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Simplified modeling of FRP wrapping of rectangular cross-sections

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ABSTRACT: The goal of this research project is to provide a simplified closed form solution to determine directly the ultimate confined concrete strength. Common cross-section shapes for RC columns are considered herein, namely square and rectangular. The simplified model is derived from a more refined iterative confinement model proposed by the same authors. Based on a detailed analysis of the stress state through Mohr's Circle, a simplified solution is proposed to account for the non-uniformly confined concrete performance exhibited in non-axisymmetric sections. The key aspect of the proposed method is the evaluation of the effective pressure to be inserted in a triaxial confinement model, based on a refined mechanical approach rather than on other "conventional" approaches. Experimental data available in literature were compared with the results of the theoretical simplified analyses to validate the proposed approach.

1 INTRODUCTION

The confining action of FRP jackets gives the best performance on circular columns, whose geometrical configuration allows the pressure due to fiber wrapping to be effective on the entire cross-section. A different behavior characterizes square and rectangular columns; in these cases, due to the presence of the corners, a part of the cross-section remains unconfined. So that square- and rectangular-section columns were found to experience less increase in strength and ductility than their circular counterparts. This is because the distribution of lateral confining pressure in circular sections is uniform, in contrast to square and rectangular sections, in which the confining pressure varies from a maximum at the corners and diagonals, to a minimum in between. Similar to the confinement with steel hoops, that loss of effectiveness is "conventionally" modeled with parabolic areas defined by the corners and eventually by longitudinal steel rebars. This conventional approach still represents an unresolved issue even in terms of code provisions. Usually confinement models proposed in International Design Codes and in scientific literature for non-circular sections are based on conventional parabolic arching action within which the concrete is fully confined and confining pressure of an equivalent circular cross-section with diameter D equal to the diagonal of the rectangular cross-section are the basis of those models.

2 THEROETICAL BASIS OF THE PROPOSED SIMPLIFIED APPROACH

Existing analytical models for predicting the behavior of FRP-confined concrete are mostly derived for cylindrical plain concrete columns. Most of existing models available for non-circular confined concrete assessment both in terms of ultimate capacity as of stress-strain relationships rely on an assumed value of an "equivalent" lateral confining pressure.

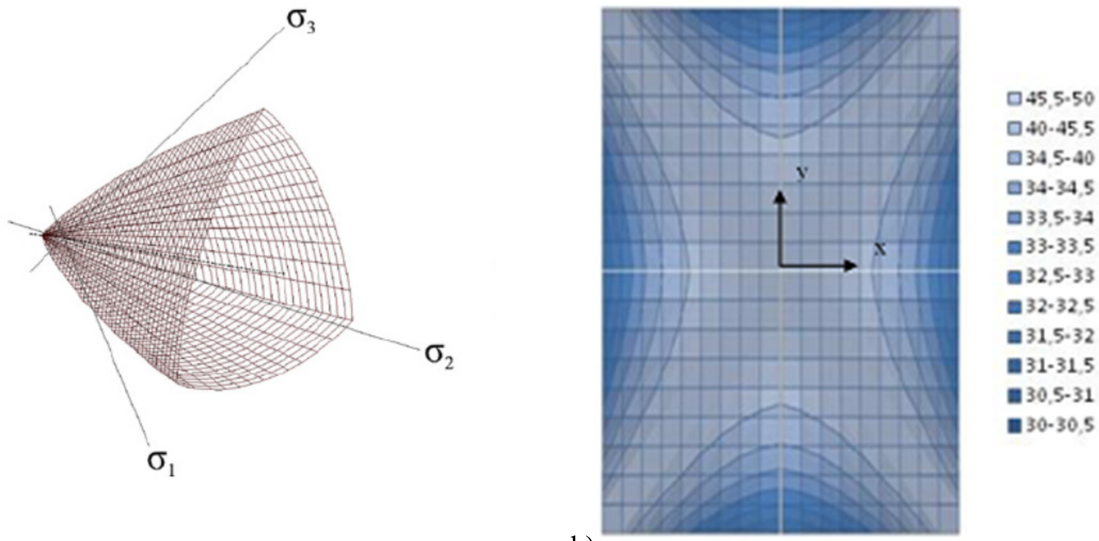
Despite the great research effort in the experimental field, considerable work is still needed to fully outline a definitive analytical model to predict the behavior of FRP confined concrete. A contribution in this direction, to deepen knowledge on non-axisymmetric or non-uniform confinement, is provided by researchers. Solid mechanics based models have been proposed by Lignola et al. (2008) to account for the confinement of hollow cross-sections and by Lignola et al. (2010) to account for the confinement of prismatic cross-sections; but due to the not uniform confining stress field inside the cross-section, they are refined iterative models.

2.1 Refined iterative models

An iterative confinement model (Lignola et al. 2010) has been proposed for rectangular concrete cross-sections, in plane strain conditions (assuming that the increment of stress due to confinement is produced without any out of plane strain). The model converges to square sections simply considering two equal sides for the rectangular cross-section. The nonlinear behavior of concrete is accounted for by adopting a secant approach.

The key innovative aspect of that model is the evaluation of the contribution of confining stress field not equal in the two transverse directions x and y, that is evaluated in each point of the cross-section explicitly considering a plasticity model for concrete under triaxial compression (Figure 1). The final value of the confined concrete strength is the weighted average over the concrete section. Rather refined values of confined concrete strength are provided by dividing the section into only a few elements.

Theoretical results, based on the proposed iterative model have been found to be in satisfactory agreement with experimental data available in scientific literature.



a) b)

Figure 1. a) Plasticity model for concrete under triaxial compression; b) a contour plot of confined concrete strength over a cross-section (f_{cc} values in MPa).

2.2 Simplified model for square sections

Previous refined iterative confinement model is simplified to account for square sections without explicitly discretizing the cross-section. To avoid the meshing of the section and consequential time consumption, the integral mean is evaluated. The main idea at the basis of the simplified model is related to the Mohr–Coulomb failure envelope theory. The confined concrete strength, $\sigma_1=f_{cc}$, yet in Richart et al. (1928) is related to the confining pressure, $\sigma_2=\sigma_3=f_l$, assumed uniform in the two principal directions orthogonal to circular column axis. By using the Mohr–Coulomb failure envelope theory (Figure 2) for concrete having unconfined strength in compression f_{co}, confinement equation can be expressed as follows for confined concrete in circular sections:

$$f_{cc} = f_{co} + f_l \cdot \tan^2\left(45° + \frac{\varphi}{2}\right) = f_{co} + k_1 \cdot f_l \qquad (1)$$

The value proposed by Richart et al. (1928) was $k_1=4.1$ corresponding to $\varphi=37°$, which is an average value for the concrete subjected to low confinement pressure. Using triaxial tests, many authors proposed different expressions for k_1. Recent studies revealed that the value of k_1 can be also assumed as a function of the confinement level and should take into account the influence of the concrete strength on the ultimate behavior.

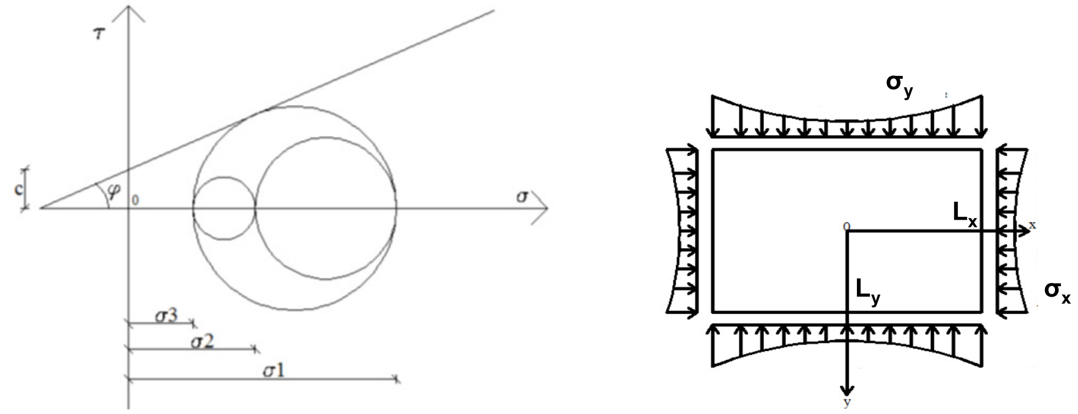


Figure 2. Confined concrete strength (related to the Mohr–Coulomb failure envelope theory) under uniform ($\sigma_2=\sigma_3$) and non-uniform ($\sigma_2>\sigma_3$) confining pressure field.

In non-circular applications, the confining pressure is not uniform in the plane of the cross-section (in two orthogonal directions $\sigma_3=f_{l,min}$ and $\sigma_2=f_{l,max}$, are different), but in any case the failure envelope can be related to the minimum confining pressure (Figure 2), being failure circle independent on σ_2. Hence, the strength of concrete is related only to minimum confining pressure continuously in each point, and integrated over the cross-section. The resultant axial force over cross-section area A is evaluated and then divided by the total area A to provide directly average confined concrete strength, $f_{cc,sq}$ (see eq. 2). In this way an average lateral pressure (term in brackets in eq. 2) is evaluated, which can be assumed as the "effective" confining pressure to be inserted in confinement model to obtain directly, and without any meshing, the confined concrete strength in square sections.

$$f_{cc,sq} = \frac{\int_{Cross-Section} (f_{cc})\, dA}{\int_{Cross-Section} dA} = \frac{\int_{Cross-Section} (f_{co} + k_1 f_l)\, dA}{\int_{Cross-Section} dA} = \\ = f_{co} + k_1 \cdot \left(\frac{\int_{Cross-Section} (f_{l,\min})\, dA}{\int_{Cross-Section} dA} \right) = f_{co} + k_1 \cdot f_{l,sq} \tag{2}$$

Given the equation of minimum confining pressure ($f_{l,min}$=min[σ_x; σ_y], with σ_x and σ_y given by Lignola et al. 2010), closed form solution of $f_{l,sq}$ is given by the solution of integrals in brackets in eq. 2):

$$f_{l,sq} = \frac{22 E_c E_f t_f \varepsilon_l}{25 E_c L + 24 E_f t_f \left(5 + 2\nu\right)} \tag{3}$$

In eq. 3 many parameters appear: L is the length of a side of the square cross-section, E_f and t_f are respectively the Young modulus and total thickness of the wrap, ε_l is the strain in the wrap at failure and finally, nonlinear mechanical properties of concrete are (secant) Young modulus E_c and apparent Poisson ratio (dilation ratio) ν at failure. However it can be easily verified that for typical values of involved parameters, influence of ν (even ranging widely between 0 and 2) is totally negligible in eq. 3, while E_c has an impact and it can be evaluated iteratively as the ratio between f_{cc} (evaluated according to eq. 2) and ultimate concrete strain, ε_{cc}, which can be assumed equal to:

$$\varepsilon_{cc} = \varepsilon_{co} \left(5 \frac{f_{cc}}{f_{co}} - 4 \right) \tag{4}$$

In previous equation ε_{cc} is the concrete strain at f_{cc}, and convergence on E_c can be found with few iterations starting with a trial value of the elastic tangent E_c.

2.3 *Simplified model for wall like sections*

Even though the refined nonlinear confinement model (Lignola et al. 2010) provides solutions also for wall like cross-sections b·h, where b>h (e.g. b/h>3 is assumed as the range of wall like columns), a simplified confinement model was also provided (Lignola et al. 2011) for wall-like cross-sections. According to this alternative simplified approach, which gives rather accurate results despite the heavily reduced computational effort, the confining stress field is only parallel to the longer side of the cross-section, thus neglecting the confinement in the shorter direction, the average confining pressure can be assumed equal to:

$$f_{l,wl} = 2 \frac{E_f t_f \varepsilon_l}{b} \tag{5}$$

In this case (figure 3) a “Biaxial” confinement is considered instead of a “Triaxial” confinement for concrete (e.g. eq. 1 with uniform confining pressure in cylindrical concrete confined members). The following approximated equation is derived (Lignola et al. 2011) according to the concrete ultimate strength surface (figure 1):

$$\frac{f_{cc,wl}}{f_{co}} = 1 + 1.42 \frac{f_{l,wl}}{f_{co}} - 1.40 \left(\frac{f_{l,wl}}{f_{co}} \right)^2 + 0.30 \left(\frac{f_{l,wl}}{f_{co}} \right)^3 \tag{6}$$

where f_l/f_{co}<1.2, otherwise it would result f_l>f_{cc}. It is highlighted that in any case, for "Biaxial" confinement, the increase in concrete strength $f_{cc,wl}/f_{co}$ is always smaller than 1.5.

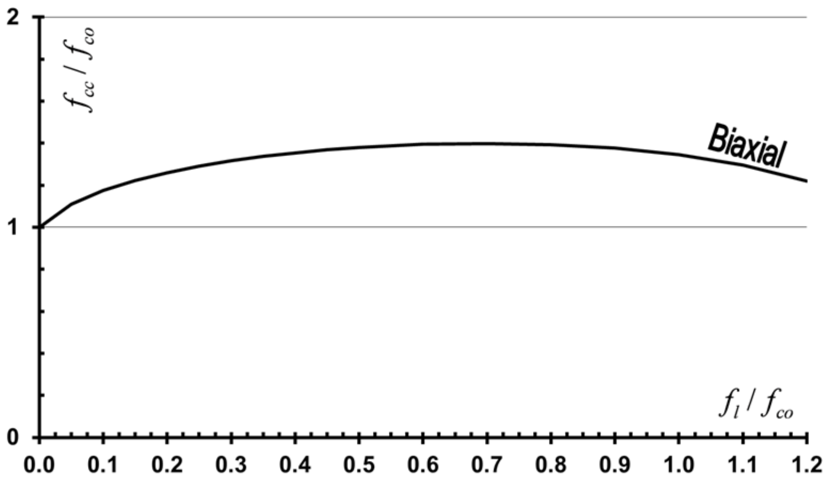


Figure 3. "Biaxial" confinement (Lignola et al. 2011).

3 PROPOSED CONFINEMENT MODEL FOR RECTANGULAR SECTIONS

3.1 Implementation of the model

The aim of this paper is to provide a direct, practical tool, oriented to the profession, to provide a simplified confinement model for rectangular cross-sections, avoiding refined nonlinear analyses mainly oriented to research (i.e. Lignola et al. 2010).

The basic idea is that a rectangular cross-section can be seen as in between a square section and a wall like section for which two simplified yet reliable confinement models have been already provided by the authors. A rectangular section having side aspect ratio ζ=b/h=1 is a square section, so that the confined concrete strength should be equal to $f_{cc,sq}$; on the other hand rectangular section having side aspect ratio $\zeta \geq 3$ is a wall like section, so that the confined concrete strength should be equal to $f_{cc,wl}$; finally a cross-section having $1<\zeta<3$ is expected to be in between the previous two limit cases. In view of this basic idea, a combination coefficient, γ, was derived, to represent these conditions (figure 4):

$$\gamma = \frac{3-\zeta}{2} > 0 \quad (7)$$

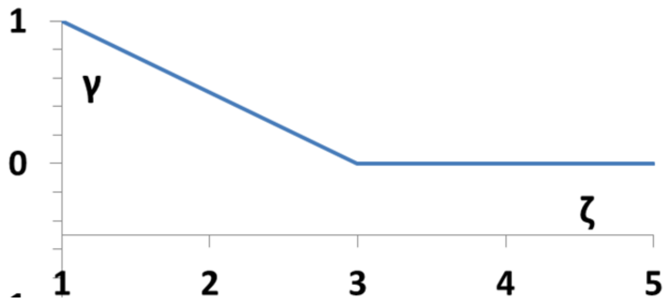


Figure 4. Combination coefficient, γ

According to these considerations, the confined concrete strength becomes (for a cross-section having a generic ζ≥1):

$$f_{cc,r} = f_{cc,sq}\left(\gamma^{\beta}\right) + f_{cc,wl}\left(1-\gamma^{\beta}\right) \tag{6}$$

Where β is a further numerical coefficient that can be calibrated, however it will be seen that a good yet simple estimate is β=1. In this procedure, in the simplified square section confinement model, the side L of the cross-section is the longest one, namely b, while in the simplified wall like confinement model, the side of the cross-section to be considered is the shorter one, namely h. The main reason for this is to reduce the "weight" of the square cross-section to the advantage of the potentially weaker wall like behavior. An outline of the proposed model is depicted in figure 5.

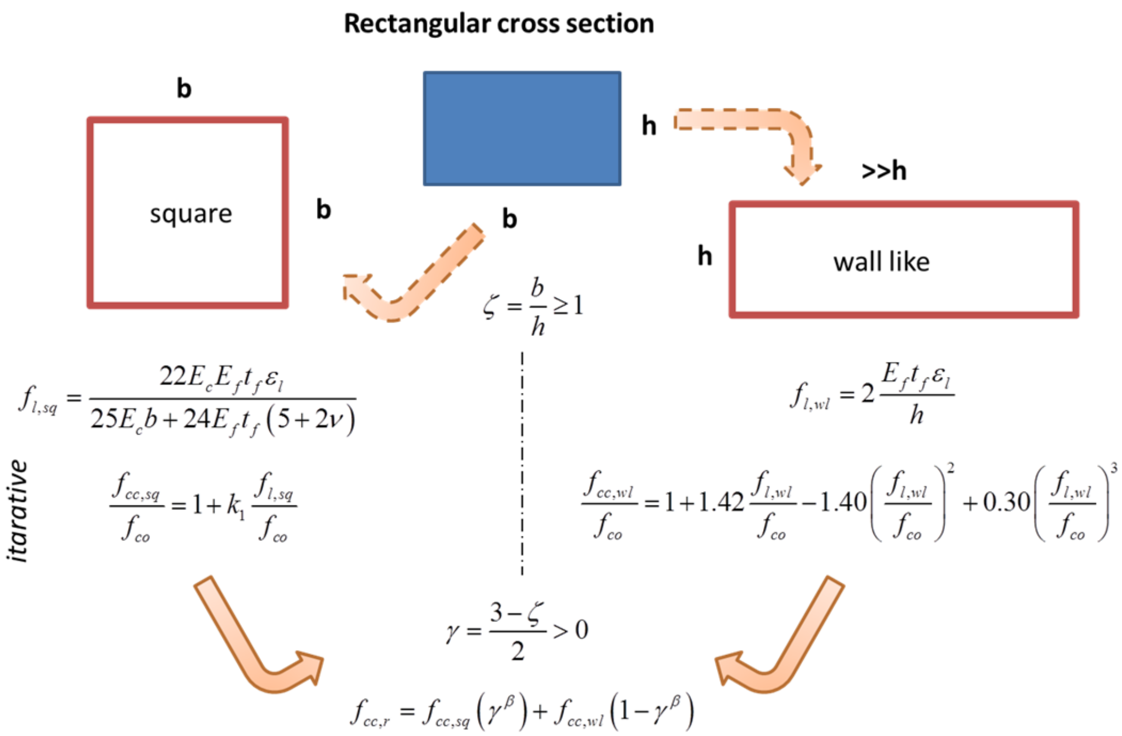


Figure 5. Outline of the proposed confinement model

3.2 *Validation of the model*

The proposed confinement model has been validated by means of experimental theoretical comparisons based on an experimental database (e.g. Demers and Neale 1994, Rochette and Labossiere 2000, Parvin and Wang 2001, Shehata et al. 2002, Harries and Carey 2002, Ilki and Kumbasar 2003, Masia et al. 2004, Berthet et al. 2005, Lam et al. 2006, Al-Salloum 2006, Harajili 2006, to cite some experimental programs) involving 74 square columns and 32 rectangular columns. Minimum and maximum dimensions of the cross-sections in the database are 188 mm and 2540 mm, always with ζ≤2. Different fibers for wraps were used with a total thickness ranging between 0.117 mm and 9.6 mm. The validation is summarized in the following figure 6, dividing, on the well-known format of the 45° line, square sections from rectangular sections. Synthetic indexes showing the reliability of the confinement model are the average, μ, of the ratios of confined concrete strength (theoretical over experimental) and their coefficient of variation, CV. It results μ equal to 0.94 and 1.07, while CV equal to 14% and

16%, respectively for square and rectangular cross-sections. These satisfactory results are based on numerical coefficient β equal to 1; in any case the sensitivity of the confinement model for rectangular sections to this parameter is represented in figure 7, and β=1 is a simple yet effective value for the considered set of tests. Obviously β is meaningless for square sections.

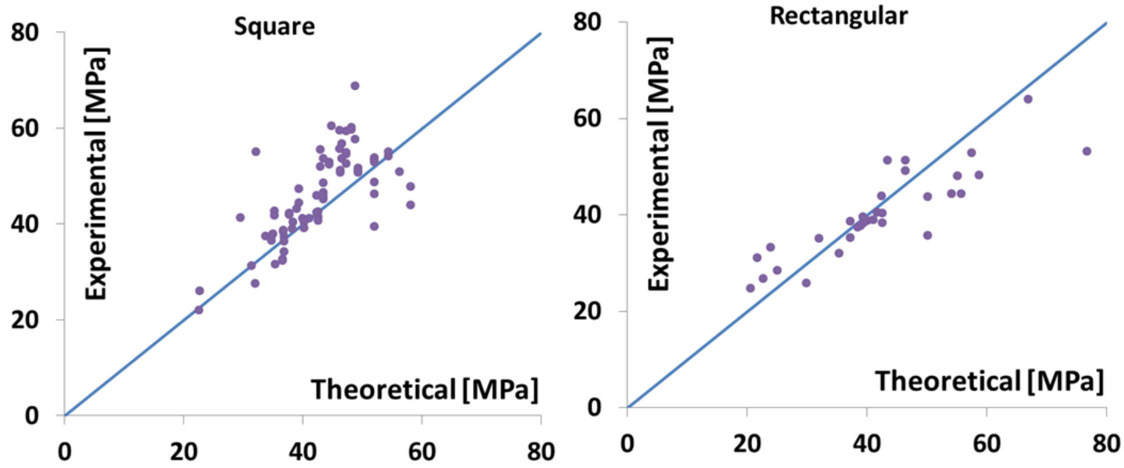


Figure 6. Validation of the proposed model.

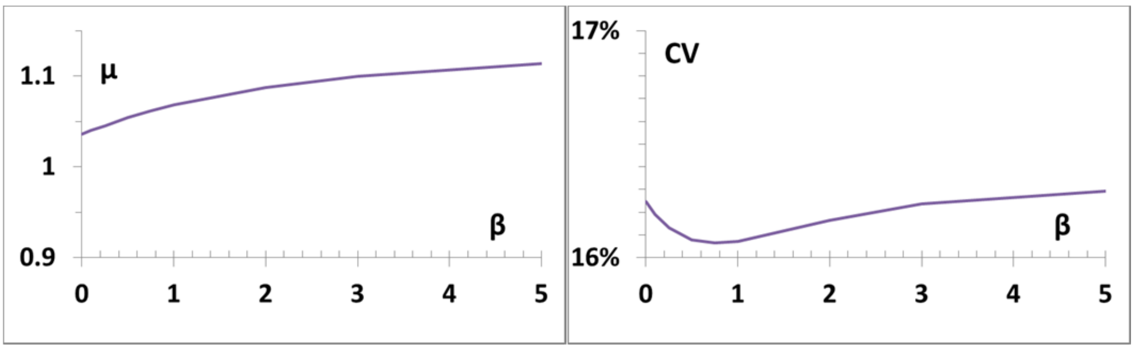


Figure 7. Sensitivity for rectangular sections to numerical coefficient β.

4 CONCLUSIONS

The proposed simplified model is derived from a more refined iterative confinement model proposed by the same authors. In this case a rectangular cross-section is considered as a combination of the behavior of a square cross-section and a wall like cross-section. An "effective" confining pressure is provided to obtain directly the confined concrete strength in square sections. In any case the model accounts for the non-uniform confined concrete performance exhibited in non-axisymmetric sections. This step is still iterative (but a few iterations provide converge) to account for nonlinear stiffness of concrete close to failure; in any case the iterations can be simply implemented in a spreadsheet. Conversely the wall like behavior is simulated by means of a simplified biaxial stress state in concrete, involving confining pressure parallel only to the longest side of the section.

To validate all the proposed approach, experimental data available in literature were compared to the results of the theoretical simplified analysis and satisfactory agreement was achieved. The simplified model gives rather accurate results despite the heavily reduced computational effort, if compared to the refined iterative algorithm proposed by the same authors.

5 ACKNOWLEDGMENTS

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6 REFERENCES

Al-Salloum, Yousef A. 2006. "Influence of edge sharpness on the strength of square concrete columns confined with FRP composite laminates". Composites Part B: Engineering, vol. 38, nº 5-6, pp. 640-650.

Berthet, J.F.; Ferrier, E. and Hamelin, P. 2005: "Compressive behavior of concrete externally confined by composite jackets. Part A: experimental study". Construction and building materials, vol. 19, nº 3, April 2005, pp. 223-232

Demers, M. and Neale, K.W. 1994. Strengthening of Concrete Columns with Unidirectional Composite Sheets, In: Mufti, A.A., Bakht, B. and Jaeger, L.G. (eds), Development in Short and Medium Span Bridge Engineering '94, Proceedings of the Fourth International Conference on Short and Medium Span Bridges, pp. 895–905, Canadian Society for Civil Engineering, Montreal, Canada

Harajli, M. H. 2006. "Axial stress-strain relationship for FRP confined circular and rectangular concrete columns." Cem. Concr. Compos. , 28 (10), 938–948.

Harries KA, Carey SA. 2002 Shape and ''gap'' effects on the behavior of variably confined concrete. Cement Concrete Res;33:881–90.

Ilki, A., and Kumbasar, N. 2003, "Compressive behaviour of carbon fibre and non-circular cross-sections", Journal of Earthquake Engineering, 7(3), 381–406

Lam L., J.G. Teng, C.H. Cheung, and Y. Xiao, 2006 "FRP-confined concrete under cyclic axial compression", Cement & Concrete Composites, 28, 949-958

Lignola, G.P., Prota, A., Manfredi, G. and Cosenza, E. 2008 Unified Theory For Confinement of RC Solid and Hollow Circular Columns. Composites Part B: eng. 39:7-8, 1151-60

Lignola, GP, Prota, A, Manfredi, G, and Cosenza, E. 2010. Non linear refined modeling of FRP Confinement on Prismatic RC Columns. 14th European Conference on Earthquake Engineering. Ohrid, Republic of Macedonia.

Lignola, GP, Prota, A, Manfredi, G, and Cosenza, E. 2011 "Modeling of RC wall-like columns FRP confinement". In CD-proceedings "1st Middle East Conference on Smart Monitoring, Assessment and Rehabilitation of Civil Structures SMAR 2011". Dubai, UAE. ISBN 9783905594584. Paper ID 125

Masia MJ, Gale TN, Shrive NG. 2004 Size effects in axially loaded square section concrete prisms strengthened using carbon fiber reinforced polymer wrapping. Can J Civil Eng 2004;31:1–13

Parvin, A. and Wang, W. 2001. Behavior of FRP Jacketed Concrete Columns Under Eccentric Loading, Journal of Composites for Construction, ASCE, 5(3): 146–152.

Richart, FE, Brandtzaeg, A, and Brown, RL. 1928. A Study of the Failure of Concrete Under Combined Compressive Stresses. Univ. of Illinois Engineering Experimental Station, Champaign, Ill, Bulletin 185

Rochette, P. and Labossiere, P. 2000: "Axial testing of rectangular column models confined with composites". ASCE Journal of composites for construction, vol. 4, nº 3, August 2000, pp. 129 –136

Shehata, I.A.E.M., Carneinro, L.A.V. and Shehata, L.C.D. 2002. Strength of Short Concrete Columns Confined with CFRPSheet, Materials and Structures, 35(1): 50–58.