1 + 1.5 \frac{T}{T_A} & 0 \leq T \leq T_A \\ 2.5 & T_A \leq T \leq T_B \\ 2.5 \left( \frac{T_B}{T} \right)^{0.8} & T_B \leq T \end{cases} \quad (2)$$

In Eq. 2,  $S(T)$  is the ordinate of the response spectrum,  $T$  is the natural period of structure.  $T_A$  and  $T_B$  are spectrum characteristic periods defined depending on the soil class in TEC. They are also defined as limiting periods of the constant spectral acceleration branch. For soil class Z3,  $T_A$  and  $T_B$  are 0.15s and 0.60s, respectively. According to TEC, territories are to be subdivided by the national authorities into seismic zones, depending on the local hazard level. Local hazard level is represented by effective ground acceleration coefficient  $A_0$ . In this study,  $A_0$  is proposed as 0.40g by TEC for seismic zone 1. Spectral acceleration coefficient,  $A(T)$ , to be considered for calculating seismic loads, is given in Eq. 3. In Eq. 3,  $I$  is building importance factor depending on the purpose of occupancy or type of building. According to TEC,  $I=1.0$  for residential and office buildings.

$$A(T) = A_0 I S(T) \quad (3)$$

In Figure 2,  $A(T)$  which is defined for soil class Z3 and seismic zone 1 in TEC is given.

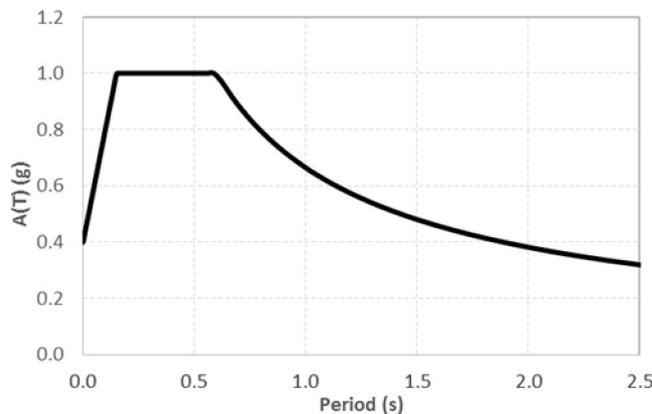


Figure 2. Elastic spectral acceleration for soil class Z3 and seismic zone 1

### 3.2 TEC definitions for input ground motion record selection

In TEC, outlines the requirements for the seismic input for dynamic analysis are given in Section 2.9. TEC allows using recorded, artificial and simulated accelerograms sets for earthquake response history analysis if the following criteria are satisfied:

- 1) The duration of strong ground motion, shall not be less than 5 times the first natural vibration period of the building and 15 seconds.
- 2) The mean of the zero-period spectral response acceleration values (calculated from the individual time histories) are not less than the value of  $A_{0g}$ .
- 3) In the range of periods between  $0.2T$  and  $2.0T$ , where  $T$  is the fundamental period of the structure in the direction along which the accelerograms will be applied, the mean spectral accelerations for 5% damping ratio shall not be less than 90% of the corresponding elastic design spectrum.
- 4) The mean value of structural response quantities from all of the analyses could be used if at least seven ground motion records are used in nonlinear dynamic analysis; otherwise, maximum value of structural response quantities should be used.

### 3.3 Selected ground motion records for analysis

Considering TEC requirements, three ground motion record sets are obtained and used for nonlinear dynamic analyses. In order to obtain record sets, the algorithm proposed by Kayhan (2012) is used. Information about obtained record sets is given below:

- 1) There are ten scaled ground motion records in each set.
- 2) A horizontal components of a ground motion is not used more than once in a set.
- 3) The scaling factor is between 0.50 and 2.00 for ground motion records.
- 4) Records are selected from the European Strong Motion Database (Ambraseys et al. 2004)
- 5) The records, obtained on sites with matching soil class C according to Eurocode-8, are used to develop ground motion record sets.
- 6) In this study, the ratio of the mean spectrum value to the target design spectrum is limited between 0.90 and 1.10 over a period range of  $0.2T$  and  $2.0T$ . Note that the period of SDOF systems are between 0.20s-0.40s. Thus, the period range of 0.04s-0.80s is selected for compatibility.

The details about three ground motion datasets compatible with elastic design spectrum are given in Table 1-Table 3. In the tables, records labels including horizontal component indices, scaling factors, earthquake names, magnitude of earthquakes, date of earthquakes and label of stations can be seen.

Table 1. Information about selected ground motion data for Set 1

Record	Scale	Earthquakes	$M_w$	Date	Station
168Y	0.546	Calabria	5.2	11.03.1978	ST44
374X	0.546	Lazio Abruzzo	5.9	07.05.1984	ST148
546Y	1.886	Mataranga	5.2	30.05.1992	ST10
2042X	1.011	Kyllini (aftershock)	4.5	22.10.1988	ST172
4343X	1.131	Izmit	7.6	17.08.1999	ST2574
5679Y	0.928	Montenegro	6.9	15.04.1979	ST2967
6920X	1.146	Duzce 1	7.2	12.11.1999	ST3257
6926Y	1.822	Duzce 1	7.2	12.11.1999	ST3263
7006X	0.575	Izmit (aftershock)	5.6	11.11.1999	ST777
7129Y	1.129	Seferihisar	5.7	10.04.2003	ST858

Table 2. Information about selected ground motion data for Set 2

Record	Scale	Earthquakes	$M_w$	Date	Station
168Y	1.543	Calabria	5.2	11.03.1978	ST44
430X	1.025	Gulf of Corinth	4.9	05.07.1988	ST121
546Y	0.763	Mataranga	5.2	30.05.1992	ST10
602X	0.726	Umbria Marche	6.0	26.09.1997	ST224
1415Y	1.117	Pulumur	5.2	15.03.1992	ST724
1705Y	1.574	Duzce 1	7.2	12.11.1999	ST576
6926Y	1.886	Duzce 1	7.2	12.11.1999	ST3263
6927X	0.804	Duzce 1	7.2	12.11.1999	ST3264
7200Y	1.405	Avej	6.5	22.06.2002	ST184
7209Y	1.790	Avej	6.5	22.06.2002	ST3332

Table 3. Information about selected ground motion data for Set 3

Record	Scale	Earthquakes	$M_w$	Date	Station
151X	0.711	Friuli (aftershock)	6.0	15.09.1976	ST33
371Y	1.830	Lazio Abruzzo	5.9	07.05.1984	ST145
479X	0.560	Manjil	7.4	20.06.1990	ST188
480Y	0.709	Manjil	7.4	20.06.1990	ST189
546Y	1.476	Mataranga	5.2	30.05.1992	ST10
1769X	1.327	Cerkes (aftershock)	5.6	14.08.1996	ST592
1958Y	1.998	Kyllini (foreshock)	4.7	30.09.1988	ST214
3726Y	1.990	Duzce 1	7.2	12.11.1999	ST775
6927Y	2.000	Duzce 1	7.2	12.11.1999	ST3264
7010Y	1.782	Izmit (aftershock)	5.6	11.11.1999	ST772

In Figure 3, the compatibility between the average spectra of three ground motion sets and target spectrum is given. It should be noted that, the ratio of the average spectrum value to the target design spectrum is limited between 0.90 and 1.10 over a period range of 0.04s and 0.80s.

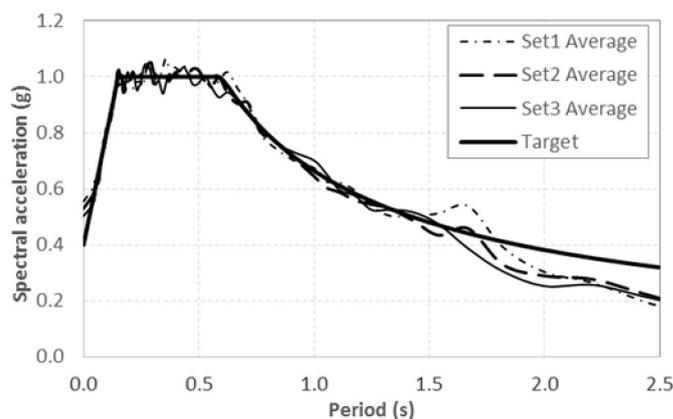


Figure 3. Average spectra of ground motion sets and target spectrum.

#### 4 ANALYSIS RESULTS

As mentioned earlier, 15 SDOF systems with three different lateral strength ratios (0.10, 0.20, 0.30) and five different natural periods (0.20s, 0.25s, 0.30s, 0.35s, 0.40s) were considered in this

study. Totally 450 nonlinear dynamic analyses were performed and maximum displacement demands for SDOF systems were calculated.

First of all, the distribution of maximum displacement demands of SDOF systems in each record set was evaluated. For this, the mean ( $\mu$ ) and standard deviation ( $\sigma$ ) of displacement demands obtained for each ground motion in a set were calculated. Then, coefficient of variation ( $CoV$ ) of demands, defined as ratio of the  $\sigma$  to the  $\mu$ , was calculated. In Table 4, basic descriptive statistics about displacement demands of SDOF systems for each set are given. As can be seen, the values of  $CoV$  for each set and each SDOF system are remarkable. Almost all of the  $CoV$  values are bigger than 1.00 and the minimum value of  $CoV$  is 0.91. These results indicate that the dispersion around the mean of displacement demands of SDOF systems in the set is very high regardless of  $T$  and  $F_y/W$ . It can be said that this is due to the nature of earthquakes and response of structural systems to earthquakes.

Table 4: Basic descriptive statistics about displacement demands of SDOF systems for each set

T (s)	$F_y/W$	Set 1			Set 2			Set 3		
		$\mu$ (cm)	$\sigma$ (cm)	$CoV$	$\mu$ (cm)	$\sigma$ (cm)	$CoV$	$\mu$ (cm)	$\sigma$ (cm)	$CoV$
0.20	0.10	8.76	10.68	1.22	8.81	8.25	0.94	8.05	8.49	1.06
	0.20	6.98	10.49	1.50	6.29	7.35	1.17	6.57	7.85	1.19
	0.30	5.84	9.62	1.65	4.28	5.54	1.29	4.52	4.31	0.96
0.25	0.10	9.84	12.52	1.27	9.79	9.39	0.96	9.00	9.65	1.07
	0.20	8.82	13.04	1.48	7.45	8.38	1.12	8.08	10.13	1.25
	0.30	6.81	10.85	1.59	5.26	6.39	1.21	5.11	5.22	1.02
0.30	0.10	10.81	13.08	1.21	11.11	10.75	0.97	9.95	10.86	1.09
	0.20	9.62	13.89	1.44	8.22	8.62	1.05	9.70	11.95	1.23
	0.30	9.13	15.72	1.72	5.99	6.33	1.06	6.60	7.20	1.09
0.35	0.10	12.01	13.56	1.13	12.68	12.23	0.96	11.27	11.90	1.06
	0.20	10.41	13.10	1.26	9.77	9.33	0.95	10.84	12.62	1.17
	0.30	10.19	15.92	1.56	7.43	6.86	0.92	7.82	7.94	1.01
0.40	0.10	12.79	14.14	1.11	13.32	13.10	0.98	11.50	11.59	1.01
	0.20	11.36	12.39	1.09	10.60	10.73	1.01	11.32	13.63	1.20
	0.30	10.49	13.84	1.32	7.87	7.19	0.91	8.52	8.34	0.98

According to TEC, the mean value of structural response quantities from all of the analyses could be used if at least seven ground motion records are used in nonlinear response history analysis; otherwise, maximum value of structural response quantities should be used. In this study, each set has ten ground motions. Thus, the mean values of displacement demands of SDOF systems (Table 4) can be considered for design and/or performance evaluation.

The mean values of displacement demands obtained SDOF systems ( $\mu$ ) are compared as the second statistical evaluation study. For each SDOF system, three values of  $\mu$  are calculated for three sets. As can be seen from the Table 4, different values of  $\mu$  are calculated for SDOF systems using different ground motion sets.

In Figure 4, the means of displacement demands of SDOF systems with  $F_y/W=0.10$  value are given. For five different values of  $T$ , the mean values belonging to sets are comparatively close but different from each other. For example, considering  $T=0.20$ s, the mean values of the demands are 8.76cm, 8.81cm and 8.85cm for Set 1, Set 2 and Set 3, respectively. An another example, considering  $T=0.40$ s, the mean values of the demands are 12.79cm, 13.32cm and 11.50cm for Set 1, Set 2 and Set 3, respectively.

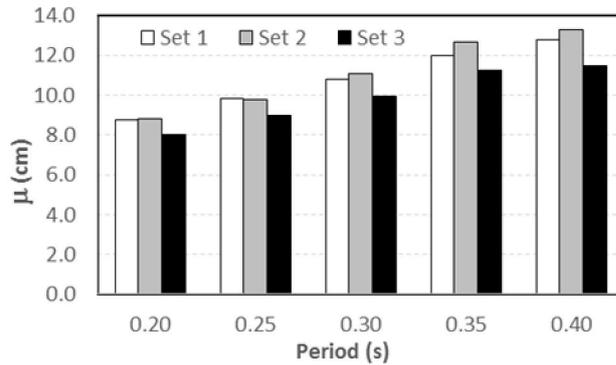


Figure 4. The mean of the maximum displacement demands ( $F_y/W=0.10$ )

In Figure 5, the means of displacement demands of SDOF systems with  $F_y/W=0.20$  value are given. Similarly Figure 4, it can be said that the mean values belonging to sets are comparatively close but different from each other. For example, considering  $T=0.25$ s, the mean values of the demands are 8.82cm, 7.45cm and 8.08cm for Set 1, Set 2 and Set 3, respectively. An another example, considering  $T=0.40$ s, the mean values of the demands are 11.36cm, 10.60cm and 11.32cm for Set 1, Set 2 and Set 3, respectively.

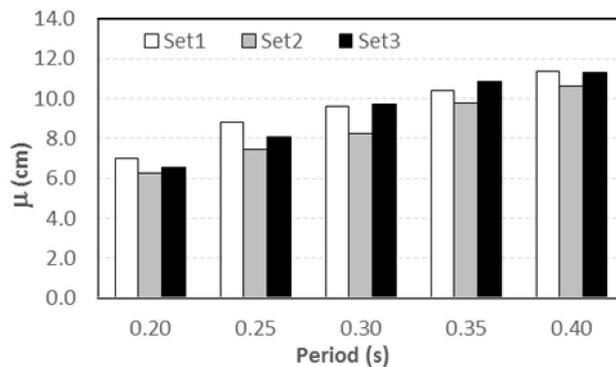


Figure 5. The mean of the maximum displacement demands ( $F_y/W=0.20$ )

In Figure 6, the means of displacement demands of SDOF systems with  $F_y/W=0.30$  value are given. As can be seen, the mean values of displacement demands for Set 1 are relatively bigger than those for the other sets. For example, considering  $T=0.30$ s, the mean values of the demands is 9.13cm for Set 1, where 5.99cm ve 6.60cm for Set 2 and Set 3, respectively.

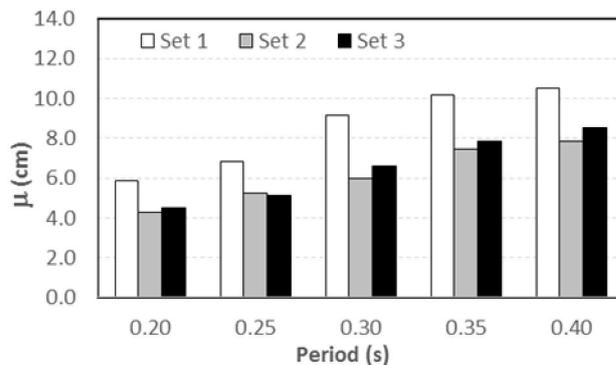


Figure 6. The mean of the maximum displacement demands ( $F_y/W=0.30$ )

## 5 CONCLUSION

In this study, the distribution of displacement demands of SDOF systems using different code-compatible ground motion record sets was evaluated. For this purpose, three different code-compatible ground motion sets were used considering soil class Z3 defined in TEC. Nonlinear dynamic analysis of 15 SDOF systems, which have various period ( $T$ ) and strength ratio ( $F_y/W$ ), were performed and maximum displacement demands were obtained. The mean ( $\mu$ ), standard deviation ( $\sigma$ ) and coefficient of variation ( $CoV$ ) of the displacement demands were calculated for each set and compared. In addition, the mean values of displacement demands ( $\mu$ ) obtained using three different ground motion sets were compared. The following conclusions were found from the results of this study.

According to values of  $CoV$  for each ground motion sets, the dispersion around the mean of maximum displacement demands of SDOF systems in a set is very high regardless of  $T$  and  $F_y/W$ . Almost all of the  $CoV$  values are bigger than 1.00 and the minimum value of  $CoV$  is 0.91. It can be said that this is due to the nature of earthquakes and response of structural systems to earthquakes.

Considering mean values of displacement demands of SDOF systems for three ground motion sets, three mean values are comparatively close to each other but different. It can be said that this is valid for SDOF systems with  $F_y/W=0.10$  and  $0.20$ . For SDOF systems with  $F_y/W=0.30$ , the mean values obtained from the sets are a bit different. Note that ground motion record sets are compatible with design spectrum but different mean values of seismic demands are calculated. Differences in the mean values of demands obtained from the sets can be at different levels.

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