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Probabilistic modeling of FRP-wrapped concrete members

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ABSTRACT: Wrapping or encasement of concrete members in Fiber-reinforced plastic (FRP) shells could significantly improve strength and ductility of these members. Due to relatively high modulus of elasticity, FRP shows a high confinement performance. Some of the available models for FRP wrapped specimens were developed based on conventional models currently used for reinforced concrete members. However, it has been demonstrated that such models many not be suitable for FRP-encased concrete. In this study, a constitutive model based on plasticity theory is applied. The model provides the stress-strain curves of axially loaded members that are confined with FRP material. Here, first a nonlinear finite element model with Drucker-Prager plasticity is performed. This model has previously been calibrated with experimental observation. Then, a probability based procedure is implemented in ANSYS program where the main parameters affecting the overall behavior of the confined member and the model error are treated as random variables. Probabilistic models for concrete and FRP materials are derived from current literature. Monte Carlo Simulation (MCS) method along with selective sampling technique are used for the probability based analysis. As expected, the uncertainties in FRP and concrete material properties and model error could considerably affect the reliability of the FRP-confined concrete. High bias factor and coefficient of variation indicates that any theoretical model for FRP-confined concrete should be performed in a reliability-based framework.

1 INTRODUCTION

An important application of FRP composites is for confining concrete members which enhances the strength of these memebrs considerably.. In seismically active regions of the world enhacement made by confinment can significantly improve the ductility capacity which is demanded by the earthquake load. Confinment may be beneficial in nonsiesmic zones for increasing the column capacity.

In circular reinforced concrete columns, FRP wrap effectively curtial the lateral expansion of concrete shortly after the unconfined strength is reached. It then reverses the direction of the volumetric response, and the concrete responds through large and stable volume contraction. As a result, the stress-strain respnse of FRP confined concrete is enhanced to be completely different from that of unconfined concrete. Figure 1 shows the schimatic stress-strain response of unconfined and FRP-confined concrete columns. As the design of such confinment requires an accurate stress-strain model for the concrete, extensive research has been carried out on the stress-strian behaviour of FRP-confined concrete (Lam and Teng, 2003, Harajli et al., 2006, Matthys et al., 2006, Wu and Wang, 2009, Fardis and Khalili, 1982, Karbhari and Gao, 1997) from which a number of stress-strian models have been suggested.

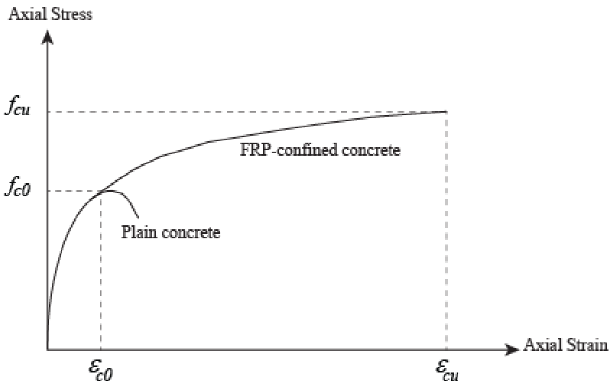

Figure 1: Axial behavior of FRP-confined and plain concrete

The ultimate state (ε_{cu}, f_{cu} point in Figure 1) is characterized by tensile fracture of the wrap. The yielding point (ε_{c0}, f_{c0} point in Figure 1) is the strength of unconfined concrete. It has been observed that at fialure, the tensile strength of the FRP wrap is generally lower than the uniaxial tensile strength of the FRP material (Shahawy et al., 2000, Lam and Teng, 2003). Several causes have been suggested for this. The two main causes are the localization in cracked concrete which leads to nonuniform distribution of stress in FRP jacket and the effect of curvature of an FRP jacket on the tensile strength of FRP. Shahawy et al. (Shahawy et al., 2000) suggested that for design specification, proper confidence levels must be set by reliability analysis of the effective hoop rupture strian of the jacket.

Finite element method has successfully been used for modelling FRP-confined circular and noncircular columns (Shahawy et al., 2000, Wu et al., 2009, Yu et al., 2010). In this study, a nonlinear finite element model is developed in the Drucker-Prager framework. Then, a probabilistic procedure is used to investigate the effect of randomness in the major contributing factors to the ultimate stress and strain of FRP-confined columns.

2 FINITE ELEMENT MODELING

In ANSYS, element SOILD65 is used to model concrete. This element is used for 3D modeling of solids with or without rebar. The solid is capable of cracking in tension and crushing in compression. Figure 1 shows the geometry of SOLID65.

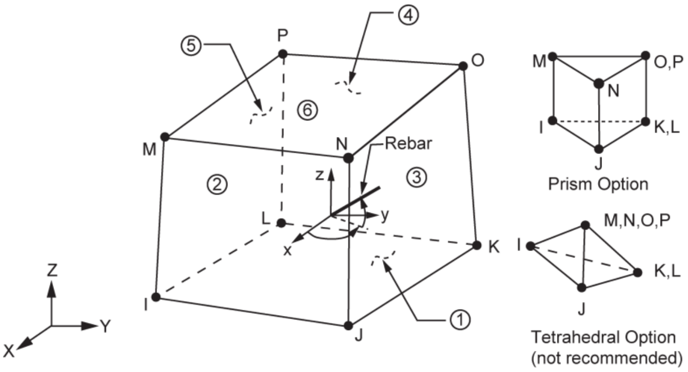

Figure 1: Element SOILD65 geometry (ANSYS, 2012)

Usually, Von-Mises or Drucker-Prager (Drucker et al. 1952) plasticity is used for concrete. Drucker-Prager yield criterion is a modification of the Von-Mises criterion which accounts for the influence of hydrostatic stress component; the higher is the hydrostatic stress (confinement pressure), the higher would be the yield strength. Equations 1 and 1 show the yield functions for Von-Mises and Drucker-Prager yield surfaces.

$$\sqrt{J_2} - \sigma_y = 0 \tag{1}$$

$$\beta I_1 + \sqrt{J_2} - \sigma_y = 0 \tag{2}$$

Where, parameters β and σ_y are the yield function parameters or material constants. I_1 and J_2 are first and second stress invariant. Figure 2 shows both of the aforementioned yield criteria in the stress invariant plane. The Von-Mises function depends on only one stress invariant and does not include the effect of hydrostatic stresses, while Drucker-Prager includes the effect of hydrostatic stresses by adding another stress invariant.

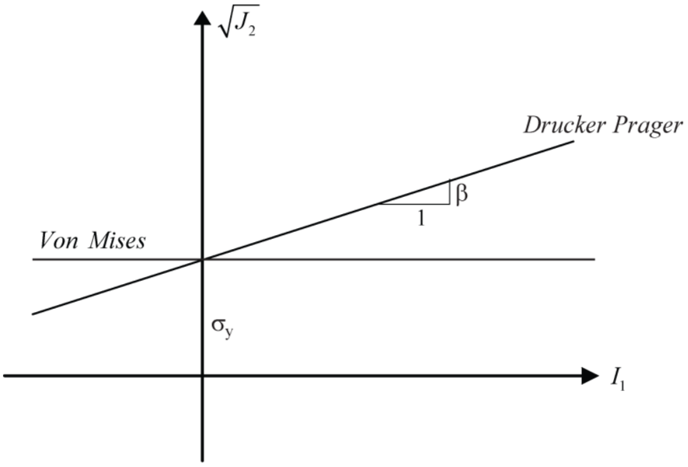

Figure 2: Drucker-Prager and Von-Mises yield criteria

Since the Drucker-Prager yield surface is a smooth version of the Mohr–Columb yield surface, it is often expressed in terms of the cohesion c and the angle of internal friction ϕ that are used to describe the Mohr–Columb yield surface. If it is assumed that the Drucker-Prager yield surface inscribes the Mohr–Columb yield surface, then the expressions for finding parameters β and σ_y ?? will be as follows.

$$\beta = \frac{2\sin\phi}{\sqrt{3}\left(3-\sin\phi\right)} \tag{3}$$

$$\sigma_y = \frac{6(c)\cos\phi}{\sqrt{3}\left(3-\sin\phi\right)} \tag{4}$$

Where in Equations 3 and 4, ϕ is the angle of internal friction and c is the cohesion value. The cohesion and the angle of internal friction for concrete are related to concrete strength as shown in Equations 6 and 7.

$$f_{c0} = \frac{2(c)\cos\phi}{3-\sin\phi} \tag{5}$$

$$k_1 = \frac{1+\sin\phi}{1-\sin\phi} \tag{6}$$

Where variable f'_{c0} is the unconfined strength of concrete and the parameter k_1 is the confinement effectiveness factor. Confinement effectiveness factor was first suggested as 4.1 by Ritchart et al (1929). It results in a friction angle of about 37 degree. Others have suggested different expressions for calculating this factor. Rochette et al. (1996) suggested a direct approach to calculate c and ϕ as given by Equations 7 and 8.

$$\phi = \sin^{-1}\left[\frac{3}{1+1.592332\, f'_{c0}(ksi)}\right] \tag{7}$$

$$c(psi) = \left(f'_{c0}(psi) - 1256\right)\frac{3-\sin\phi}{6\cos\phi} \tag{8}$$

The internal friction angle and cohesion shown in Equations 7 and 8 were used by other researchers (Mirmiran, Zagers et al. 2000; Shahawy et al. 2000).

For modelling FRP material, researchers (Mirmiran, Zagers et al. 2000) have used a tension-only membrane element type to model FRP. In ANSYS, element SHELL41 is suitable for this purpose. With this element type, membrane behavior with an orthotropic elastic material is implemented. One possibility to better model FRP in ANSYS which has not been tried previously by researchers is to use its reinforced shells and solids elements. These elements constitute a base element that can be reinforced with additional elements. In the case of FRP, the saturant can be used as the base element while fibers are added as reinforcing elements. Reinforcing elements can be defined as discrete or smeared, and they can act as tension-only, compression only or tension and compression elements. In fact, for FRP, the tension-only fibers are used. Element SHELL181 can be used as the base element for FRP composite material. Then, it can be reinforced using REINF265 smeared element. Each layer of reinforcement behaves as a unidirectional material. All layers including the base element perform like a parallel system. Perfect bond is assumed amongst the layers. Each layer can have its own thickness (defined as fiber area and space), orientation and local axis coordinate system. This option seems to be the most appropriate to model FRP sheets in ANSYS.

Maximum mesh sizes of 15 mm along with 15 degrees angle are used for meshing the model. Figure 3 shows the meshed model. All used element types are shown in this Figure. No bond slip is modeled. Thus, all link and shell elements are directly connected to concrete solid elements.

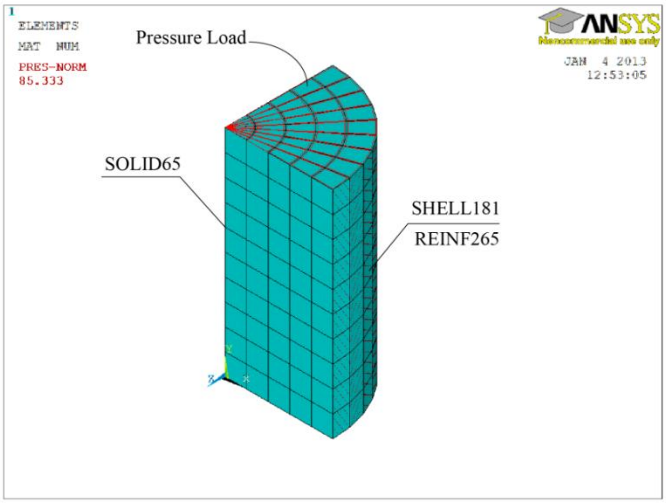

Figure 3: Geometry, meshing and element type in ANSYS model

3 RELIABILITY ANALYSIS

In this study, based on reliability analysis, statistical properties of ultimate state of concrete cylinders wrapped with FRP sheets are investigated. The stress and strain corresponding to the ultimate load capacity of the cylinder are function of concrete and FRP materials properties. All material properties are treated as random variables.

For concrete material properties, statistical models used in Attard and Stewart (1998) study are used. Attard and Stewart in their probabilistic analysis of concrete stress block used the lognormal distribution as the probability density function for the concrete compressive strength. The probabilistic model for concrete modulus of elasticity is driven from Setunge (1993) study. It was modeled using Normal distribution with 0.15 coefficient of variation. The mean value of the modulus of elasticity is shown in Table 1.

Statistical models for FRP material are taken from the study by Atadero et al. (2009) and Atadero and Karabahari (2008). Although the statistical models for different layers of FRP sheets are not completely similar; for simplicity it is assumed that statistical models for one layer of FRP can be used for multilayer FRP sheets. Results presented by Atadero et al. (2009) showed that there are correlations amongst the properties of FRP. In this study, average values of correlation coefficients for a three layer laminate are used. The correlation between the thickness and the ultimate strength and the thickness and the modulus of elasticity are -0.50 and -0.20, respectively. The correlation between modulus of elasticity and ultimate strength is 0.10.

Table 1. Statistical models of main random variables

Variable		Mean/Bias	STD^1/COV^2	PDF^3	Reference
Concrete	f_c	$f'_{cm} = f_{c0} + 7.5MPa$	6.0MPa	Lognormal	(Attard and Stewart, 1998)
	E_c	$4730.3\left(f'_{cm}\right)^{0.5164}$	0.15	Normal	
Composite	t_f	1.27mm (one layer)	0.05	Lognormal	(Atadero and Karbhari, 2008)
	E_f	68.9GPa	0.20	Lognormal	
	σ_{uf}	896.3MPa	0.15	Weibull	
Model Error	X_{fcu}	1.10	0.10	Lognormal	Based on Samaan et al. (1998) study
	$X_{\varepsilon cu}$	1.21	0.12	Lognormal	

1 Standard Deviation 2 Coefficient of Variation 3 Probability Density Function

The model error is the result of simplifications and assumptions made for the derivation of the theoretical model like those proposed by the codes. The model error is defined as shown in Equation 9.

$$X_M = \frac{Exprimentally\ measured\ value}{Theoritical\ predicted\ value} \tag{9}$$

Estimation of the model error requires experimental results. Test results used for calibration of Samaan et al. (1998) model are used here to find the statistical distribution of model error for ultimate strain and strength. After analyzing all specimens in ANSYS program, the mean and coeffceint of variation of model errors are derived. It is assumed that the model error can be reasonably modeled using Lognormall distribution. It is worth mentioning that the model error used here inncludes the uncertianties in parameters of Drucker-Prager plasticity model as well.

In the absence of probabilistic placticity parameters, this way of dealing with theoritical uncertientins seems quite reasonable.

Reliability analysis in this study is performed using the MCS method. In the MCS method, the statistical properties such as mean and standard deviation are calculated using random number generation. In this study, selective MCS based on Latin Hypercube method is implemented. For each of the considered specimens 1000 simulations are generated.

4 RESULTS AND DISCUSSION

In this study, using a calibrated nonlinear finite element procedure, which is based on Drucker-Prager plasticity model, the ultimate state of FRP-confined concrete cylinder is evaluated. Then using selective MCS method, the ultimate stress and strain of FRP-wrapped concrete cylinder are statistically investigated and the statistical properties of these parameters are extracted. Using a parametric program in ANSYS, the whole investigation (including the nonlinear finite element analysis and probabilistic simulation) is carried out. Non-associative Drucker-Prager plasticity is developed for this study. Drucker-Prager parameters are taken from Rochette et al. (1996) study. The calculated model error includes the uncertainties involved in the Drucker-Prager parameters and the model errors. A standard concrete cylinder with 300 mm height and 150 mm diameter is considered for the reliability analysis. The nominal concrete compressive strength of 25 MPa is considered for concrete material. Mean value of FRP material properties as well as statistical models for all considered random variables are shown in Table 1. For comparison, three different layers of confinement (1, 2 and 3 layers) are applied.

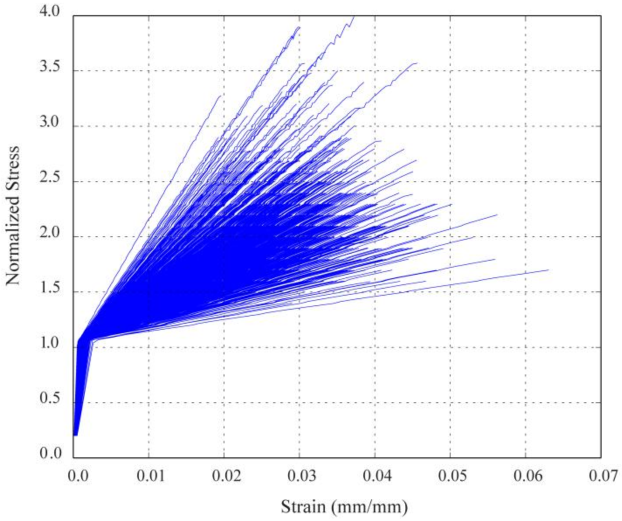

Figure 4: Stress-strain behavior of simulated specimens (Specimen with one layer of FRP)

Figure 4 shows the stress-strain curves for all simulated data for the specimen with one layer of FRP. The stress is normalized with regards to ultimate concrete compressive strength. As can be seen, there is disparity in both stress and strain. Each of the shown graphs in Figure 4 is a result of nonlinear finite element analysis in ANSYS. These graphs are used to derive the statistical properties for ultimate stress and strain of considered FRP-confined specimens.

Based on the simulated data, mean and standard deviation of the ultimate stresses and strains can be found. In Table 2, the statistical properties of normalized (with respect to ultimate concrete compressive strength) ultimate strength for all three considered specimens are shown. In addition to the statistical properties, for comparison, the ultimate strength and strain obtained based on Samaan et al. (1998) are also shown in Table 2. The nominal value in Table 2 is based on 95% chance of exceedance. Bias factor is the ratio of mean to nominal and COV is the coefficient of variation.

Table 2. Statistical properties of normalized ultimate stress of considered specimens

	Experimental	This study (Based on FEM)			
Specimen	Samaan et al. (1998)	Mean	Nominal	Bias	COV
1 Layer	2.24	2.28	1.63	1.40	0.21
2 Layer	3.38	3.38	2.23	1.51	0.25
3 Layer	3.67	4.50	2.82	1.60	0.29

Figure 5 shows the empirical cumulative density function (CDF) of simulated ultimate strain for all considered specimens. The mean and nominal value are graphically shown on the Figure. Furthermore, the corresponding coefficients of variation for each graph are also shown. The results in Figure 5 show that the cumulative density function for the ultimate strain of confined cylinder is not sensitive to the number of wrapped layers.

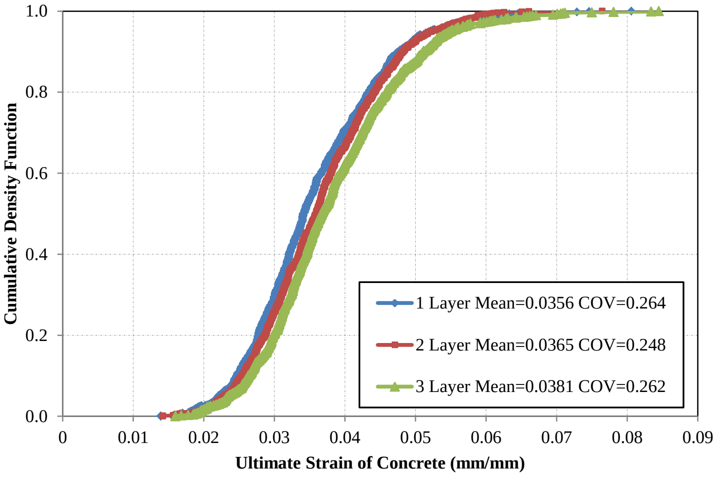

Figure 5: Empirical CDF of ultimate strain of simulated specimens

Results shown in Figure 5 suggest that the statistical models for FRP-confined specimens with different number of layers are not very different from one another while the uncertainties in the material properties and the theoretical model have considerable impact on the ultimate strain and strength of FRP-confined cylinders. As shown in Table 2, the difference between the mean and nominal values of ultimate strength could be more than 60%. The order of coefficient of variation in both ultimate strain and stress is about 0.25 showing noticeable disparity in predicting the ultimate state of FRP-confined concrete. This confirms that instead of using any average or lower bound values for theoretically predicting the behavior of FRP-wrapped columns, reliability-based values should be evaluated.

CONCLUSION

In this study, using the Drucker-Prager plasticity model for concrete, the nonlinear behavior of FRP-confined concrete cylinder under compressive force was investigated in ANSYS program. Then based on the calibrated finite element model, a reliability-based nonlinear finite element procedure using ANSYS software was implemented. Taking advantage of parametric design language and probabilistic module in the ANSYS program, Monte Carlo Simulation based on selective sampling using Latin Hyper Cube technique was performed leading to probabilistic evaluation of FRP-confined concrete cylinder behavior. It was shown that a well-calibrated ANSYS model can be effectively used for probabilistically evaluating the nonlinear behavior of FRP-confined concrete structures. As expected, the inherent uncertainties in FRP and concrete material properties and systematic uncertainties in the theoretical model considerably affected the reliability of the final results. High bias factor and coefficient of variation around 25% indicates that any theoretical model for FRP-confined concrete should be performed in a reliability-based framework.

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