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# Improving Dynamic Feature Extraction with Frequency Domain Decomposition for Structural Health Modelling

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ABSTRACT: This paper presents two methods including time domain analysis and frequency domain analysis for extracting the dynamic features of frequencies and modal shapes. Time domain analysis involves the auto-regressive moving-averaging (ARMA) in conjunction with the natural excitation technique (NExT) for processing ambient vibration. Frequency domain analysis is to estimate the power spectral density (PSD) matrix at discrete frequencies. The output PSD matrix is then decomposed by singular value decomposition (SVD) to calculate the singular vectors and also the scalar singular values, from which the natural frequency is identified as the dominant peak value, and the corresponding modal shape is found accordingly as the singular vector. Both time domain and frequency domain methods have been tested with a highway bridge monitoring case. The results obtained show that time domain analysis is able to automatically identify frequency but sensitive to the input parameters, frequency domain analysis is stable and effective at extracting modal shapes. With two methods implemented as an integrated framework, the dynamic feature extraction is significantly facilitated for SHM practitioners and structural engineers to assess structural integrity for condition-based maintenance.

## 1 INTRODUCTIONS

As the deteriorated infrastructure is one of the great challenges for sustaining America's economic growth, proactively maintaining and updating the critical infrastructures is imperative to extend the asset life for providing sufficient service to local communities. To do so, infrastructure asset performance must be thoroughly and timely evaluated so that engineers are able to derive the cost effective solutions for asset life cycle management. For civil infrastructure such as bridges, conventional approach is to conduct visual inspection on regular basis and rank the system in a scale of 1 to 10. It is a subjective process, unable to uncover the internal hidden defects, therefore unlikely to establish the rational basis for good maintenance or upgrade decisions. Instead, placing various sensors onto a structure, so-called structural health monitoring (SHM), enables engineers to collect data for comprehensive assessment of structural integrity, dynamic characteristics and service performance.

SHM is an emerging approach for monitoring civil structures, and yields large datasets in a short time. For instance, hundreds of data records are captured every second by an accelerometer, and the raw data must be processed to attain the useful information including frequency and modal shapes. This paper presents both time domain and frequency domain analysis methods have been employed for extracting dynamic features. The methods are elaborated as follows.

# 2 METHODS

## *2.1 Time domain methods*

### 2.1.1 Natural Excitation Technique

Conventional modal parameter extraction practice utilizes frequency response functions (FRFs) which requires measurements of both input excitation and the resulting response measurement. However, ambient vibration does not lend itself to FRF calculations because the input excitation cannot be accurately measured or conveniently simulated. This is often true of large civil engineering structures such as tall building, long bridge and offshore platform. In order to handle the situations with immeasurable input excitation, the natural excitation technique is commonly used to calculate the correlation functions and, in conjunction with a system identification technique such as the ARMA (Zhao and Wu 2012), to extract modal parameter from the ambient vibration data.

The derivation brings out clearly that the correlation functions between the response vector and the response of a selected reference DOF satisfy the homogenous differential equation of motion. In other words, the correlation functions display the same damped frequencies and damping ratios as the FRFs of free vibration. In the implementation of the NExT, the correlation functions can be estimated using two different methods: (1) a direct procedure; and (2) via taking the Fourier transform in terms of spectral density function and then taking its inverse Fourier transform. Once the correlation functions are obtained, a variety of methods may be applied to extract the modal parameters. Herein the ARMA model is employed to represent the system and to identify modal properties as discussed in the following section.

### 2.1.2 Auto-Regressive-Moving-Average Model

The auto-regressive-moving-average (ARMA) model is known as a good model for representing output-only systems. Previous researches (Pandit, 1991; DeRoeck et al, 1995) concluded that it gives stable, reliable and accurate results when applied to ambient acceleration measurement. Andersen (1997) further demonstrated that the ARMA method is a robust and reliable system identification method for linear time-invariant structure subject to unmeasured Gaussian white noise excitation.

For a linear time-invariant system given a general time-series excitation, the observed output data $X_t$ at $t$ can be expressed as

$$\sum_{k=0}^{N} b_k F_{t-k} = X_t + \sum_{k=1}^{N} a_k X_{t-k} + e(N), \ \ (t = 0, \dots, N) \qquad (1)$$

where $X_{t-k}$ is the time-series observed data and $F_{t-k}$ is the time-series excitation force, $e(N)$ is the measurement noise, $a_k$ is the AR(auto-regressive) coefficients and $b_k$ is the MA(moving average) coefficients of the system, respectively. For an output-only system, with the assumption of the unobserved white noise input and indistinguishable measurement noise, the system can be approximated by

$$\sum_{k=0}^{N} b_k \sigma^2 \delta_{t-k} = b_t = R_t + \sum_{k=1}^{N} a_k R_{t-k} \tag{2}$$

where $b_t = 0$ (when $t > N$), therefore $\sum_{k=0}^{N} a_k R_{t-k} = 0$ $(t > N)$ which can be rewritten as

$$\begin{bmatrix} R_M & R_{M-1} & \dots & R_1 \\ R_{M+1} & R_M & \dots & R_2 \\ \vdots & \vdots & \dots & \vdots \\ R_{L-1} & R_{L-2} & \dots & R_{L-M} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} R_{M+1} \\ R_{M+2} \\ \vdots \\ R_L \end{bmatrix} \tag{3}$$

and the AR(auto-regressive) coefficients $a_k$ can thus be solved. As for the the MA(moving average) coefficients $b_k$, they can be solved by the following equations:

$$\begin{cases} b_0^2 + b_1^2 + \cdots + b_M^2 = c_0 \\ b_0 b_1 + b_1 b_2 + \cdots + b_{M-1} b_M = c_1 \\ \vdots \\ b_0 b_M = c_M \end{cases} \tag{4}$$

where $c_k = \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j R_{k-i-j}$ $(k = 0,1, \dots, N)$. With the coefficients $a_k$ and $b_k$, the transfer function of the system can be written as

$$H(z) = \frac{\sum_{k=0}^{N} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \tag{5}$$

and the natural frequencies can be derived from the poles of the transfer function

$$f_k = \frac{\ln(z_k)}{\Delta t} \tag{6}$$

the vibration modes are

$$V_k = \lim_{z \to z_k} H(z)(z - z_k) \tag{7}$$

### *2.2 Frequency domain decomposition*

The frequency domain decomposition (FDD) technology implemented in this research was introduced first by Brincker et al. (2001). It overcomes the disadvantages of the classical frequency domain methods in terms of accuracy, but keeps the important features such as user-friendliness and easy to implement. It has been widely used for modal identification of output-only systems due to its reliability, straightforward and effectiveness.

The theoretical background of the FDD technology is presented as following. The relationship between the unknown input *x(t)* and the measured response *y(t)* can be expressed in the frequency domain through the frequency response function (FRF):

$$G_{yy}(j\omega) = \bar{H}(j\omega) G_{xx}(j\omega) H(j\omega)^T \tag{8}$$

where $T$ denote conjugate and transpose operations, $G_{xx}(j\omega)$, and $G_{yy}(j\omega)$ is the power spectral density (PSD) matrix of the input and output, respectively, and $H(j\omega)$ is FRF matrix. The FRF can be written in partial fraction form:

$$H\left(j\omega\right)=\sum_{k=1}^{n}\frac{R_k}{j\omega-\lambda_k}+\frac{\bar{R}_k}{j\omega-\lambda_k} \quad (9)$$

where n is the number of modes, $\lambda_k$ is the pole and $R_k$ is the residue:

$$R_k=\phi_k\gamma_k^T \quad (10)$$

where $\phi_k$, $\gamma_k^T$ is the modal shape vector and modal participation vector, respectively. Suppose the input is white noise, then PSD is a constant matrix $G_{xx}(j\omega)=C$.

The estimate of the output PSD matrix at discrete frequencies $\omega=\omega_i$ is then decomposed by taking the singular value decomposition (SVD) of the matrix:

$$\hat{G}_{yy}\left(j\omega_i\right)=U_iS_iU_i^H \quad (11)$$

where the matrix $U_i=[u_{i1}, u_{i2}, K, u_{im}]$ is a unitary matrix holding the singular vectors $u_{ij}$, and $S_i$ is a diagonal matrix holing the scalar singular value $s_{ij}$. Near a peak corresponding to the *kth* mode in the spectrum this mode or may be a possible close mode will be dominating. If only the *kth* mode is dominating there will only be one term in Equation (21). Thus, in this case, the first singular vector $u_{i1}$ is an estimate of the modal shape:

$$\hat{\phi}=u_{i1} \quad (12)$$

and the corresponding singular value is the auto PSD function of the corresponding single degree of freedom system (SDOF), refer to Equation (21). This PSD function is identified around the peak by comparing the modal shape estimate $\hat{\phi}$ with the singular vectors for the frequency lines around the peak. As long as a singular vector is found has high modal assurance criterion (MAC) value with $\hat{\phi}$ the corresponding singular value belongs to the SDOF density function. From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping ratio can be obtained via inverse Fourier transform from univariate PSD function back to time domain. (Brincker et al. 2001).

## 3 TEST CASE

Meriden Bridge was used as the use case to test the effectiveness of the time domain analysis and frquency domain decompsition for dynamic feature extyraction. The bridge was built in 1964 and is located on I-91 Northbound in Meriden, Connecticut. The bridge is a simple-support single-span eight-girder steel composite bridge with 85 feet long and 55 feet wide. It has less than a 12% skew and 3% longitudinal slope. According to the Connecticut Department of Transpiration (ConnDOT), the bridge carries three lanes with an annual average daily traffic of 57,000 vehicles with 9% trucks. Details of the bridge can be found in (Christopher et al 2009, Christenson et al 2011, Christenson et al 2012). A wired long-term SHM system with a total number of 38 sensors was installed on the Meriden Bridge for both bridge health monitoring (BHM) and bridge weight in motion (BWIM) purposes. A detail of the gage plan on the Meriden Bridge is presented in Figure 1. The sensors include 18 foil strain gages, 4 piezoelectric strain sensors, 8 piezoelectric accelerometers, 4 capacitance accelerometers (with additional temperature sensing capability), and 4 resistance temperature detectors. Among the gage plan installed sensors on the Meriden Bridge is presented bridge, three of them are focused, shown in Figure 1. These three piezoelectric accelerometers are used to extract the natural frequencies and data acquisition system can be found in (Li 2014).

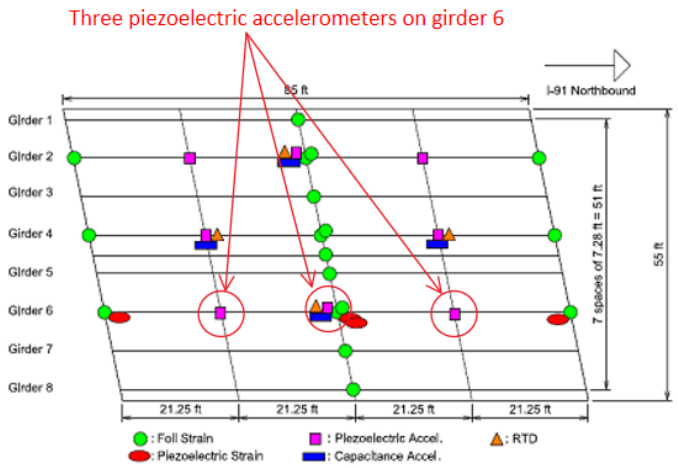



Figure 1 Sensor placement layout for Meriden Bridge

Both the time domain and frequency domain algorithms are integrated into DarwinSHM Extractor (Wu et al. 2015) and employed for identifying the modal properties of the Meriden Bridge in terms natural frequencies, modal shapes, and damping ratio. A data set with 30 minutes long between 6 PM and 7 PM on Sep 12th, 2013 is used. The singular value plot from all the piezoelectric accelerometers is presented in Figure 1.

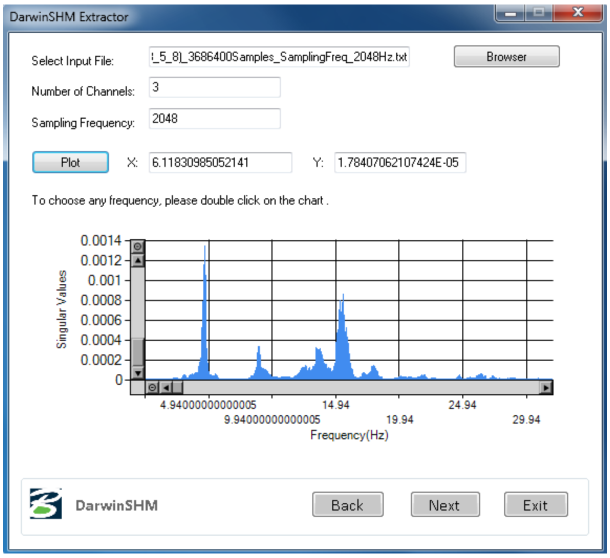



Figure 2 Meriden Bridge singular value plot by frequency domain decompsoition

The peaks of singular value on Figure 2 implies that natural frequencies are around 4.5Hz, 8.9Hz, 13.3Hz and 15.2Hz. For each frequency selected, a modal shape can be plotted as shown in figure 3 (a) by FDD. In contrast, time domain method automatically identified the frequency for each mode and the corresponding modal shape as shown in Figure 3 (b). The frequencies identified by time domain analysis are compared with FDD in Table 1. Apart from mode 1, the frequencies of other modes are quite different.

Table 1 Comparison of extracted frequencies (Hz)

| Mode No. | Frequency Domain Decomposition | Time Domain Analysis |
|---|---|---|
| 1 | 4.5 | 4.23 |
| 2 | 8.9 | 11.54 |
| 3 | 13.3 | 17.62 |
| 4 | 15.2 | 25.39 |

It should be noted that extracted frequencies of time domain analysis in Table 1 are dependent upon proper selection of input software parameters. Herein, input parameters of time domain analysis are selected based on trial and error.

(a) Frequency domain decomposition (b) Time Domain Analysis

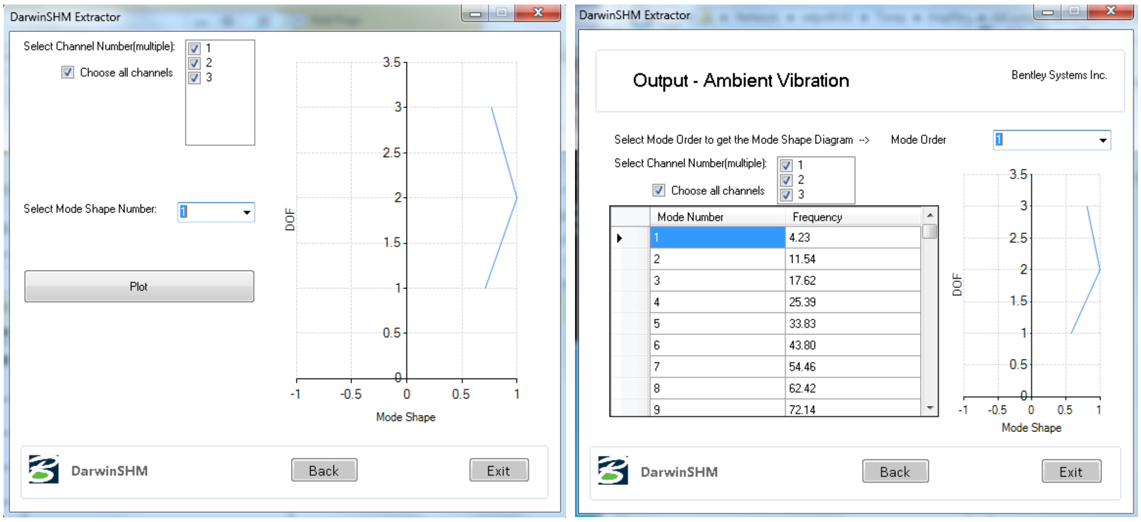
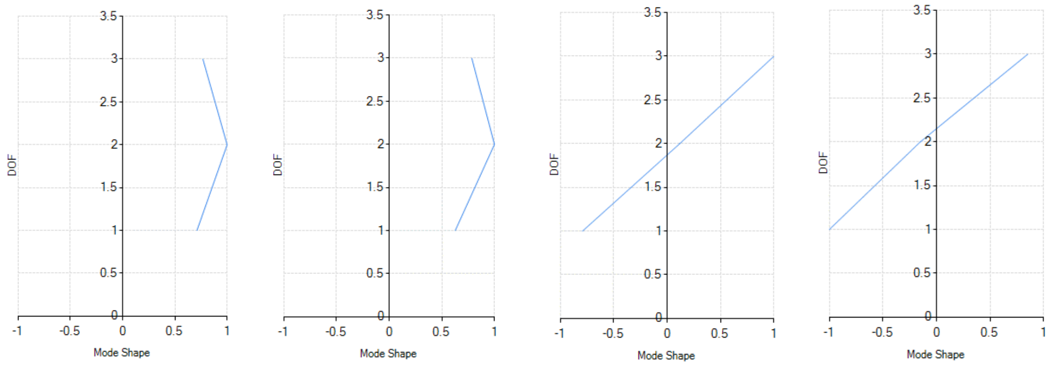



Figure 3 Meriden Bridge modal shape plots

(a) Frequency domain decomposition: first four modal shapes




(a) Time domain analysis: first four modal shapes

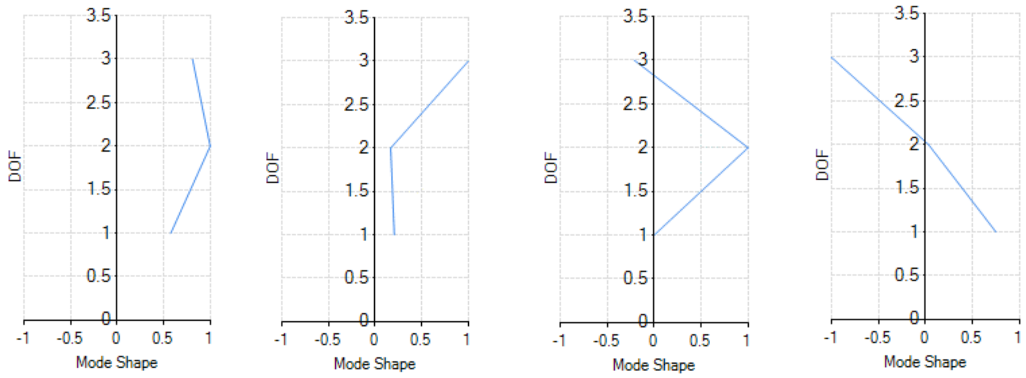



Figure 4 Modal shapes by both time domain analysis and frequency domain decomposition

Figure 4 (a) illustrates the modal shapes at 4.5Hz and 8.9Hz, which are perpendicular to each other in 3D plane. Also, modal shapes at 13.3Hz and 15.2Hz are perpendicular, i.e. they lie inside two perpendicular 3D planes. As noted earlier, apart from mode 1, the modal shapes identified by time domain approach are not consistent with the frequencies as identified by FDD, thus appear very differently as shown in Figure 4 (b).

## 4 CONCLUSIONS

Two feature extraction methods have been developed and tested on a real highway bridge. The results obtained show that FDD is flexible such that users can examine the singular value plot and apply engineering judgment in determining the natural frequency, but the process can be subjective to user judgement for choosing the frequency. Although the time domain method does not require user's judgement, it requires for a good set of reference coordinates for extracting the features, the method can be improved with stabilization diagrams. The modal shapes are plotted in a 2D plot, which needs to be enhanced with 3D in future for intuitive and accurate presentation of modal shapes.

## 5 ACKNOWLEDGEMENTS

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