Second Conference on Smart Monitoring,
Assessment and Rehabilitation of Civil Structures

SMAR 2013

Predicting the Deflection of RC Beams Strengthened with FRP Laminates

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ABSTRACT: The primary concerns in rehabilitation of existing structures are strength and ductility of the system but sometimes the deformation of a member may become critical. The flexural behavior of a RC beam strengthened with externally bonded reinforcement (EBR) such as Fiber Reinforced Polymer (FRP) sheets is well understood but the deflection of the member is a topic that not many researchers have addressed over these years. Design guidelines have pointed out to some simple and approximate ways to calculate the deflection but a more accurate relationship is needed to prevent unexpected behavior under service and ultimate loads.

In this study, an effort was made to find a suitable relationship for estimating the deflection of RC beams strengthened with FRP laminates using EBR technique. Prediction of the deflection is made at service load level by determining different values for effective moment of inertia. Results of several experimental works done by other researchers were used to validate the accuracy of the proposed equations. The experimental results correlated well with the values predicted using the proposed simple equations with less than 17% error.

Keywords: Deflection, FRP laminates, RC Beams, Rehabilitation.

1 INTRODUCTION

Today, considering high cost of reconstruction of old buildings, the issue of strengthening and repair of existing buildings and those, which are damaged due to different reasons, are discussed frequently. By studying the failure of RC building, it is noticed that several factors such as: the mistake in original design, deterioration of materials and construction error, may cause the problem. In addition, the change in building codes cause the re-evaluation of structures design and if it is needed, these structures need retrofit and rehabilitation. The use of fiber reinforced polymer (FRP) sheets for strengthening and repair of RC structures becomes very attractive. The installation procedure is quick and does not require heavy machinery. Therefore, the performance of members strengthened with FRP laminates should be investigated and studied.

Many investigations were done in flexural behavior of RC beams strengthened with FRP sheets and it is proven that externally bonding FRP sheets to RC beams increase their ultimate load carrying capacity. Only a few studies were conducted to determine methods for prediction of deflection of them. In this paper, the short time deflection of RC beams strengthened with FRP sheets is empirically evaluated and the findings compare with existing experimental data.

2 CALCULATION OF DEFLECTION OF RC BEAMS WITH FRP SHEETS

In most studies, the deflection of RC beams strengthened with FRP sheet numerically calculated using integration technique using the moment-curvature diagram. This method is very accurate and has been adopted by the guidelines offered by several building codes such as ACI 440.2R-08, CNR-DT 200/2004 and fib-14-2001. Unfortunately, the method is very time consuming and may not be very practical for design engineers. Therefore, it should be find more simple and fast method for public whom using it.

The deflection of an elastic and homogenous beam which is depend on span length, type of loading, boundary conditions, modulus of elasticity and moment of inertia, is calculated by using structural analysis techniques. For example, the maximum deflection at the mid-span of a beam in a four points load system can be calculated as follow:

$$\delta_{\max} = \frac{\mathrm{P.La}}{48\,\mathrm{E\,I}}\left(3L^2 - 4{L_a}^2\right) \tag{1}$$

Where L is the span of the beam; P is the total concentrated load divided into two concentrated loads (P/2), each applied at a distance L_a from the support; E is the modulus of elasticity that should be considered the modulus of elasticity of concrete for RC beams; and I is moment of inertia of beam section that should be used as I_e in RC beam which means the effective moment of inertia of the beam section after cracking. According to the ACI 318-11specifications, the effective moment of inertia suggested by Branson, I_e, can be determined by following equation:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{2}$$

In Equation 2, M_{cr} is the cracking moment; M_a is the maximum service moment; I_{cr} is the moment of inertia of the cracked transformed section, and I_g is the moment of inertia of the gross section neglecting the steel reinforcement.

In RC beams strengthened with FRP sheets all parameters of span length, type of loading, boundary conditions and modulus of elasticity can be considered as same as before. Now, if a simple relationship can be found to estimate a suitable value for moment of inertia of retrofitted beam, one can predict the amount of deflection without significant hardship. It is obvious that finding the proper value of, I_e, should be done by using existing experimental data and then it can be used in Equation 2 to determine the deflection.

3 COLLACTING AVAILABEL DATA

A wide range of experimental test results of bending test on retrofitted beams were gathered that contain over 513 data points. The data points were drawn from load-displacement diagrams of 57 RC beams strengthened with FRP sheets tested under four-point loading by other researchers. An overview of different experimental studies is summarized in Table 1. A wide range of different parameters such as concrete strength, modulus of elasticity, reinforcement ratio, load level, FRP sheet termination point and span length were considered.

Since the control of deflection at service load level is important, most deflection's relationships are defined that level of loading. As a result, corresponding deflection relationship cannot be used for higher levels of loading. Ultimate strength of a member, shall be taken as the nominal strength calculated in accordance with concrete design provision such as ACI 318-11, multiplied by the strength reduction factors, φ. The basic requirement for strength design may be expressed as follows:

$$\varphi\, M_n \geq M_u \tag{3}$$

Where M_n is nominal moment (design moment), M_u is ultimate moment that should be calculated based on sum of 1.2 times dead load and 1.6 times live load (average 1.4) and φ is reduction factor that can be calculated using Figure 1. So, the service load level can be estimated with ultimate load of a beam multiplied by value of $\varphi/1.4$.

Table 1.Experimental Studies of RC Beams Strengthened with FRP Laminates

Study	Number of beam specimens	Number of data points
Alusalloum et al. (2002)	2	11
Rahimiet al. (2001)	8	59
Dong et al. (2002)	1	9
Ahmed et al. (2001)	7	68
Pham et al. (2004)	14	180
Gao et al. (2006)	7	91
Esfahani et al. (2007)	3	18
Demakos (2008)	4	16
Ceroni et al. (2001)	6	27
Algusundaramoorthy et al. (2003)	5	34

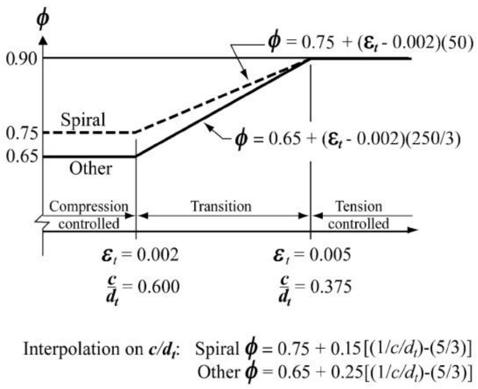

Figure 1.Variation of φ with Tensile Strain in Rebar for Grade 60 Steel (ACI 318-11).

In Fig. 1, c is the depth of the neutral axis at nominal strength, d_t is the distance from the extreme compression fiber to the extreme tension steel, and ε_t is the tensile strain in extreme tension steel. In this research, ε_t is calculated for all the experimental beams collected and φ is calculated accordingly. Ultimately, it was observed that strain in steel rebar exceeds 0.005 and therefore the φ is considered as 0.9. Therefore, the service load level consider 60% (0.9/1.4 = 0.64) of the experimental ultimate load and all of 513 data points were considered fell within this range. For all of the points, M_{cr}, I_g, I_{cr}, and I_e (obtained from Bronson's equation), are calculated. In addition, theoretical deflections were calculated by using different moment of inertia and were compared with empirical deflections value.

4 ANALYSIS OF DATA

In order to determine the best relationship for estimating the deflection of strengthened beam and find effective ways to improve those several steps were taken.

First Step: I_g is considered as effective moment of inertia in deflection relationships similar to Equation 1 and the deflection was calculated. For every data point, the error values were calculated by following relationship.

error = (calculated deflection – experimental deflection)/ experimental deflection (4)

As shown in Table 2 and Figure2, the average value of deflection error, which is calculated with I_g, is equal to 60.8%. These values were positive and negative for different data points. Therefore, it is more useful to consider the error as the absolute value of error at each data point. The average of absolute errors was 61.3%, which could be considered very high. Conducting a statistical approach to find a coefficient for correcting the value of moment of inertia, αI_g, a correction value of $\alpha = 0.4$ was obtained. The average error of 2.2% and the average absolute error of 31% was obtained that were still high.

Second Step: I_{cr} is considered as effective moment of inertia. As shown in Table 3 and Figure 3, the observed average error and absolute error were 25% and 34%, respectively. Conducting a similar approach as before, to find a correction factor for the value of moment of inertia, βI_g, a correction value of $\beta = 1.2$ was obtained. The average error of 4% and the average absolute error of 28% was obtained that were still high.

Table 2.average of error and error absolute for calculating deflection if consider $I=\alpha\ I_g$

$I = \alpha\ I_g$		
absolute error (abs(e))	error (e)	α
0.581	0.5646	.25
0.386	0.3038	0.3
0.32	0.1176	.35
0.313	-0.0221	0.4
0.327	-0.1308	.45
0.352	-0.2177	0.5
0.41	-0.3481	0.6
0.469	-0.4412	0.7
0.525	-0.5111	0.8
0.573	-0.5654	0.9
0.613	-0.6089	1

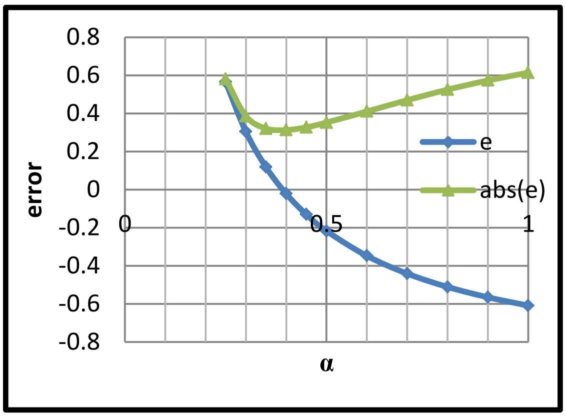

Figure 2.Variation of error and absolute error for calculating deflection if consider $I=\alpha I_g$ with change of α

Table 3.average of error and error absolute for calculating deflection if consider $I=\beta\ I_{cr}$

$I = \beta\ I_{cr}$		
absolute error (abs(e))	error (e)	β
0.434	0.387	0.9
0.344	0.249	1
0.294	0.135	1.1
0.283	0.086	1.15
0.279	0.041	1.2
0.283	-0.039	1.3
0.319	-0.168	1.5

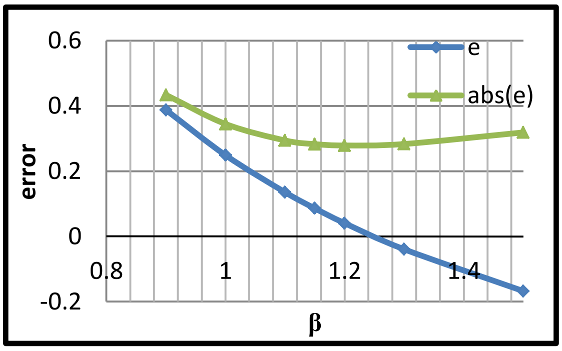

Figure 3. Variation of error and absolute error for calculating deflection if consider $I=\beta I_{cr}$ with change of β

Third Step: A combination of two moments of inertia, I_g and I_{cr}, were considered as follows:

$$I = \gamma I_g + (1-\gamma) I_{cr} \quad (5)$$

By similar approach, as shown in Table 4 and Figure 4, it was found that with $\gamma = 0.1$ the average absolute error was minimized at 27.5% and the average value for error became 1.3%. It was apparent that the effect of the correction factor in this case was even less significant.

Forth Step: When in Equation 5, γ is put equal to $(M_{cr}/M_a)^3$, it change to Bronson's relationship used to estimate I_e. As shown in Table 5 and Figure 5, using this relationship and a correction factor for I_e, nI_e, least absolute error value (n = 1) was calculated as 17.4%, which showed significant improvement in comparison with previous steps.

Table 4.average of error and error absolute for calculating deflection if consider $I=\gamma\ I_g+(1-\gamma)\ I_{cr}$

$I = \gamma\ I_g + (1-\gamma) I_{cr}$		
absolute error (abs(e))	error (e)	γ
0.33	0.225	0.008
0.329	0.222	0.009
0.327	0.219	0.01
0.312	0.192	0.02
0.3	0.166	0.03
0.292	0.141	0.04
0.285	0.117	0.05
0.2758	0.052	0.08
0.2752	0.013	0.1
0.276	-0.005	0.11
0.278	-0.023	0.12
0.305	-0.144	0.2
0.353	-0.257	0.3

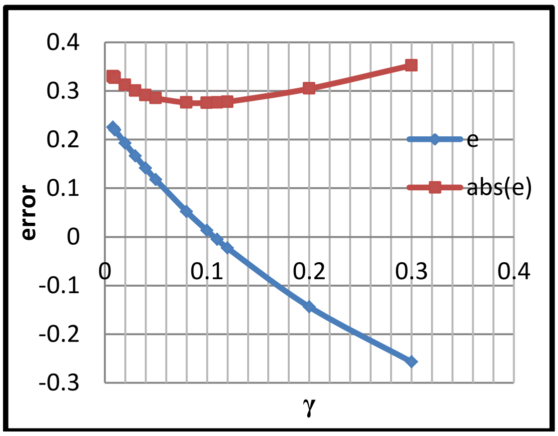

Figure 4. Variation of error and absolute error for calculating deflection if consider $I=\gamma\ I_g+(1-\gamma)\ I_{cr}$ with change of γ

Table 5.average of error and error absolute for calculating deflection if consider $I= n\ I_e$

$I = n\ I_e$		
absolute error (abs(e))	error (e)	n
0.945	0.942	0.5
0.418	0.386	0.7
0.198	0.079	0.9
0.1736	-0.029	1
0.189	-0.117	1.1
0.278	-0.253	1.3

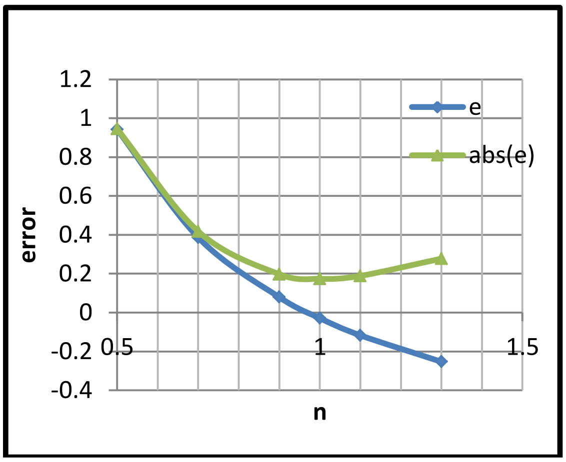

Figure 5. Variation of error and error absolute for calculating deflection if consider $I= n\ I_e$ with change of n

Table 6.average of error and error absolute for calculating deflection if consider I= $(m\frac{M_{cr}}{M_a})^3 I_g + [1-\left(m\frac{M_{cr}}{M_a}\right)^3] I_{cr}$

$\boldsymbol{I = (m\frac{M_{cr}}{M_a})^3 I_g + [1-\left(m\frac{M_{cr}}{M_a}\right)^3] I_{cr}}$		
absolute error (abs(e))	error (e)	m
0.289	0.188	0.5
0.183	0.019	0.9
0.1736	-0.029	1
0.178	-0.076	1.1
0.225	-0.168	1.3

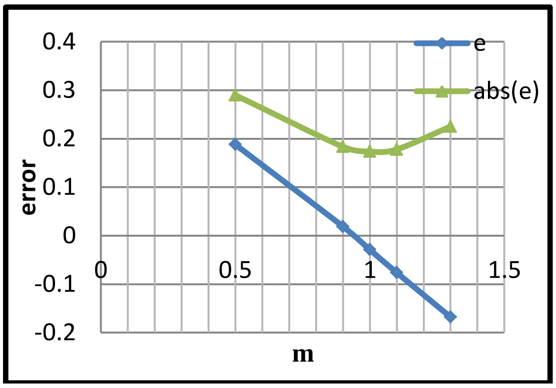

Figure 6. Variation of error and error absolute for calculating deflection if consider I= $(\boldsymbol{m\frac{M_{cr}}{M_a}})^3 \boldsymbol{I_g} + [\mathbf{1}-\left(\boldsymbol{m\frac{M_{cr}}{M_a}}\right)^3] \boldsymbol{I_{cr}}$ with change of m

Fifth Step: In order to show if the Bronson's relationship could be improved by using a correction factor, m(M_{cr}/M_a) in Bronson's equation, observing values shown in Table 6 and Figure 6, the best value for the correction factor is determined as m = 1, which indicated that the Bronson's relationship is the best estimate.

Sixth Step: The final attempt to improve Bronson's relationship was the use of different power value instead of traditional value of three. Two statistical approaches were considered to show and test the effectiveness of the approaches:

- The power is varied in order to minimize the absolute error
- The power is varied in order to maximize the correlation factor of straight line regression

Following the first approach, as shown in Table 7 and Figure7, it was observed that when using power value of p = 2.8 the average absolute error became minimized and close to 17.23%.

In second approach, the values of experimental deflection values and estimated values calculated with Bronson's relationship with different power values, r, were plotted and the correlation factor, R^2, of a best-fit straight line was maximized and the slope of the line, s, became close to one. As shown in Table 8 and Figure 8, it was clear that the power values larger that p = 3.5 provides similar values for slope, correlation factor and error.

5 CONCLUSIONS

In this study, by gathering 57 experimental results from flexural test of RC beams strengthened with FRP laminate and obtaining over 513 data points, the effectiveness of existing relationships to estimate the deflection of strengthened beam under service load was examined. Several corrections were tested to see if the estimation error could be decreased.

The result showed that for a simple calculation of deflection of strengthened RC beams with FRP laminates under service load:

- Using Bronson's relationship with power values of above 3.5, deflection can be estimated with about 17% absolute error.
- Using 0.4 times I_g as effective moment of inertia, deflection can be estimated with 31.2% absolute error.

- Using 1.2 times I_{cr} as effective moment of inertia, deflection can be estimated with 27.9% absolute error.
- Using $0.1I_g+0.9\ I_{cr}$ as effective moment of inertia, deflection can be estimated with 27.5% absolute error.

Table 7.average of error and error absolute for calculating deflection if consider $I=(\frac{M_{cr}}{M_a})^p I_g + [1 - (\frac{M_{cr}}{M_a})^p]\ I_{cr}$

absolute error (abs(e))	error (e)	p
0.376	-0.358	1
0.215	-0.152	2
0.177	-0.081	2.5
0.1724	-0.058	2.7
0.1723	-0.048	2.8
0.1727	-0.038	2.9
0.1736	-0.028	3
0.179	-0.003	3.3
0.183	0.011	3.5
0.194	0.041	4
0.205	0.064	4.5
0.215	0.082	5

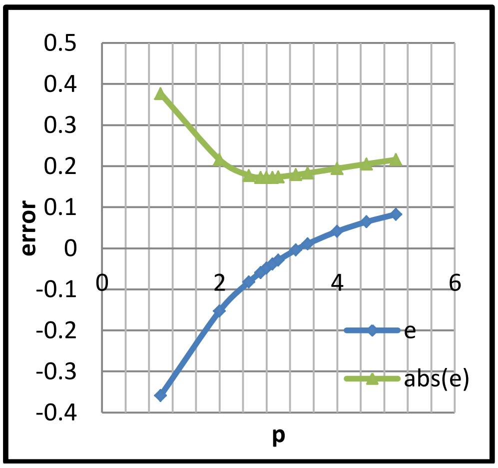

Figure 7. Variation of error and error absolute for calculating deflection if consider $I=(\frac{M_{cr}}{M_a})^p I_g + [1 - (\frac{M_{cr}}{M_a})^p]\ I_{cr}$ with change of p

Table 8.slope (s) of trend line and data correlation (R^2) and % error by changing of p

% error	s	R^2	p
1.1205	0.7835	0.8475	1
1.1225	0.895	0.8784	1.5
1.0892	0.9564	0.8976	2
1.0616	0.9862	0.9075	2.5
1.0472	0.9985	0.9118	3
1.0423	1.0019	0.9130	3.5
1.0424	1.0012	0.9129	4
1.0448	0.9988	0.9122	4.5
1.0477	0.9958	0.9112	5
1.0533	0.99	0.9094	6
1.0575	0.9853	0.908	7
1.0604	0.9816	0.9069	8
1.0645	0.9763	0.9053	10

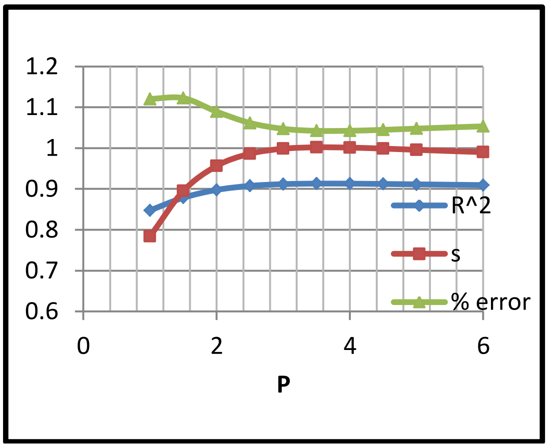

Figure 8. Variation of slope (s) of trend line and data correlation (R^2) and % error with power (p)

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