

## An Advanced Method to Estimate Damping Values from Long Term Measurements

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**ABSTRACT:** Based on the improved sensor techniques and the comparatively cheap computer equipment, structural monitoring has become an important tool in the field of structural lifetime assessment. Compared to the development in sensor and computer techniques the development of evaluation and interpretation methods is not able to keep up with.

The objective of this paper is to present a method to evaluate dynamic data (e.g. acceleration measurements) in order to get the damping. The focus of the method is the automatically evaluation of damping values from long term monitoring data. The method is generally based on the cross-correlation of signals. From an initial estimate of the damping, which can be based on laboratory test, or values from the literature, a set of time histories is derived. These time histories are compared with the measured data; the time history which fits best, based on the cross-correlation, provides the estimation for the damping values. The method was tested on a long term measurement on a railway bridge.

### 1 INTRODUCTION

Structures that are exposed to dynamic loads respond with vibrations. Especially for structures, which are made by using inadequate materials or must comply with special architectural demands, perceptible vibrations are reported frequently (Spengler 2010). The benefits of modern materials necessarily lead to a design with reduced stiffness and reduced weight. This leads to an increased yielding and thus large, frequent and highly dynamic structural movements under loads such as temperature, wind and traffic (Tilly & Cullington 1984, Bachmann 1997).

The damping of structures and structural parts is an important factor in the damage identification among other system parameters (Strauss et al. 2009) such as frequency, modal forms, deflection. Beams in cracked state compared to the uncracked state, show beside a lower natural frequency usually an increased damping (Dieterle 1981). For a further increase of the load the damping is decreasing again.

The present work will introduce an approach to estimate the damping from dynamic measurements such as velocity or acceleration. This approach is based on the cross-correlation method and was especially developed for the automatic evaluation of long term monitoring data.

## 2 STATE OF THE ART

### 2.1 General

There are several different methods to calculate the damping value from a given time history of acceleration or velocity. Some of them are described below.

Generally they are divided in time domain methods and frequency domain methods (Flesch 1993, Wenzel 2005). Within the time domain methods the damping is calculated directly from the time history of the observed values. In contrast to the time domain methods, for the frequency domain methods the observed time history has to be transferred to the frequency domain. This is usually done by a frequency transformation (Cooley & Tukey 1965). Both groups of methods are widely spread among engineers but are with restrictions for the use in automatic evaluation within a long term monitoring work. A short description of the most common used methods for both groups is described in the following.

#### 2.1.1 Methods in time domain

The basis for this group of methods is the time history of a dynamic measurement (e.g. acceleration measurement). One of the most common used method is the logarithmic decrement method. The logarithmic decrement basically describes the rate at which the amplitude of a damped vibration decreases. The logarithmic decrement is defined as:

$$\delta = \frac{1}{n} \ln\left(\frac{x_1}{x_{n+1}}\right)$$

where  $x_1$  is the first amplitude value taken and  $x_{n+1}$  the amplitude after n-cycles.

The logarithmic decrement can be measured experimentally and it gives a simple method to determine the damping ratio of a system using the following relation between logarithmic decrement and damping ratio

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

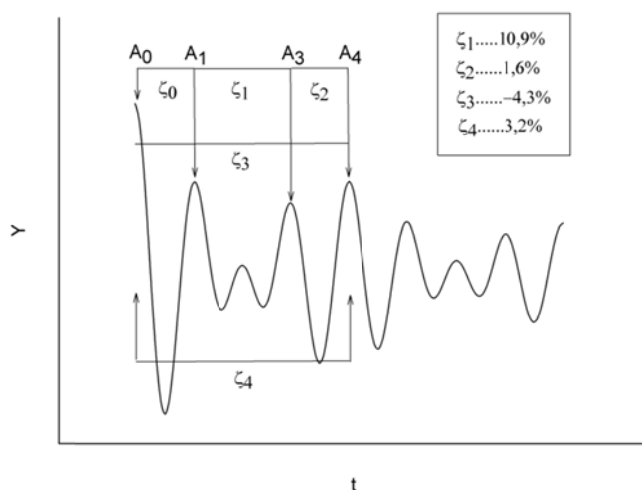


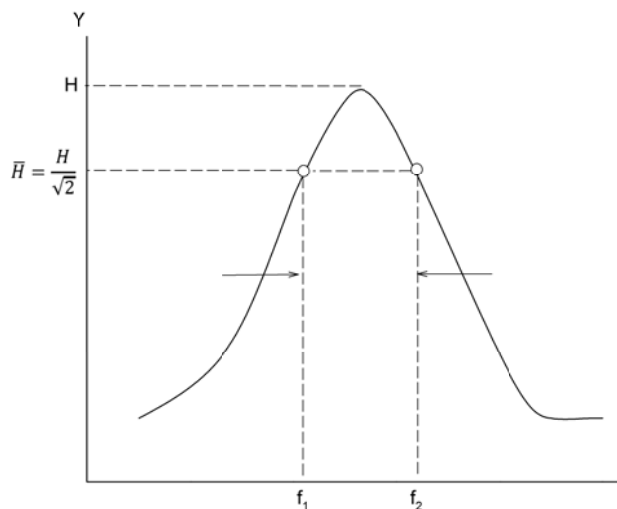
Figure 1 shows the problems that can occur using this method on vibration data of complex structures. The method, in this case, strongly depends on the time interval, starting point cycles count respectively, which is used for calculating the damping value.

Figure 1: Calculated Damping for different time windows

As shown in Figure 1 the calculated damping values for different periods are completely different. The calculated damping values from different time periods are scattered between -4.3 % and 10.9 %. This example shows that this method is sensible to the starting point and the time interval observed for calculating the damping.

### 2.1.2 Methods in frequency domain

One method which belongs to the general group of frequency domain methods is the damping estimation by frequency domain decomposition (Brincker et al. 2001). This method is based on the power spectral density analysis (PSD) of a given time history. The damping estimation, as a single-degree-of-freedom PSD function of each mode has to be identified using the first spectral eigenvalues and based on its frequency peak and selected MAC value (Brincker et al. 2001). From identified univariate PSD function of the equivalent single degree of freedom system (SDOF), logarithmic decrement and damping ratio of each mode can be estimated via inverse Fourier transform from univariate PSD function back to time series.



Another method to estimate the damping in the frequency domain is the half power bandwidth method. The damping value of the *i*-th mode is achieved by using the spectrum analysis of the time history of the dynamic signal (Flaga et al. 2008). It is based on interpreting values of frequency, see Fig. 2. Based on the selected frequency, the logarithmic decrement of damping can be calculated by:

$$\zeta = \frac{f_2 - f_1}{f_2 + f_1}$$

Figure 2. Half power bandwidth method

Both frequency domain methods described in this section are basically reliable methods to estimate the damping value but there are some disadvantages in using them for the automated evaluation of the damping within a long term monitoring work.

## 3 APPROACH BASED ON CROSS-CORRELATION

### 3.1 General

The main disadvantages of the existing methods of estimating the damping are:

- The boundary conditions regarding time interval for the time domain methods
- Boundary conditions regarding frequency for the frequency domain methods

### 3.2 The cross-correlation method

In signal processing, cross-correlation is a method of estimating the similarity of two waveforms (time histories) as a function of a time-lag applied to one of them. This is also known as a sliding dot product or sliding inner-product. It is commonly used for searching a long-signal for a shorter, known feature (Nikolić et al. 2012).

For a given function the cross correlation is defined as:

$$h(t) = \int_{-\infty}^{+\infty} f(t) * g(t + \tau) d\tau$$

As an example, consider two real valued functions  $f$  and  $g$  differing only by an unknown shift along the  $x$ -axis. One can use the cross-correlation to find how much  $g$  must be shifted along the  $x$ -axis to make it identical to  $f$ . The formula essentially slides the  $g$  function along the  $x$ -axis, calculating the integral of their product at each position. When the functions match, the value of  $(f * g)$  is maximized. This is because when peaks (positive areas) are aligned, they make a large contribution to the integral. Similarly, when troughs (negative areas) align, they also make a positive contribution to the integral because the product of two negative numbers is positive.

To calculate the cross-correlation of two time histories there are two possibilities. One is to calculate the cross-correlation with the convolution of the two discrete functions. Another possibility is to use the relation between the convolution in the times domain and the multiplication in the frequency domain which can be described as:

$$f(t) \leftrightarrow F(\omega) \quad (1)$$

$$g(t) \leftrightarrow G(\omega) \quad (2)$$

$$h(t) = f(t) * g(t) \leftrightarrow H(\omega) = F(\omega)G(\omega) \quad (3)$$

As stated in the equations 1,2 and 3 the convolution in the time domain can be replaced by a normal multiplication if both functions are transferred to the frequency domain. A further inverse fourier transformation of the result  $H(\omega)$  leads to the cross-correlation value  $h(t)$ .

### 3.3 Estimation of damping values based on cross-correlation

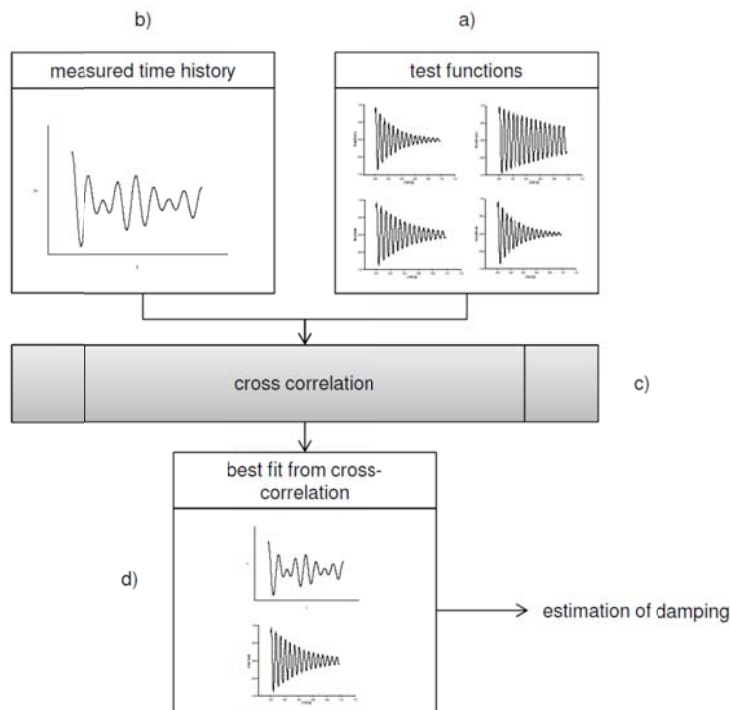
The basic idea of the cross-correlation method of estimating the damping is based on a comparison of the measured time history with a set of test functions. This is the first step to do (Figure 3 a). The test functions are calculated synthetically based on the equations for the damped single-mass system (Östereicher 2013).

$$y(t) = y_0 * \sin(2\pi f_0 * t + \varphi_0) * e^{(-\xi\pi f_0 t)} \quad (4)$$

Input values for the test functions are the damping  $\delta$  and the natural frequency  $f_0$ . As a starting point for the objective method it is necessary to have an estimation for both parameters. These initial values can be found by evaluating some data manually before starting with the automated evaluation. Once these initial values are found, a set of test functions can be derived. The results from several evaluations based on this method show that the frequency should be known within an interval of about 1 Hz. The initial damping values should be known within an interval of about 30 % (Östereicher 2013).

The next step is to get a significant time window from the measured time history (Figure 3 b). One method is to find local maxima in the time history and use these maxima as starting point for the time interval.

The main step is to compare the extracted time history with the calculated test functions. Each test function is compared with the extracted time history. The comparison between the extracted time history and the test functions is made by means of cross-correlation. The result is a correlation value for each comparison step.



The highest correlation value

$$s = \max|h(t)|$$

gives the test function which fits best.

Figure 4 shows a simulation example. The time history is created with a natural frequency of 15 Hz and a damping value of 3 % (Figure 4 a). Additionally some noise is added. The results in Figure 4 b shows that the damping is estimated with 3 % described by a correlation value of  $h(t)= 0.978$ .

Figure 3. New approach in damping estimation

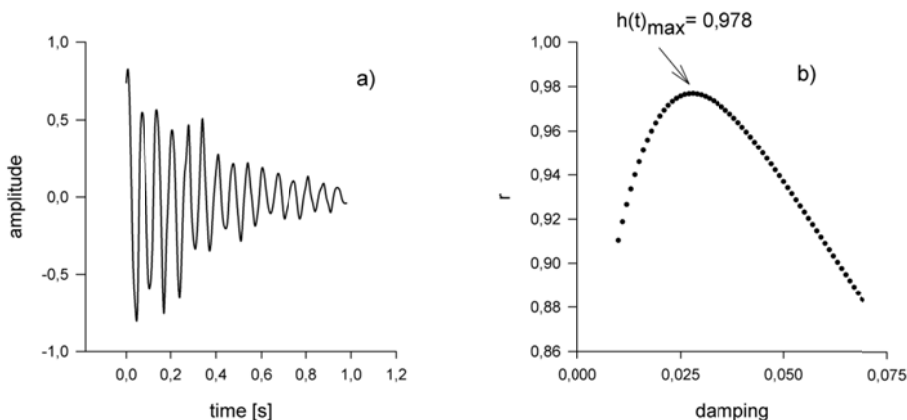


Figure 4. Simulation example for the damping estimation based on the cross-correlation

## 4 APPLICATION OF THE METHOD TO A RAILWAY BRIDGE

### 4.1 General

To test the method the data of a long term measurement on a railway bridge was used. The measurement was performed in 2008/2009, during a time interval from November 2008 to February 2009. The railway bridge is situated in the district of Tulln in lower Austria at the Franz Josefs Bahn km 24.985. The bridge was built in 1982/1983. For the measurement among other sensors, 3 acceleration sensors were applied along the bridge. For the present evaluation the data of the sensor in the middle of the first span was used. The sampling rate for the acceleration

measurement was chosen with 1000 Hz. The reason for this rather high sampling rate is that the measurement originally was performed to test the influence of digital filtering on the maximum acceleration.



Figure 5. Picture of the bridge structure (North-east view)

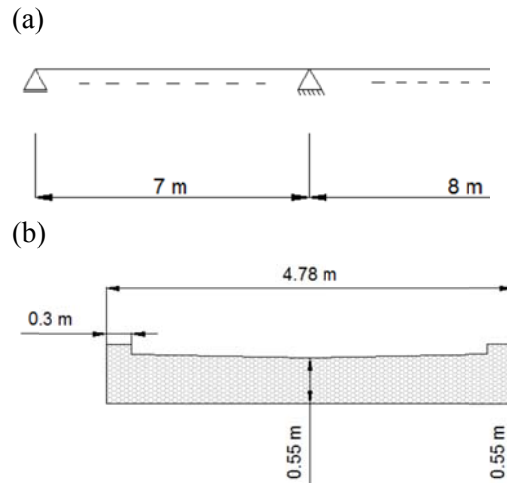


Figure 6. Static system of the bridge structure (a) span width of lateral and middle field , (b) cross-section of the bridge deck

#### 4.2 Static system

The static system of the bridge is a 3-span continuous reinforced concrete beam. The length of the outer spans is 7m, the central span is 8m. The crossing inclination of the bridge deck is  $85^\circ$ , see Figure 5. The concrete type of the bridge deck is B400, which is equivalent to C30/37 according to ÖNORM EN 1992-2 (Reiter 2005). The bridge deck thickness is between 0.55m in the middle to 0.60m at the boundaries, as shown in Figure 6(b). The bearings of the bridge deck are elastic neoprene bearings.

#### 4.3 Pre- estimation of the natural frequency and damping

As stated in Section 3.3, to estimate the damping initial values for the damping and the natural frequency are necessary. To get an estimation for the natural frequency the *Random Window Fourier Transformation (RWFT)* (Österreicher 2013) was used. This method allows an automatic evaluation of the natural frequency for long term monitoring data. The RWFT calculates the FFT in  $n$  Windows within a time history. Afterwards a summation is applied to all the FFT's to get frequency estimation.

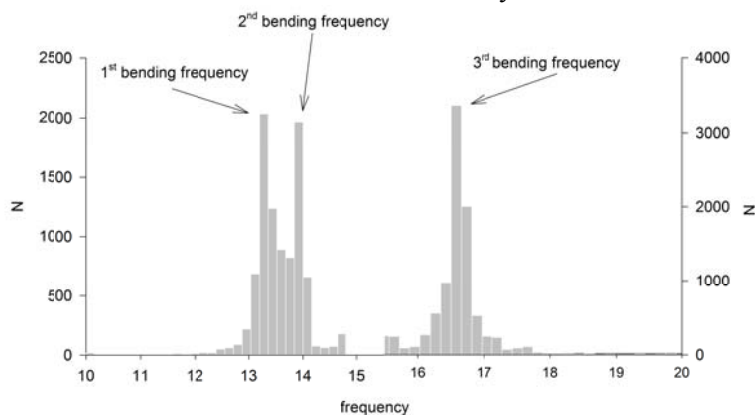


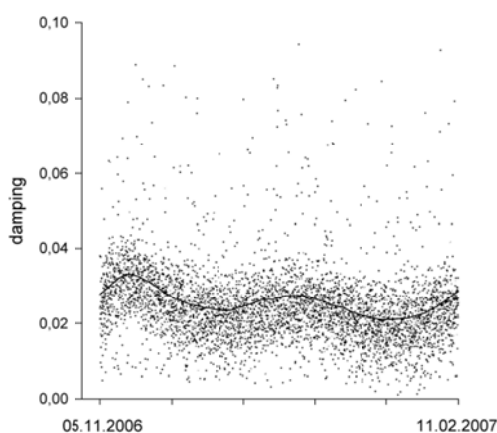
Figure 7. Pre-estimation of the natural frequencies

The advantage of this method is that a trigger can be realized. This means, that after selecting a time window from the time history, predefined trigger conditions can decide if the data are used for a FFT and the summation afterwards.

Using predefined trigger conditions several load models (train load, ambient load) can be evaluated separately.

Figure 7 shows the result of the natural frequency estimation. The trigger conditions were defined such that just the ambient vibration behavior is considered. As shown in Figure 7 there are 3 significant natural frequencies (bending frequencies) within 10Hz and 20Hz. For the further estimation of the damping the frequency at about 13.2 Hz was considered. To get a first estimation of the damping some data were evaluated manually with the method of logarithmic decrement.

#### 4.4 Damping estimation of the long term monitoring data



With the given pre-estimation of the damping and the natural frequency the damping over the whole observation time was calculated. The same trigger conditions for choosing the time windows were chosen for the damping estimation.

The results of the damping estimation shows that the damping values are scattered between 0,3% and 9%, but a significant frequency of occurrence is between 2% and 4%. Additionally Figure 8 shows that the main damping values are changing during the observation time.

Figure 8. Result of the damping estimation

## 5 CONCLUSIONS AND FURTHER DEVELOPMENT STEPS

The present work shows that the present method to estimate the damping is a reliable method to get damping values for a structure. The main advantage of this method is that for each time window estimation of the damping a correlation value is given. This correlation values describe “quality” of the estimations. In the present work, damping values with a correlation value below 0.7 were further not considered. Another advantage is that this method can be applied in automatic monitoring systems. With the trigger conditions different load models can be considered separately. The results depend on the frequency and the damping. In this paper, just the damping of the test functions was changed to get the best correlated test function and thus an estimation of the damping. Basically it is also possible to change the frequency and the damping together so that the result is a frequency and damping dependent correlation value. Figure 9 shows the result of a 2D simulation of damping and frequency. The sensitivity of this method to frequency is significantly higher than to the damping.

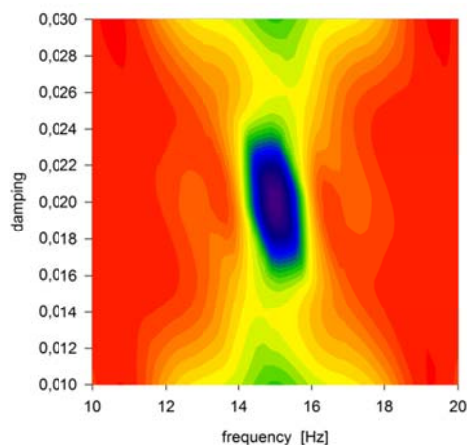


Figure 9. 2D Sensitivity of the damping estimation

This means that changes in the frequency lead to significant changes in the correlation value.

## ACKNOWLEDGEMENT

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