Second Conference on Smart Monitoring,
Assessment and Rehabilitation of Civil Structures

SMAR 2013

Ultrasonic waves for health monitoring of composite structures

U. Gabbert, S. Duczek, S.M.H. Hosseini

Otto-von-Guericke-University of Magdeburg, Germany

ABSTRACT: Ultrasonic waves offer a great potential for structural health monitoring (SHM) in many industrial applications. The application of ultrasonic waves takes advantage of mode conversions and wave reflections at distinguished discontinuities, which can be used to identify structural failures and damages. The high sensitivity of Lamb waves with respect to small structural changes and the low costs to build a network of surface attached piezoelectric transducers to excite and to receive the waves makes such a system very attractive for industrial applications. The numerical modeling of waves in complex structures with commercial finite element software tools, applying standard finite elements with linear or quadratic shape functions, requires an enormous computational power, which exceeds today´s computer resources. Therefore, we proposed to use higher order finite element schemes, which may result in much lower computational costs for ultrasonic wave propagation analysis in complex fiber reinforced composites. In the paper the advantage of higher order finite elements is demonstrated with help of a benchmark example. Finally it is shown that the numerical results can also be experimentally verified by applying a laser-scanning vibrometer.

1 INTRODUCTION

In 1917 a special type of elastic ultrasonic guided waves, propagating in thin-walled structures with free boundaries, has been described by Horace Lamb (1917), named Lamb waves later on. Such waves travel in at least two basic modes, a symmetric and an anti-symmetric one (Figure 1), Viktorov (1967). A characteristic feature of ultrasonic guided waves (Lamb waves) is their dispersive behavior, Giurgiutiu (2008). The dispersion curves in Figure 2 are calculated analytically by Perez (2012) and with a semi-analytical (SAFE) numerical approach by Ahmad (2011). Under certain conditions a conversion of the symmetric mode into the anti-symmetric one or vice versa can be observed. Mode conversions occur when the propagating wave packet encounters damages or structural discontinuities, like holes, cracks, stringers, stiffeners etc.

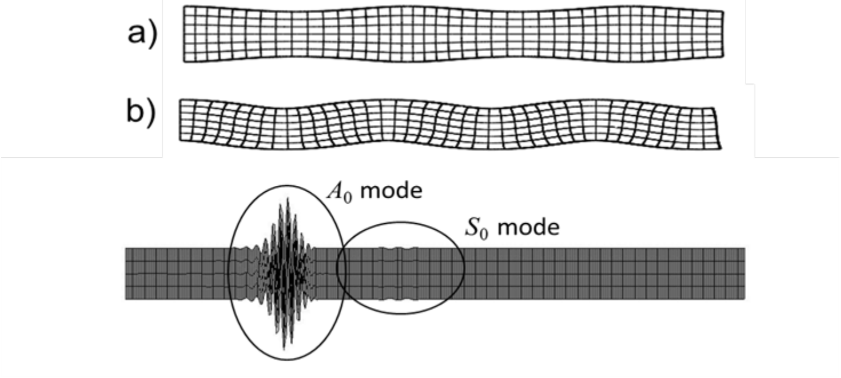

Figure 1: Lamb modes: a) symmetric mode S_i and b) anti-symmetric mode A_i

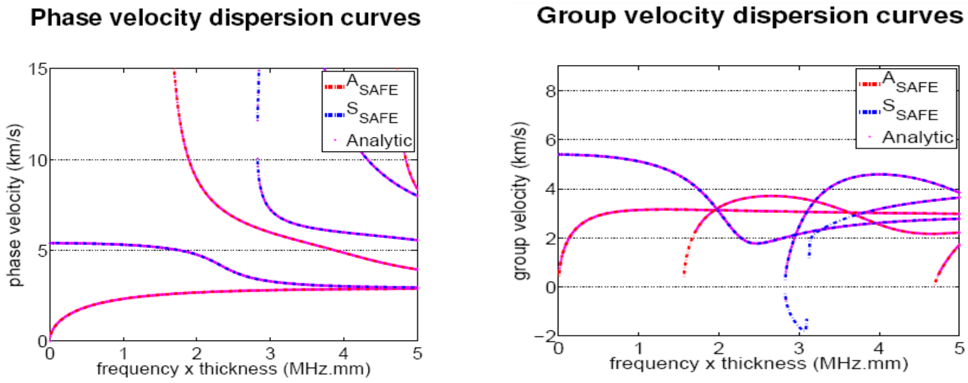

Figure 2: Phase velocity and group velocity of a thin aluminum plate

Despite the mentioned complex propagation properties there are important features making Lamb waves very interesting for SHM purposes and account for their popularity, Boller et.al. (2004). The general concept of a SHM system is shown in Figure 3. One characteristic of Lamb waves are their small wavelengths in higher frequency ranges making them sensitive towards small structural damages. Another property is the low amplitude loss, which is proportional to $r^{-0.5}$ only, compared to other types of elastic waves (*r* is the travelled distance from the source), Su & Ye (2009). From a numerical point of view the simulation of ultrasonic guided waves is a highly demanding task. Due to the short wavelength a finite element analysis requires a rather refined temporal as well as spatial discretization. If applying 3D finite elements with linear or quadratic shape functions (e.g. Abaqus, Ansys, Nastran, Marc) the required model size exceeds quickly the today available computer resources. To overcome these limitations we propose the application of higher order finite element schemes. Quite popular are the normalized integrals of the Legendre-polynomials, resulting in the hierarchical *p*-FEM, Szabo & Babushka (1991), Düster (2002), Duczek & Gabbert (2012). Another approach is to employ finite elements with Lagrange-Polynomials on the Gauss-Lobatto-Legendre grid points, also referred to as spectral elements (*S*EM), Komantitsch & Tromp (2002), Kudela & Ostachwicz (2009), Schmicker (2011). Recently, finite elements based non-uniform rational B-splines have been developed (*N*-FEM), Bazilevs (2006), Cotrell et al. (2009), Cox (1972). The mentioned higher order finite elements schemas schemes have been extended recently by the authors to accommodate the simulation of electrically excited ultrasonic guided waves. An anisotropic polynomial degree distribution is proposed to be able to vary the polynomial order in each co-ordinate direction in three-dimensional applications, Duczek and Gabbert 2013; Willberg and Gabbert 2012, Perez 2012. So far, after the best of our knowledge, only the *S*EM has been very successfully applied to solve high frequency wave propagation problems.

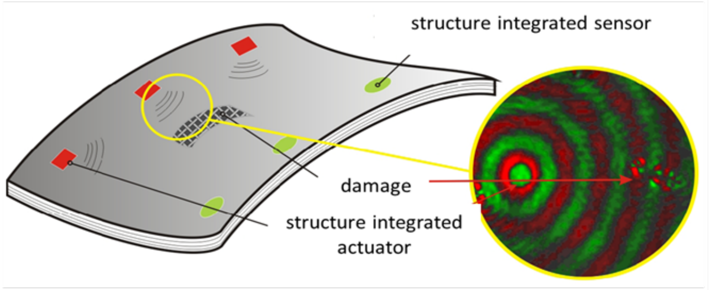

Figure 3: Concept of a structural health monitoring (SHM) system based on ultrasonic waves

Analytical and semi-analytical methods are restricted to special problems and simple geometries, e.g. to calculate dispersion diagrams, Giurgiutu (2008), Ahmad (2011). An alternative approach to solve the 3D wave equation is the application of the finite difference method. The LISA (Local Interaction Simulation Approach) software, Lee & Staszewski 2007, and the EFIT (Elastodynamic Finite Integration Technique) software, Fellinger et al, 1995, are based on the FDM. To the authors knowledge no fair comparison between the FDM and the FEM with respect to wave propagation analysis has been published yet. From the authors experiences the FDM has several restrictions compared with the FEM. The adaptation of the polynomial degree to a given task, resulting in an exponential convergence rate, the flexibility to adapt curved boundaries and to fulfill the required boundary conditions, the high condition number of the resulting systems matrices are some advantages of higher order FEM schemes. From our perspective, higher order FEM schemas are the most suitable approach if complex engineering structures have to be analyzed and optimally designed. In the following the *p*-FEM, the *S*EM and the *N*-FEM are tested based on a benchmark problem in order to show their advantages in comparison with conventional finite elements; for details we refer to Willberg et al. (2012).

2 COMPARISON OF DIFFERENT HIGER ORDER FINITE ELEMENT SCHEMAS

2.1 Finite element analysis

In the standard finite element approach the shape functions are introduced as polynomials in the following way

$$\mathbf{u}(\mathbf{x},t) = \mathbf{N}(\mathbf{x})\mathbf{U}(t)\,. \tag{1}$$

Here **u(x**, *t*) contains the unknown functions, **x** is the vector of co-ordinates, *t* is the time, **N**(**x**) is a matrix containing the shape functions of the finite element, **U**(*t*) is the vector of unknown (nodal) parameters, which are functions of time *t*. Based on the standard finite element procedure, Szabo & Babushka (1991), the semi-discrete form of the equation of motion is derived as

$$\mathbf{K}\mathbf{U}(t) + \mathbf{C}\dot{\mathbf{U}} + \mathbf{M}\ddot{\mathbf{U}}(t) = \mathbf{F}(t)\,. \tag{2}$$

Here **K** is the stiffness matrix, **C** is the damping matrix, **M** is the mass matrix, and **F** is the load vector. In commercial software packages, such as Ansys, Abaqus, Nastran and Marc, only finite elements are available using linear or quadratic polynomials as shape functions. For the simulation of ultrasonic waves we recommend the application of higher order polynomials as shape functions. Here we focus on three different types of polynomials, namely the Lagrange polynomials with special supporting points (*S*EM), the Legendre polynomials (*p*-FEM) and the non-rational B-spline polynomials (*N*-FEM). In one dimension the shape functions can be written as

$$N_n(x) = \Phi_n^{type}(x), \qquad \text{type} = \begin{cases} \text{Lagrange}: & S\text{EM} \\ \text{Legendre}: & p - \text{FEM} \\ \text{NURBS}: & N - \text{FEM} \end{cases} \tag{3}$$

The shape functions in two dimensional or three dimensional problems can be established as tensor product of the one-dimensional polynomials Eq. (3) as

$$\begin{aligned} N_n^{3D}(x,y,z) = N_i(x)\cdot N_j(y)\cdot N_k(z), \quad & n = 1,2,\ldots,(p_x+1)(p_y+1)(p_z+1), \\ & i = 1,2,\ldots,(p_x+1),\ j = 1,2,\ldots,(p_y+1), \\ & k = 1,2,...,(p_z+1) \end{aligned} \tag{4}$$

For details regarding the special shape functions, the development and implementation of the different finite element schemas we refer to Willberg et al. (2012), Duczek et al. (2012), Duczek & Gabbert (2013) and Willberg & Gabbert (2011).

2.2 Analysis of Lamb waves - A convergence study

The above mentioned higher order finite elements are applied to solve a 2D benchmark problem. The computational results are compared with an analytical reference solution given by Perez (2012). The geometry and the boundary conditions are depicted in Figure 4 including also the material properties and the geometry of the benchmark example. The length of the plate is chosen such that no reflection from the right boundary is affecting the signals at the two measuring points A, B during the simulation time. The excitation is performed with time dependent forces in form of a sine-function modulated by a Hann-window as described in Figure 4, resulting in a narrow banded frequency content of the signal. Due to its smaller wavelength it is sufficient to study the behavior of the A_0-mode only. If the discretization for the A_0-mode is sufficient, the S_0-mode is captured as well with at least the same accuracy.

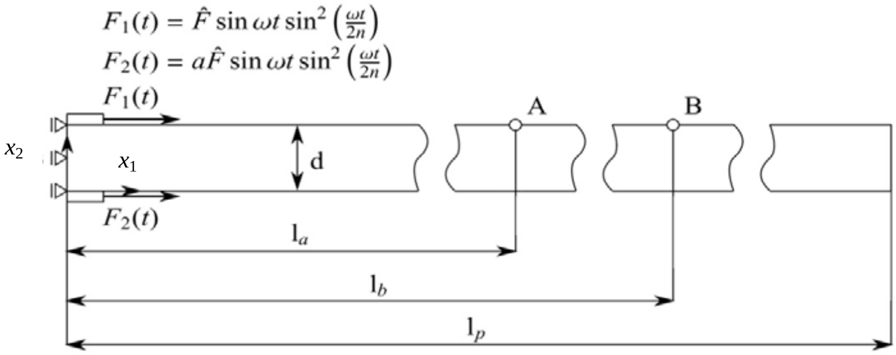

Figure 4: Two-dimensional model of an aluminum plate with loads and boundary conditions:
a=1: excitation symmetric mode, a=-1: excitation anti-symmetric mode;
Dimension: l_a=100 mm, l_b=200 mm, l_p=500 mm, d=2 mm;
Material properties: E=7 10^{10} N/mm^2; ν=0.33; ρ=2700 kg/m^3;
Longitudinal speed: c_1=6197 m/s, transversal speed: c_2=3121 m/s.

The time of flight (t_c) is an appropriate quality indicator. To extract the time of flight a Hilbert-transform is applied to the time history of the displacements saved at points A (u_A) and B (u_B) in Figure 4. Several different investigations regarding the accuracy, the convergence behavior and the efficiency have been performed by Willberg et al. (2012). Figure 5 displays the results of a convergence study with respect to a varying polynomial degree in the global x_1-direction. To avoid any influence of the mesh density in the x_2-direction a fixed polynomial degree of $p_{x2} = 4$ has been applied, which guarantees that the measured signal quality is only influenced by p_{x1}. For the different polynomial degrees in the x_1-direction pseudo-nodes per wavelength versus relative error (E_{rel}) in the time of flight curves are plotted. In the *S*EM the pseudo-nodes correspond to physical nodes. In the *N*-FEM and the *p*-FEM they are a distinct measure of the mesh density. The investigations have shown, that the convergence rate is the faster, the higher the polynomial orders are. Figure 5 shows, that p_{x1}=3 results in a low number of pseudo-nodes per wavelength (<10) to approximate the A_0-mode with a relative error of about 1 %. We also note that the *p*-FEM and the *S*EM behave nearly the same. In terms of accuracy there is no advantage of the *S*EM as proposed in the literature (see e.g. Kudela & Ostachovicz, 2009). The *N*-FEM exhibits a faster convergence rate in relation to the *S*EM and the *p*-FEM. The *N*-FEM exhibits a $C_{p\text{-}1}$ continuity in comparison to a C_0 continuity of the other elements. When looking at the results for *N*-FEM with C_0 continuous elements we have observed the same rate of convergence as in the other two methods. This suggests that the higher order continuity is the reason for the higher convergence rate as seen in the Figure 5. Evaluating all results a polynomial degree template of $p_{x1} = 3$ and $p_{x2} = 4$ is recommended for all three investigated higher order finite element approaches (see Willberg, Duczek et al. 2012).

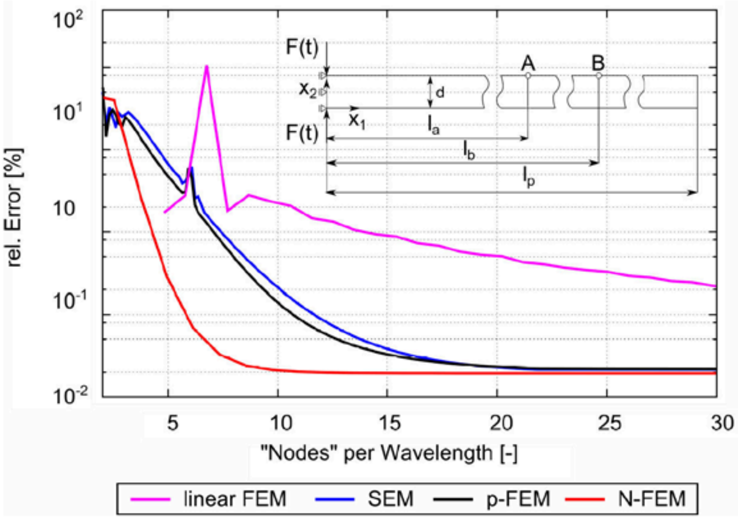

Figure 5: Convergence curves for the A_0 mode for different "nodes" per wave length for $p_{x1} = 3$, $p_{x2} = 4$; the ordinate shows the relative error in the group velocity; for details see Willberg et al. (2012).

Finally, the wave propagation in a 3D plate is analyzed to test the above given recommendations. The plate problem has been calculated with all three types of finite elements applying the above given optimal template of polynomial degrees. Figures 6 and 7 show a quarter of the plate. Due to the double-symmetry this quarter is sufficient to analyze the wave field.

Figure 6: Simulation of the wave propagation in a plate (*p*-FEM: $p_{x1}=p_{x2}=3$, $p_{x3}=4$; mesh density:25×25×1, dof: 69312); see Duzek & Gabbert (2013)

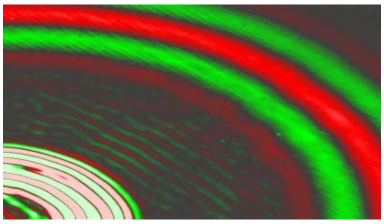

Figure 7: Wave field of the plate measured with a laser-scanning vibrometer

The plate is excited with a circular piezoelectric patch actuator (PIC151) of 10 mm diameter and a thickness of 1mm posed on top in the middle of the plate (the lower corner at Figures 67). The excitation signal is the same as given in Figure 4. Figure 6 shows a snap-shot of the propagating waves, with the fast symmetric wave (longer wave length) and the slow anti-symmetric bending wave (shorter wave length). The results have been verified with the commercial finite element package Abaqus, where a mesh with more than one million degrees of freedom (dof) is required to achieve the same accuracy as it is received with, e.g., the *p*-FEM.

3 EXPERIMENTELL INVESTIGATIONS

An experimental investigation of the wave propagation can be performed, e.g., with help of a high accurate laser-scanning vibrometer as shown in Figure 8. This method is widely used to study the wave propagation phenomenon at the surface of structures. A great advantage is the application of three laser heads, which allow recording the three-dimensional wave field with a very high precision. We have applied the 3D scanning-laser technique from Polytec (PSV 400 3D), which results in a very high accuracy, especially if the in-plane component is important.

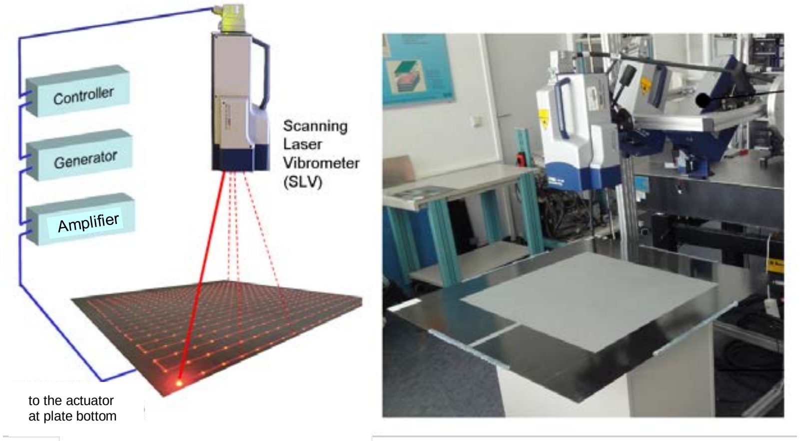

Figure 8: Measured wave field in a plate similar to Fig. 6, see Willberg, Mook et al. 2012

With this technique the simulated plate structure from Figure 6 was also experimentally investigated resulting in a good agreement between the simulation results and the measurements. Several further plates have been numerically investigated, such as aluminum and steel plates, plates manufactured from carbon reinforced plastics, sandwich plates with core layers made from honeycomb and hollow spheres, and other particle reinforced materials, Willberg et al. (2012), Hosseini & Gabbert (2012), Hosseini et al (2013). We have investigated several typical damages and compared experimental and numerical results successfully, Pohl et al. (2012), Pohl & Mook (2013). Figure 9 shows the Lamb wave propagation in three CFRP plate with different types of "damages". The main feature of Figure 9 is the mode conversion from the fast S_0-mode into the slow A_0 mode, which can be recognized at the sensor. Very recent investigations deal with the so called "continuous mode conversion", which was first time experimentally observed and described at a fabric reinforced plate, Willberg, Koch et al. (2012).

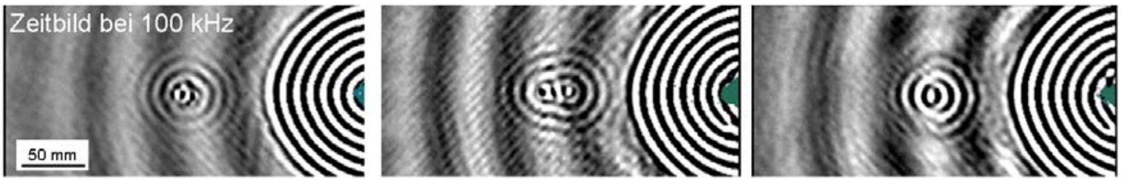

Figure 9: Mode conversion at different types of "damages" (from left: circular and elliptic thickness reductions, circular thickening), see Willberg, Mook et al., 2012.

4 CONCLUSION AND OUTLOOK

The paper presents the evaluation of three recently developed higher order 3D finite element schemas to analyze the propagation of ultrasonic waves (Lamb waves) in different kinds of thin-walled structures. It has been shown that the application of higher order finite elements leads to a significant reduction of the overall computational costs. Especially the isogeometric finite elements (*N*-FEM) have shown excellent performance and accuracy. The finite element results are also verified with measurements applying a 3D laser-scanning vibrometer. Based on these finite elements the development of general purpose simulation software for ultrasonic guided waves is in progress to optimally design future SHM systems.

Figure 11: Carbon reinforced structure covered with a net of piezoelectric patches as actuators and sensors

The development of a SHM system is an ongoing research project. We envision that a future SHM system consists of a network of surface-bonded encapsulated piezoelectric actuators and sensors (Figure 11). Each sensor and actuator themselves may constitute a single, autonomous and intelligent subsystem, which communicates with other subsystems via the structure, e.g. by applying ultrasonic waves. Local energy harvesting units, integrated into the subsystems, autonomously provide the subsystems with the required energy. Such a viable option could be the use of a piezoelectric resin as proposed by Arlt, et al, 2008.

Acknowledgement: The authors like to thank the German Research Foundation (DFG) and all partners of the joint project PAK357 for their support (GA 480/13-2).

REFERENCES

Ahmad, Z. A. B., 2011. *Numerical Simulations of Lamb Waves in Plates Using a Semi-Analytical Finite Element Method*. PhD thesis, Otto-von-Guericke-University of Magdeburg, VDI Fortschrittberichte, 20, Nr. 437, VDI-Verlag, Düsseldorf.

Ahmad, Z.A.B., Gabbert, U. 2012. Simulation of Lamb wave reflections at plate edges using the semi-analytical finite element method. *Ultrasonics*, 52, pp. 815-820.

Arlt, C., Röstermund, D., Riedel, U., Sinapius, M. 2008. Functionalized ceramic nanoparticle in epoxy resin as high performance matrices for CFRP, in *Nonofair 2008 New Ideas for Industries*, VDI-Verlag GmbH 2008, pp. 53-59 (ISBN 978-3-18-092027-6, ISSN 0083-5560).

Bazilevs, Y. 2006. *Isogeometric Analysis of turbulence and fluid-structure interaction*. PhD thesis, University of Texas at Austin.

Boller, C., Staszewski, W., Tomlinson, G. 2004. *Health monitoring of aerospace structures*. John Wiley & Sons.

Cottrell, J.A., Hughes, T. J. R., Bazilevs, Y. 2009. *Isogeometric analysis: Toward integration of CAD and FEA*. John Wiley & Sons.

Cox, M.G. 1972. The numerical evaluation of B-splines. *Journal of the Institute of Mathematics and its Applications*, 10:134–149.

Duczek, S., Willberg, C. Schmicker, D., Gabbert, U. 2012. Development, Validation and Comparison of Higher Order Finite Element Approaches to Compute the Propagation of Lamb Waves Efficiently. *Key Engineering Materials*, Vol. 518, pp. 95-105.

Duczek, S., Gabbert' U. 2013. Anisotropic hierarchic finite elements for the simulation of piezoelectric smart structures. *Engineering Computations*, Vol. 5 (online available).

Düster, A. 2002. *High order finite elements for three-dimensional, thin-walled nonlinear continua*, PhD thesis, Technical University of Munich.

Fellinger, P., Marklein, R., Langenberg, K. J., Klaholz, S. 1995. Numerical modelling of elastic wave proagation and scattering with EFIT – Elastodynamik Finite Integration technique, *Wave Motion*, Vol. 21, pp. 47-66.

Giurgiutiu, V. 2008. *Structural health monitoring with piezoelectric wafer active sensors*, Academic Press (Elsevier).

Ha, S., Chang, F.-K. 2010. Optimizing a spectral element for modelling PZT-induced lamb wave propagation in thin plates. *Smart Materials and Structures*, 19:1–11.

Hosseini, S.M.H., Gabbert, U. and Lammering, R.: Influence of the cover plate thickness on the Lamb wave propagation in honeycomb sandwich panels. *CEAS Aeronautical Journal*, Vol.4, Number1, April 2013, pp. 69-76.

Hosseini, S. M., Kharaghani, A., Kirsch, C., Gabbert, U. 2013. Numerical simulation of Lamb wave propagation in metallic foam sandwich structures: a parametric study. *Composite Structures*, Vol. 97, pp. 387-400.

Hosseini, S.M., Gabbert, U. 2013. Numerical simulation of the Lamb wave propagation in honeycomb sandwich panels: a parametric study. *Composite Structures*, Vol. 97, pp. 189-201.

Komatitsch, D., and Tromp, J. 2002. Spectral-element simulations of global seismic wave propagation validation, *International Journal of Geophysics*, 149:390–412.

Kudela, P., Ostachowicz, W. 2009. 3D time-domain spectral elements for stress waves modelling, *Journal of Physics*, 181:1–8.

Lamb, H. 1917. On waves in an elastic plate. *Royal Society of London Proceedings Series A*, 93, pp. 114–128.

Lee, B., Staszewski, W. J. 2007. Lamb wave propagation modelling for damage detection in metallic structures: Part i and Part ii, *Smart Materials and Structures*, Vol. 16, pp. 249-259, pp. 260-274.

Peng, H., Meng, G., Li, F. 2009. Modelling of wave propagation in plate structures using three-dimensional spectral element method for damage detection. *Journal of Sound and Vibration*, 320, pp. 942–954.

Perez, J. M. V. 2012. *Numerical methods in the simulation of Lamb waves for structural health monitoring applications in composites.* PhD Thesis, Otto-von-Guericke-University of Magdeburg, VDI Fortschrittberichte, 20, Nr. 441, VDI-Verlag, Düsseldorf.

Pohl, J., Willberg, C., Gabbert, U., Mook, G. 2012, Experimental and theoretical analysis of Lamb wave generation by piezoceramic actuators for structural health monitoring. *Experimental Mechanics*, pp. 429-438.

Pohl, J., Mook, G. 2013. Laser vibrpmetric analysis of propagation and interaction of Lamb waves in CFRP plates, *CEAS Aeronautical Journal*, online January 2013.

Schmicker, D. 2011. *Development and testing of higher order finite elements based on Lagrange polynomials for the analysis of guided ultrasonic waves in thin-walled structures*. Diploma-Thesis, Otto-von-Guericke-University Magdeburg.

Su, Z., Ye, L. 2009. *Identification of damage using Lamb waves: From fundamentals to applications*. Springer.

Szabo, B., Babushka, I. 1991. *Finite Element Analysis*. John Wiley and Sons.

Viktorov, A. 1967. *Rayleigh and Lamb Waves*. Plenum Press New York.

Weber, R., Hosseini, S.M.H, Gabbert, U. (2013). Numerical simulation of the guided Lamb wave propagation in particle reinforced composites. *Composite Structures*, Volume 94, pp. 3064-3071.

Willberg, C., Duczek, S., Vivar-Perez, J.M., Schmicker, D., Gabbert, U. 2012. Comparison of different higher order finite element schemes for the simulation of Lamb waves. *Computer Methods Appl. Mech. Engrg*., 241-244, pp. 246-261.

Willberg, C., Gabbert, U. 2012. Development of a three-dimensional piezoelectric isogeometric finite element for smart structure applications. *Acta Mechanica*, Volume 223, pp. 1837-1850.

Willberg, C., Koch, S., Mook, G., Pohl, J., Gabbert, U. 2012. Continuous mode conversion of Lamb waves in CFRP plates, *Smart Materials and Structures*, Volume 21, 075022.

Willberg, C. Mook, G., Pohl, J., Gabbert, U. 2012. Laser-vibrometric measurement and numerical modelling of local and continuous mode conversion of Lamb waves in CFRP plates, in Proc. of the 6th *European Workshop on Structural Health Monitoring*, Dresden, July 3– 6, 2012, Th.2.E.1 (7 pages).