

## Finite element modeling of thermal deformations in concrete beams reinforced with FRP bars

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**ABSTRACT:** The use of fiber reinforced polymer (FRP) bars in concrete structures is becoming as the best solution to the steel corrosion problem. However, the thermal incompatibility between concrete and FRP bars in the transverse direction that produces tensile stresses within concrete under high temperatures may cause splitting failure of concrete cover and consequently the reduction of durability and serviceability of concrete structures. Numerous experimental tests and analytical investigation were carried out on thermal effects on FRP reinforced concrete structures. Nevertheless, the finite element modeling of thermal behavior of FRP bars embedded in concrete is not more investigated. This paper presents a nonlinear numerical study using ADINA finite element software to investigate the effect of the ratio of concrete cover thickness to FRP bar diameter ( $c/d_b$ ) on the distribution of transverse thermal deformations in concrete cover and FRP bars for an asymmetric problem using prismatic concrete beams reinforced with FRP bars submitted to high temperatures up to + 60 °C. The results of thermal deformations in FRP bars-reinforced prismatic concrete beams predicted from non linear finite element model are compared with those obtained from analytical model and experimental tests.

### 1 INTRODUCTION

In the last decade, the use of fiber reinforced polymer (FRP) in concrete structures has increased because of their high tensile strength, high stiffness to weight ratio, light weight, and their resistance to corrosion and fatigue. However, the thermal incompatibility between FRP bar and concrete particularly in transverse direction consists a big problem in reinforced concrete structures. This thermal incompatibility induces tensile stresses, within concrete under temperature increase, that may cause splitting cracks and, eventually, deterioration of the bond between FRP bars and concrete. Extensive experimental and analytical research were carried out to analyse the thermal effect on FRP bars embedded in concrete (Rahman et al. 1995, Aiello et al. 2001, Masmoudi et al. 2005, Zaidi and Masmoudi 2007). Nevertheless, the finite element modeling of thermal behavior of concrete structures reinforced with FRP bars was not sufficiently investigated, many parameters need more investigation such as concrete cover thickness, FRP bar diameter, spacing between FRP bars, the shape of concrete elements, and temperature variations. In this paper, a non linear finite element model is developed to analyze the distribution of thermal deformations in concrete cover surrounding FRP bars for prismatic concrete beams subjected to temperature increase up to 60°C using ADINA program. Also, this investigation permits to evaluate

the ratio of concrete cover thickness to FRP bar diameter ( $c/d_b$ ) that reduces splitting cracks in concrete for an asymmetric problem. A comparison between numerical, analytical and experimental results in terms of thermal deformations is presented.

## 2 FINITE ELEMENT INVESTIGATION

### 2.1 Finite element model

In order to analyse the distribution of transverse thermal deformations in concrete cover surrounding glass FRP (GFRP) bars, a nonlinear finite element study was carried out using ADINA program for prismatic concrete beams reinforced with GFRP bars under a temperature increase ( $\Delta T$ ) up to  $60^\circ\text{C}$ . Each beam has been reinforced with only one GFRP bar so as to obtain an asymmetric concrete confining action, as shown in figure 1. The ratio of concrete cover thickness to FRP bar diameter ( $c/d_b$ ) varied from 1 to 2.5. The cross-section of the prismatic concrete beam was modeled by means of two dimensional plane stress elements because of the constant of axial deformations. The finite element investigation was performed just for the half of the concrete cross-section (Figure 1b) for the reason of its symmetry with respect to z-z axis. The meshing of both concrete cover and FRP bar was carried out by utilizing triangular element with 6 nodes as shown in figure 2. GFRP bars used in this study were considered to have a linear elastic behavior. The bars used in this study had a nominal diameter of 9.5, 12.7, 15.9, 19.1, or 25.4 mm. GFRP bars coated with sand are composed from E-glass fibers and vinyl-Ester resin. The mechanical properties of GFRP bars determined by experimental tests (Zaidi and Masmoudi 2007) are presented in Table 1. The modulus of elasticity ( $E_t$ ) and Poisson's ratio ( $\nu_t$ ) of GFRP bars in the transverse direction have been evaluated theoretically using the rule of mixture and were found to be equal to 7.1 GPa and 0.38, respectively. The average values of the transverse and the longitudinal coefficients of thermal expansion (CTE) of GFRP bars for the five FRP bar diameters tested were found to be equal to  $\alpha_t = 33 \pm 4 \times 10^{-6}/^\circ\text{C}$  and  $\alpha_l = 8 \pm 1 \times 10^{-6}/^\circ\text{C}$ , respectively. The concrete used had a non-linear behavior. Mechanical and physical properties of the concrete are reported in Table 2. The average tensile strength ( $f_{ct}$ ) of concrete was determined by splitting tests and the average compressive strength ( $f'_c$ ) has been evaluated by standard compression tests. The modulus of elasticity of concrete ( $E_c$ ) has been determined according to CAN/CSA-S806-02 guidelines, while the Poisson's ratio of concrete ( $\nu_c$ ) was assumed 0.17. The coefficient of thermal expansion (CTE) of concrete determined by experimental tests was found to be equal to  $11.6 \pm 2.1 \times 10^{-6}/^\circ\text{C}$ .

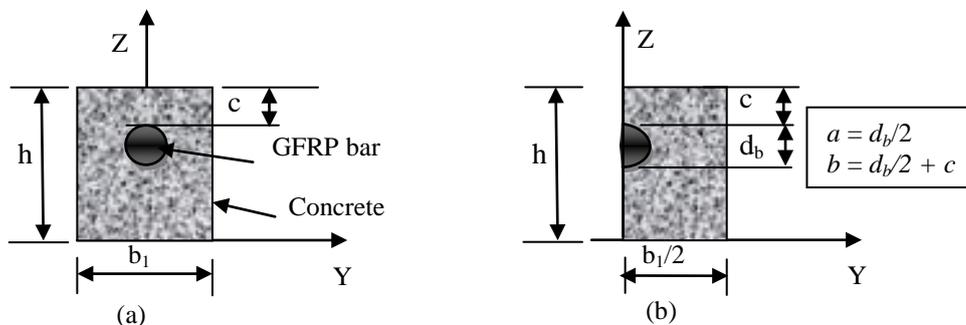


Figure 1. Cross-section of modeled prismatic concrete beam reinforced with GFRP bar

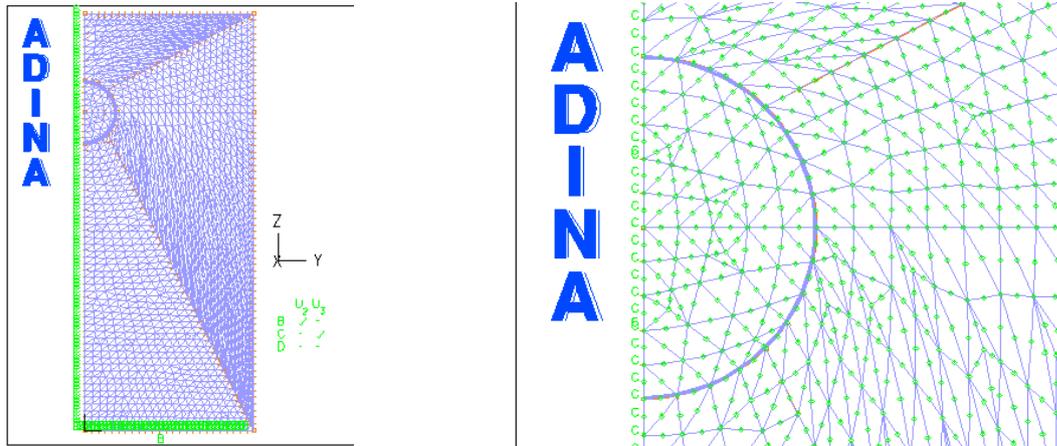


Figure 2. Meshing of the both concrete and GFRP bar

Table 1. Mechanical properties of GFRP bars

Bar diameter $d_b$ (mm)	Ultimate tensile strength $f_{fu}$ (MPa)	Longitudinal modulus of elasticity, $E_l$ (GPa)	Longitudinal Poisson's ratio, $\nu_l$
9.5	$627 \pm 22$	$42 \pm 1$	$0.28 \pm 0.02$
12.7	$617 \pm 16$	$42 \pm 1$	$0.28 \pm 0.02$
15.9	$535 \pm 9$	$42 \pm 1$	$0.28 \pm 0.02$
19.1	$600 \pm 15$	$40 \pm 1$	$0.28 \pm 0.02$
25.4	N/a	N/a	$0.28 \pm 0.02$

Table 2. Mechanical and physical properties of concrete

Compressive strength $f'_{c28}$ (MPa)	Tensile strength $f_{ct28}$ (MPa)	Modulus of elasticity $E_c$ (GPa)	Poisson's ratio ( $\nu_c$ )	Coefficient of thermal expansion $\alpha_c$ ( $\times 10^{-6}/^\circ\text{C}$ )
$40 \pm 3$	$4.1 \pm 0.1$	$28 \pm 2$	0.17	$11.6 \pm 2.1$

## 2.2 Non-linear numerical results and discussion

Figure 3 shows transverse thermal strains versus temperature variations ( $\Delta T$ ) at FRP bar/concrete interface of prismatic concrete beams reinforced with GFRP bars and having a ratio of concrete cover thickness to FRP bar diameter ( $c/d_b$ ) varied from 1 to 2.5. It can be observed that the transverse thermal strains are linear and similar until  $\Delta T$  equal to 30, 35, and 45°C corresponding to ratios of  $c/d_b$  ranged from 1 to 1.2, 1.3 to 1.9, and 2 to 2.5, respectively. From these thermal loads, the thermal strains increase suddenly because of the producing of the circular crown of splitting cracks, around FRP bars, which are great and profound. Also, it can be seen that for  $c/d_b < 1.6$  the transverse thermal strains are important when  $\Delta T \geq 30^\circ\text{C}$  and can be reached 6000  $\mu\epsilon$  in average which is 3 times greater than that of specimens having  $c/d_b \geq 1.6$ . Figure 4 illustrates the

effect of FRP bar diameter on the transverse thermal strain at the interface of FRP bar/concrete of an asymmetric problem with a constant concrete cover thickness equal to 20 mm varying FRP bar diameter. It can be seen that the FRP bar diameter has no big effect on the transverse thermal for  $\Delta T \leq 30^\circ\text{C}$ . While for  $\Delta T > 30^\circ\text{C}$ , the transverse thermal strain increases with the increase in the FRP bar diameter, as shown in Figure 4. The same observation can be noted for transverse thermal strains at the FRP bar/concrete interface of prismatic concrete beams reinforced with constant FRP bar diameter varying concrete cover thickness (20 , 25, 30 mm), as shown in figure 5.

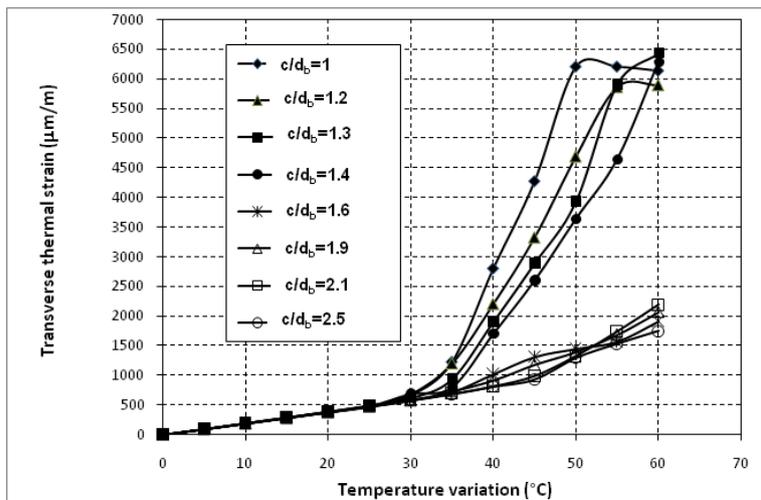


Figure 3. Numerical transverse thermal strains at FRP bar/concrete interface for reinforced prismatic concrete beams having different ratios of  $c/d_b$

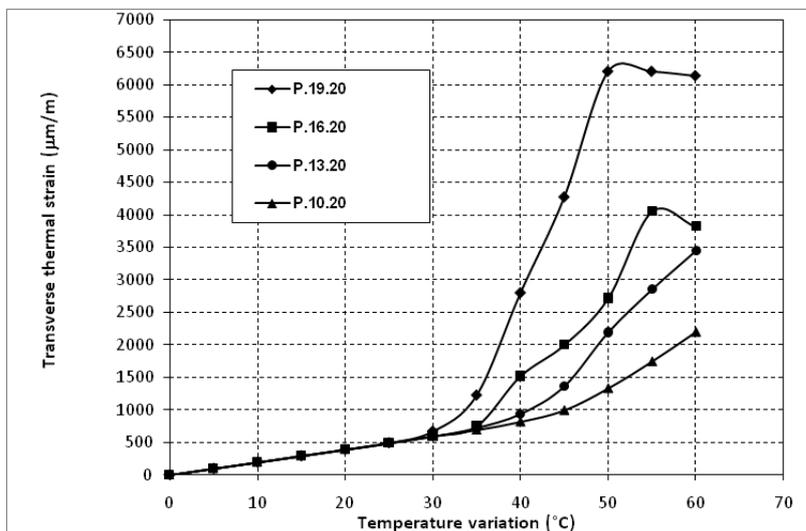


Figure 4. Numerical transverse thermal strains at FRP bar/concrete interface for prismatic concrete beams reinforced with different FRP bar diameters and having concrete cover thickness  $c = 20$  mm. (P.19.20 = P:Prismatic specimen ; 19 : nominal size of the bar (19.1mm) ; 20 mm : concrete cover thickness)

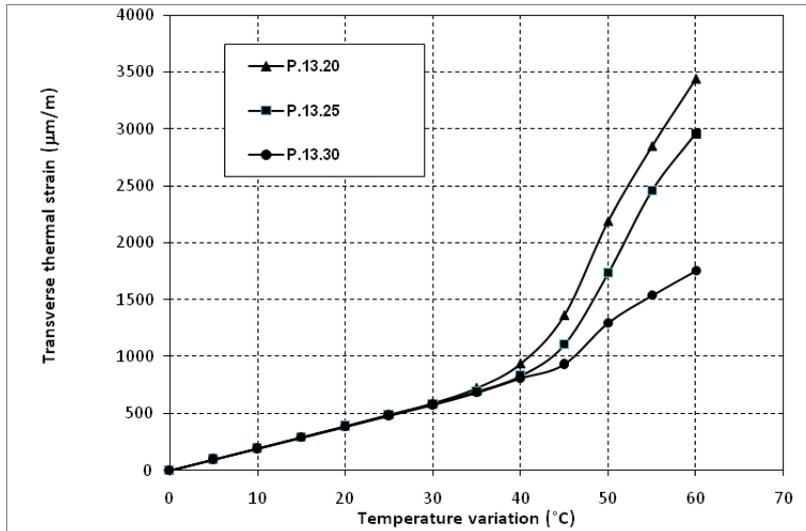


Figure 5. Numerical transverse thermal strains at FRP bar/concrete interface for prismatic concrete beams reinforced with 12.7mm FRP bar diameter and having different concrete cover thickness. (P.13.20 = P:Prismatic specimen ; 13 : nominal size of the bar (12.7mm) ; 20 mm : concrete cover thickness)

Figure 6 and 7 present the transverse thermal strains curves at the external surface of concrete cover of prismatic concrete beams reinforced with GFRP bar having a ratio  $c/d_b$  varied from 1 to 2.5. From figure 7, it is observed that for a ratio  $c/d_b \leq 1.6$  transverse thermal strains curves are linear and similar until temperature increase of 45 °C and 55 °C corresponding to ratios  $c/d_b = 1$ , and 1.2 to 1.6, respectively, from which the concrete thermal strains raise abruptly because of splitting cracks which reach the external surface of concrete cover or too close (Figure 8b and 8c). Nevertheless, for  $c/d_b > 1.6$ , transverse thermal strains curves are linear and similar, as shown in figure 6. It can be concluded that ratios of  $c/d_b > 1.6$  have no big influence on transverse thermal strains at the external surface of concrete cover of prismatic concrete beams reinforced with GFRP bars and subjected to temperature increase up to +60 °C.

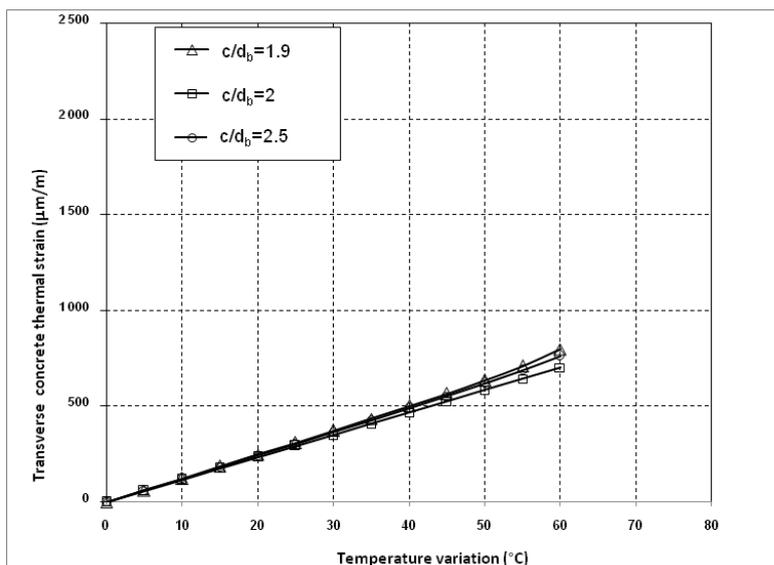


Figure 6. Numerical transverse thermal strains versus temperature variation at external surface of concrete cover prismatic concrete beams having ratios  $c/d_b > 1.6$

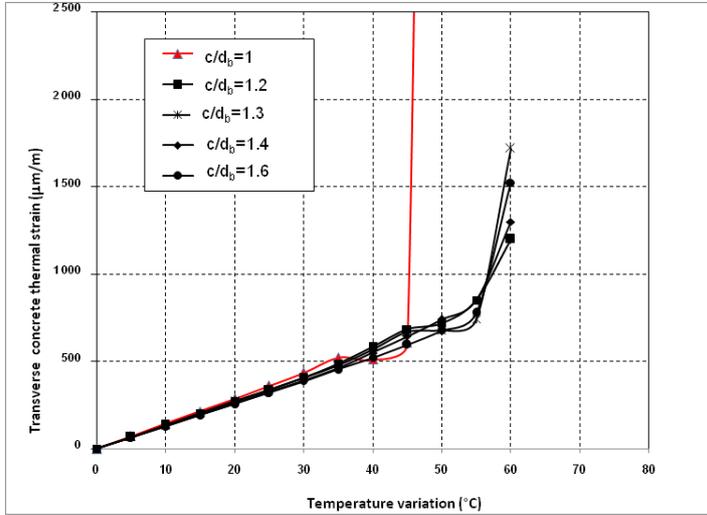
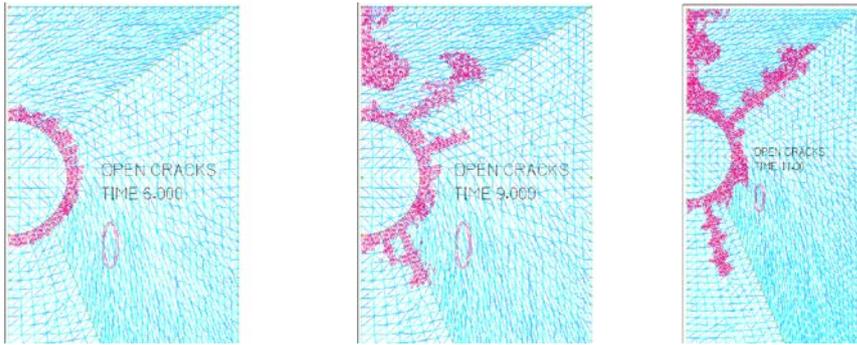


Figure 7. Numerical transverse thermal strains versus temperature variation at external surface of concrete cover prismatic concrete beams having ratios  $c/d_b \leq 1.6$



(a) For  $c/d_b = 1$  ;  $\Delta T = +30^\circ\text{C}$       (b) For  $c/d_b = 1$  ;  $\Delta T = +45^\circ\text{C}$       (c) For  $c/d_b = 1.2$ ;  $\Delta T = +55^\circ\text{C}$

Figure 8. Radial cracks within concrete cover at FRP bar/concrete interface.

### 3 LINEAR ANALYTICAL MODEL

The analytical model used in this study is based on the theory of elasticity. Masmoudi et al. (2005), Aiello et al. (2001), and Rahman et al. (1995) were developed an analytical models for concrete cylinder reinforced with FRP bar submitted to temperature variation  $\Delta T$ . The difference between transverse thermal expansions of FRP bar and concrete induces radial pressure  $P$  at the interface of FRP bar/concrete when  $\Delta T$  increases. This radial pressure generates circumferential tensile stresses that may cause splitting failure of concrete cover if the confining action of concrete is not sufficient. The transverse thermal strains in concrete ( $\varepsilon_{ct}$ ) and in FRP bar ( $\varepsilon_{ft}$ ), at the interface of FRP bar/concrete, due to the radial pressure  $P$  and the temperature variation  $\Delta T$ , are given by the following equations:

$$\varepsilon_{ct}(a) = \frac{P}{E_c} \left( \frac{r^2 + 1}{r^2 - 1} + \nu_c \right) + \alpha_c \Delta T \quad [1]$$

$$\varepsilon_{ft}(a) = \alpha_t \Delta T - \frac{(1 - \nu_t)}{E_t} P \quad [2]$$

where  $E_t$  is the modulus of elasticity of the FRP bar in the transverse direction;  $\nu_{tt}$  is the Poisson's ratio of FRP bar in the transverse direction;  $\alpha_t$  is the transverse coefficient of thermal expansion of FRP bar;  $r = b/a$  is the ratio of circumscribed concrete cylinder radius to FRP bar radius (see Figure 1b);  $E_c$  is the modulus of elasticity of concrete;  $\nu_c$  is the Poisson's ratio of concrete; and  $\alpha_c$  is the coefficient of thermal expansion of concrete;  $P$  is the radial pressure exerted by FRP bar on the concrete, obtained from the compatibility equation of transverse deformations using Eq. (1) and Eq. (2).

$$P = \frac{(\alpha_t - \alpha_c)\Delta T}{\frac{1}{E_c} \left( \frac{r^2 + 1}{r^2 - 1} + \nu_c \right) + \frac{1}{E_t} (1 - \nu_{tt})} \quad [3]$$

However, the transverse thermal strain of concrete  $\varepsilon_{ct}(b)$ , at the external surface of concrete cover of prismatic concrete beams, due to the radial pressure  $P$  and the temperature variation  $\Delta T$ , is given by:

$$\varepsilon_{ct}(b) = \frac{2P}{E_c(r^2 - 1)} + \alpha_c \Delta T \quad [4]$$

#### 4 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS

Figures 9 and 10 compare typical analytical and numerical transverse thermal strains, at the external surface of concrete cover, with those obtained from experimental tests carried out by Zaidi and Masmoudi (2007) on prismatic concrete beams reinforced with GFRP bars under thermal loads up to +60 °C, having ratios of  $c/d_b$  equal to 1.3 and 1.9, respectively. For a ratio  $c/d_b = 1.3$  as shown in figure 9, numerical and analytical results predicted from analytical and numerical models are in good agreement with experimental results until a temperature increase of 45 °C from which numerical and experimental results are greater than those evaluated from analytical model because of profound splitting cracks, within concrete beams, which are not considered in the linear analytical model. While, for a ratio  $c/d_b = 1.9$ , numerical, analytical and experimental strain curves are linear and similar, as shown in figure 10, since there is no crack near the external surface of concrete beams.

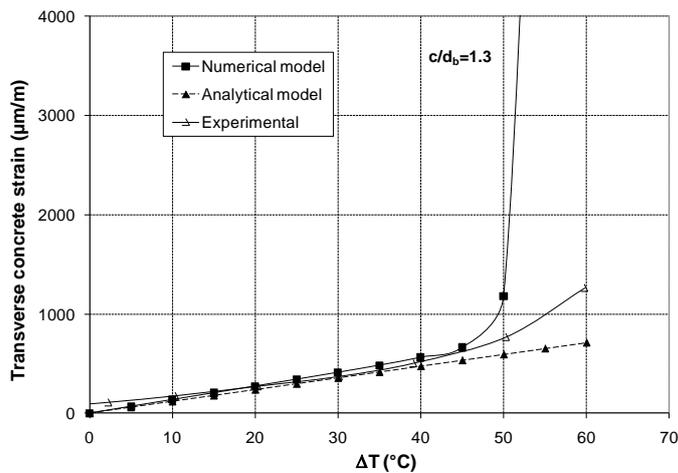


Figure 9. Transverse thermal strain at external surface of rectangular concrete beam having a ratio  $c/d_b=1.3$  - Comparison between numerical, analytical and experimental results.

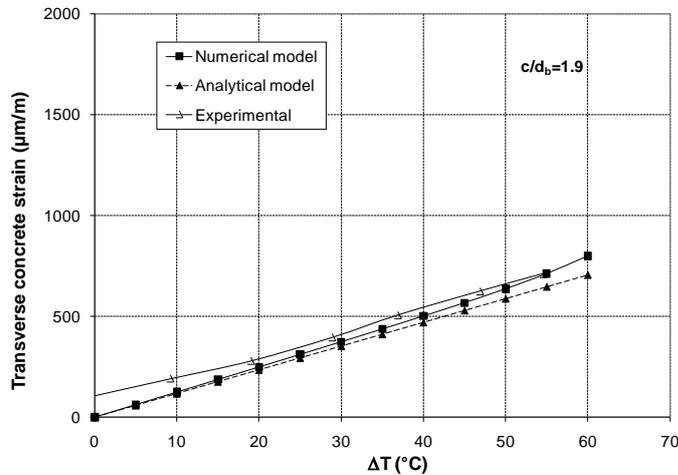


Figure 10. Transverse thermal strain at external surface of rectangular concrete beam having a ratio  $c/d_b=1.9$  - Comparison between numerical, analytical and experimental results.

## 5 CONCLUSIONS

- FRP bar diameter variation has no big influence on transverse strains of prismatic concrete beams under a temperature increase  $\Delta T \leq 30^\circ\text{C}$ . However, for  $\Delta T > 30^\circ\text{C}$  the transverse thermal strain increases with the increase in FRP bar diameter. The same conclusion can be noted for the concrete cover thickness effect on transverse thermal strains.
- The numerical transverse strain curves, at FRP bar/concrete interface of prismatic concrete beams under thermal loads up to  $+60^\circ\text{C}$  having  $c/d_b$  varied from 1 to 2.5, are linear and similar until thermal loads  $\Delta T$  varied from  $+30$  to  $+45^\circ\text{C}$  depending of  $c/d_b$  ratio. From these thermal loads, the transverse strain curves increase abruptly because of the circular crown of splitting cracks developed within concrete around FRP bars.
- A ratio of concrete cover thickness to FRP bar diameter  $c/d_b > 1.6$  with  $\Delta T \leq 30^\circ\text{C}$  seems to be sufficient to avoid harmful radial cracks within concrete surrounding GFRP bar under high temperatures for an asymmetric problem studied in this investigation.
- For a ratio  $c/d_b > 1.6$ , the transverse thermal strains of concrete, at the external surface of concrete cover, predicted from numerical and analytical models are in good agreement with experimental results obtained from prismatic concrete beams reinforced GFRP bar submitted to thermal loads up to  $+60^\circ\text{C}$ .

## 6 REFERENCES

- Aiello, M.A., Focacci, F., and Nanni, A. 2001. Effects of thermal loads on concrete cover of fiber reinforced polymer reinforced elements: Theoretical and experimental analysis. *ACI Materials Journal*, 98: 332-339.
- Canadian Standards Association. 2002. *Design and construction of building components with fiber – reinforced polymers*. CAN/CSA-S806-02, Toronto, Ontario, Canada,
- Masmoudi R, Zaidi A, Gérard P. 2005. Transverse thermal expansion of FRP bars embedded in Concrete. *Journal of Composites for Construction*. 9: 377-387.
- Rahman HA, Kingsley CY, Taylor DA. 1995. Thermal stress in FRP reinforced concrete. *Proceedings of Annual Conference of the Canadian Society for Civil Engineering*. Ottawa, 605-614.
- Zaidi, A., and Masmoudi, R. 2007. Effect of Concrete Cover Thickness and FRP-Bars Spacing on the Transverse Thermal Expansion of FRP Bars. *8<sup>th</sup> International Symposium on Fiber Reinforced Polymer Reinforcement for Concrete Structures*, University of Patras, Department of Civil Engineering, Patras, Greece.