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A novel numerical approach for modelling the monotonic and cyclic response of FRP strips glued to concrete

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ABSTRACT: The mechanical behaviour of the adhesive interface between the FRP strips and the concrete substrate often controls the response of FRP-strengthened RC members. Plenty of studies devoted to understanding the mechanical behaviour of FRP strips glued to concrete mainly focused on their response under monotonic actions, which are certainly relevant in a wide class of practical applications. On the contrary, few contributions are currently available to better understand the response of FRP-to-concrete interfaces under cyclic actions, such as those deriving by either seismic excitations or traffic loads. This paper presents a novel numerical approach to simulate such a response. Particularly, a damage-based approach is formulated to simulate the fracture behaviour of FRP-to-concrete joints under loading/unloading cycling tests. The model is formulated within the general framework of Fracture Mechanics and is based on assuming that fracture at the FRP-to-concrete interface develops in (pure shear) mode II, as widely accepted in similar problems. Two alternative expressions of the bond-slip behaviour are considered herein and a preliminary validation is finally proposed.

1 INTRODUCTION

Fiber-Reinforced Polymer (FRP) materials recently gained popularity in a variety of retrofitting solutions aimed at upgrading structural members in existing civil engineering structures, such as concrete columns (Pan et al., 2007), wooden floor beams (Corradi et al., 2006) and masonry panels (Marcari et al., 2007). As a matter of fact, the mechanical response of the adhesive interface often controls the structural performance of Reinforced Concrete (RC) members strengthened by Externally-Bonded (EB) FRP strips. Thus, plenty of researches aimed at investigating the bond behaviour of FRP strips glued to concrete were carried out in the last decades and are currently available in the literature. Particularly, the FRP-to-concrete fracture and debonding process was thoroughly investigated via both experimental (see, for instance, Chajes et al., 1996; Czaderski et al., 2012) and theoretical (Ferracuti et al., 2006, Cornetti & Carpinteri, 2011; Martinelli et al., 2011; Caggiano et al., 2012; Caggiano & Martinelli, 2013) studies. However, such studies, intended at investigating either the behaviour of FRP-to-concrete adhesive joints or the response of EB-FRP strengthened RC beams, were generally carried out by only considering monotonic actions applied to the members under consideration.

Nevertheless, FRP strips are widely used in practical applications with the aim of enhancing the structural performance of RC beams under cyclic actions possibly induced by either traffic loads or earthquakes. Despite the significant differences between the two aforementioned load cases, the state of knowledge about the actual behaviour of both the adhesive FRP-to-concrete interface and the performance of EB-FRP strengthened RC members under cyclic actions is still in need for dedicated investigations under both the experimental and theoretical standpoints. In fact, few studies are nowadays available on this topic. Particularly, Mazzotti & Savoia (2009) and Nigro et al. (2011) reported the results of low-cycle fatigue tests carried out by assuming a single shear test set-up, whereas the results of high-cycle fatigue tests were recently documented by Carloni et al. (2012). Regarding theoretical modelling, Ko & Sato (2007) proposed an empirical bond-slip model intended for simulating the behaviour observed in a series of monotonic and cyclic tests carried out on Aramid (A), Carbon (C) and Polyacetal (P) FRP strips glued to concrete blocks and tested in double shear. The model was based on assuming a Popovics-like law and involved seven mechanical parameters, which should be calibrated experimentally as a result of the empirical nature of the model under consideration.

This paper is intended as a further contribution to the modelling of FRP-to-concrete adhesive interface under cyclic actions: it presents a theoretical model formulated within the general framework of Fracture Mechanics (FM) to describe the post-elastic behaviour of the aforementioned adhesive interface. Particularly, the model is based on the assumption that fracture occurs in “mode II” (i.e. pure shear) and two alternative expressions (i.e. exponential and linear softening) are considered to describe the bond stress release in the post-peak regime. As generally accepted in FM, the unloading branch before the peak load is unaffected by damage mechanisms, whereas in post-peak regime the variations of the elastic (unloading) stiffness are driven up by means of a fracture-based damage modelling. Firstly, Section 2 outlines the key theoretical foundations of the proposed model and proposes some closed-form expressions of the fracture work which can be derived once having assumed “a priori” an analytical expression (either exponential or linear) for the post-peak branch of the bond slip law. Then, Section 3 proposes some comparisons between the model simulations and a series of monotonic and cyclic test results available in the scientific literature. Finally, concluding remarks as well as future developments of the present work are highlighted in Section 4.

2 THE THEORETICAL MODEL

A simplified theoretical model is proposed to model the cyclic response of FRP strips glued to brittle substrates, made of materials such as concrete or masonry. Particularly, the present proposal is based upon the following key assumptions:

- the crack develops at the FRP-to-concrete interface in (pure shear) "mode II";
- the analytical expression of the monotonic softening branch of the bond-slip relationship is described "a priori" by assuming an analytical expression (either exponential or linear in shape);
- stiffness degradation in the unloading stages depends upon the actual value of the "fracture work" developed in each interface point;
- “small” displacements are assumed at the interface and axial strains possibly developing in the concrete substrate are neglected.

The four assumptions listed above lead to define the general governing equations for the mechanical behaviour of FRP strips glued to a brittle substrate. They are derived by writing the classical “equilibrium”, “compatibility” and “(generalised) stress–strain” relationships, in both monotonic and cyclic response.

2.1 Basic assumptions

The proposed model is intended at modelling the FRP strip glued to a brittle support and schematically depicted in Fig. 1.

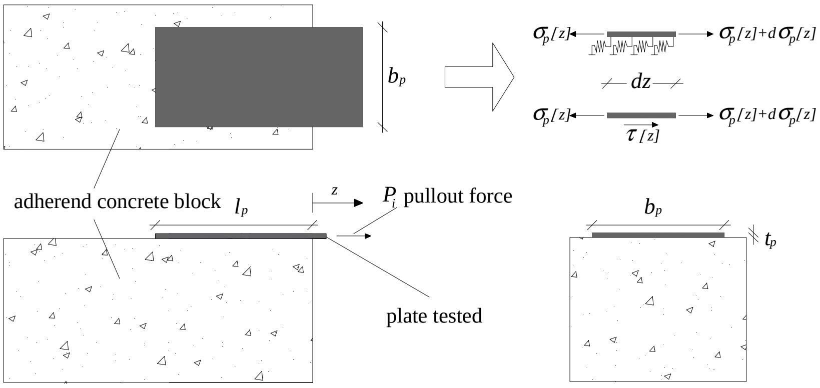

Figure 1. Single-lap shear test of a FRP-to-concrete bonded joint.

The assumptions of uniform width and thickness, b_p and t_p respectively, and a unique bond relationship throughout the adhesive interface, lead to the following equilibrium condition:

$$\frac{d\sigma_p[z]}{dz} = -\frac{\tau[z]}{t_p} \tag{1}$$

being $\tau[z]$ the interface bond stress and $\sigma_p[z]$ the axial stress in its cross section. The bond-slip equations for the adhesive behaviour can be expressed through two alternative bond-slip laws (even though under the simplified hypothesis of mode II response). The first one is given by the following negative exponential law:

$$\begin{cases} \tau[z] = -k_E s[z] & \text{if } s[z] \le s_e \\ \tau[z] = -\tau_0 e^{-\beta(s[z]-s_e)} & \text{if } s[z] > s_e \end{cases} \tag{2.a}$$

where k_E is the tangential bond stiffness in pre-peak response of the interface shear-slip relationship, $s[z]$ the shear slip at the considered z abscissa, $s_e = \tau_0/k_E$ represents the elastic slip value, τ_0 is the shear strength, while β is the exponential parameter of the post-peak τ-s relationship. Then, a linear softening interface model can be alternatively defined by means of the following expressions:

$$\begin{cases} \tau[z] = -k_E s[z] & \text{if } s[z] \le s_e \\ \tau[z] = -\tau_0 + k_S\left(s[z]-s_e\right) & \text{if } s_e < s[z] \le s_u \\ \tau[z] = 0 & \text{if } s[z] > s_u \end{cases} \tag{2.b}$$

being k_S the negative stiffness in the post-peak branch and $s_u = \tau_0/k_E + \tau_0/k_S$ the ultimate slip. The linear elastic behaviour of the FRP strip can be easily represented by the following relationship:

$$\sigma_p[z] = E_p \varepsilon_p \tag{3}$$

where E_p is the Young modulus of the composite, whereas the strain field can be calculated by means of the following compatibility condition:

$$\varepsilon_p = \frac{ds[z]}{dz}. \tag{4}$$

Finally, the following differential equation can be obtained by introducing eqs. (3) and (4) into the equilibrium condition (eq. 1):

$$\frac{d^2 s[z]}{dz^2} + \frac{\tau[z]}{E_p t_p} = 0 . \qquad (5)$$

2.2 Fracture-based damage modelling

The unloading/reloading stiffness is modelled within the framework of FM theory by considering, for each point of the adhesive interface, the fracture work w_{sl} and the corresponding fracture energy in “mode II” G_f^{II}. The fracture work, w_{sl}, developed during the sliding fracture process, controls the evolution of damage. Particularly, the variable $w_{sl}[s]$ represents the “inelastic portion” of the enclosed area of the τ-s curve in the range *[0-s]* (Fig. 2). Particularly, the dissipated work was obtained through the following relationships in the cases of EXPonential (EXP) and LINear (LIN) softening branches, respectively:

$$w_{sl} = \int_0^s \left|\tau[s]\right| ds - \frac{\tau^2[s]}{2k_E} = \begin{cases} \dfrac{k_E s_e^2}{2}\left(1 - e^{-2\beta(s[z]-s_e)} - \dfrac{2\left(e^{\beta(s_e - s[z])} - 1\right)}{\beta s_e}\right) & \text{EXP} \\ \dfrac{(k_E + k_S)(s - s_e)\left(2k_E s_e + k_S(s_e - s)\right)}{2k_E} & \text{LIN} \end{cases}, \qquad (6)$$

and, clearly, $w_{sl} = 0$ for $s[z] = s_e$.

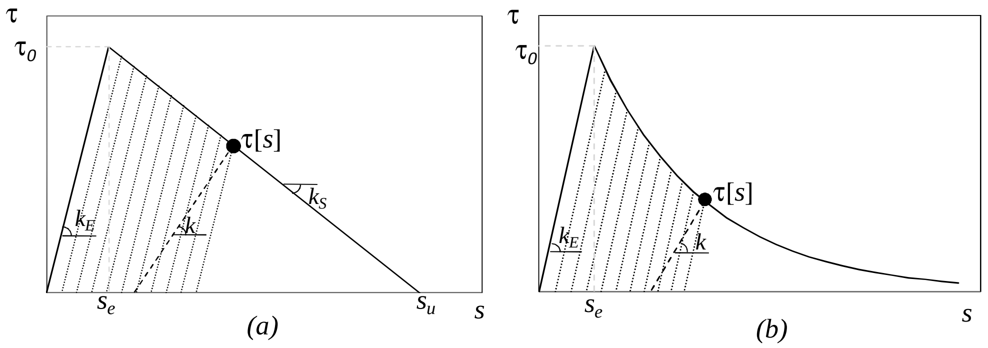

Figure 2.Fracture work spent as defined in eq. (6): (a) linear and (b) exponential softening branches.

Since a unique bond-slip law, possibly defined by eqs. (1) and (2), is assumed through the bond length, the value of G_f^{II} is uniform throughout such a length and depends on the key parameters involved in the two expressions (2.a) and (2.b):

$$G_F^{II} = \int_0^\infty \left|\tau[s]\right| ds = \begin{cases} \dfrac{k_E s_e^{\,2}}{2} \cdot \left(1 + \dfrac{2}{\beta s_e}\right) & \text{EXP} \\ \dfrac{k_E s_e^{\,2}}{2} \cdot \left(1 + \dfrac{k_E}{k_S}\right) & \text{LIN} \end{cases} . \qquad (7)$$

Finally, the damage parameter d can be defined in each point of the adhesive interface:

$$d = \xi^{\alpha_d}, \text{ with } \xi = \frac{w_{sl}}{G_f^{II}} , \qquad (8)$$

where α_d controls the shape of the damage curve and the loading/unloading stiffness k is related to the elastic one through the following relationship:

$$k = k_E\left(1-d\right). \tag{9}$$

2.3 Outline of the numerical procedure

A Finite Difference (FD) procedure is developed for integrating equation (5) under monotonic and cyclic actions. Particularly, a Central-Difference (CD) expression is assumed to express the second derivative of eq. (5) in the internal nodes of the FD mesh represented in Fig. 3:

$$\Delta s_i^{\ j} = \frac{\Delta s_{i+1}^{\ j} + \Delta s_{i-1}^{\ j}}{2 + \dfrac{k_{T,i}}{E_p t_p}\cdot \Delta z^2} \qquad \text{for } i=0,..n-1, \tag{10}$$

where j is the current analysis step, i the node number and k_T the corresponding tangential stiffness of the local bond-slip law depending on s_i^{j-1}. Since the analyses are intended to proceed in displacement control, the following boundary conditions are applied at the unloaded and loaded end of the FRP strip, respectively:

$$\Delta s_{-1}^{\ j} = \Delta s_1^{\ j} \tag{11}$$

$$\Delta s_n^{\ j} = \Delta s_c^{\ j} \tag{12}$$

where eq. (11) corresponds to the condition of zero stress (and strain) at the unloaded end, and eq. (12) to the imposed slip increment at the loaded end (i.e., node n).

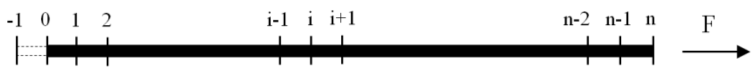

Figure 3. Finite difference discretisation of the FRP-to-concrete interface.

The set of ($n+2$) simultaneous equations (10)-(12) can be solved in terms of slip increment vector $\boldsymbol{\Delta s}^j$ and, in principle, the final solution in the *j-th* analysis step can be obtained iteratively to take into account the possible interface nonlinearity. Particularly, the trial solution at the *k-th* iteration of the *j-th* incremental analysis step can be obtained in terms of both interface slip and bond stress vectors (which collect the $n+2$ components of both quantities):

$$\boldsymbol{s}^j\Big|_k = \boldsymbol{s}^{j-1} + \boldsymbol{\Delta s}^j\Big|_k \tag{13}$$

$$\boldsymbol{\tau}^j\Big|_k = \boldsymbol{\tau}^{j-1} + \boldsymbol{\Delta\tau}^j\Big|_k = \boldsymbol{\tau}^{j-1} - \boldsymbol{k}_T \cdot \boldsymbol{\Delta s}^j\Big|_k \tag{14}$$

where $\boldsymbol{s}^{j-1}$ and $\boldsymbol{\tau}^{j-1}$ are slip and bond stress vectors, at the convergence of the *j-th* incremental analysis step, and $\boldsymbol{k}_T$ a vector collecting the tangential stiffnesses $k_{T,i}$ at the various nodes of the FD discretisation. If the node *i-th* ended up the *(j-1)-th* analysis step in the elastic stage, the following condition should be met by the trial solution (14) for the same node to remain in elastic stage:

$$\left|\tau_i^{\ j}\Big|_k\right| \le \left|\tau\left(s_{cr,i}\right)\right| \tag{15}$$

where τ is the bond-slip law expressed by either of equations (2) and $s_{cr,i}$ a state variable which represents the total slip developed in the node i during the fracture process and, in monotonic conditions, could be simply expressed as $s_{cr,i}=s_i-s_{el}$. If eq. (15) is satisfied in all nodes at the first iteration ($k=1$), then they hold their elastic status and the force ΔF^j increment, corresponding to the imposed slip increment $\Delta s_c^{\,j}$, can be derived by equilibrium:

$$\Delta F^{j} \overset{[k=1]}{=} \Delta F^{j}\Big|_k = \left[\sum_{i=1}^{n} \frac{\tau_i^{\,j}\big|_k + \tau_{i-1}^{\,j}\big|_k}{2}\right] \cdot b_p \cdot \Delta z \, . \tag{16}$$

If this is not the case, the slip increment $\Delta s_i^j|_k$ should be subdivided in an elastic part $\Delta s_i^j|_{k,el}$, corresponding to the achievement of the equality in equation (15) and the cracking part $\Delta s_i^j|_{k,cr} = \Delta s_i^j|_k - \Delta s_i^j|_{k,el}$. Then, an iterative search of the equilibrium for the *j-th* can be carried out by employing eqs. (10)-(12) as an elastic predictor and the equality in eq. (15) to obtain the corrector. Once convergence is achieved (i.e. in terms of unbalanced forces at the *k-th* iteration of the *j-th* increment), the vector $\boldsymbol{s}_{cr}$, collecting the state variable $s_{cr,i}$, can be updated as follows:

$$\boldsymbol{s}_{cr}^{\ j} = \boldsymbol{s}_{cr}^{\ j-1} + \boldsymbol{\Delta s}_{cr}^{\ j-1}\Big|_k \tag{17}$$

and the corresponding force determined through eq. (16). Then, in the following incremental analysis steps, the same node *i* will keep the cracking status if no sign change occurs between the increment slip at the previous step *(j-1)-th* and the one obtained by solving eqs. (10)-(12):

$$\Delta s_i^{\,j-1} \cdot \Delta s_i^{\,j}\Big|_k > 0 \, . \tag{18}$$

If this is the case for all the nodes, the corresponding force can be determined through eq. (16) and the status variable updated via eq. (17). Otherwise, an unloading stage starts in the nodes where the inequality (18) is not satisfied and an iterative predictor-corrector search leads to the new system status. The incremental analysis proceeds up to the achievement of a given failure condition which could be formulated in terms of maximum slip occurring at the unloaded end.

3 EXPERIMENTAL COMPARISONS

The formulation presented in Section 2 needs to be validated in its soundness and capability to simulate the FRP-to-concrete pull-out behaviour under both monotonic and cyclic conditions. Experimental data characterising both of the above mentioned experimental situations, on three types of FRP sheets are available in Ko & Sato (2007). The results of some tests carried out on a single ply of A-FRP strips are considered to achieve a preliminary validation of the proposal.

Three equal specimens were tested under monotonic and cyclic actions. They are characterised by an A-FRP strip with relative axial stiffness $E_p t_p = 10.4$ *kN/mm* and width $b_p = 50$ *mm*. Then, the values of the bond-slip material parameters are identified for the two (alternative) softening laws (namely, the exponential and linear one). Particularly, $k_E = 52.22$ *MPa/mm*, $\tau_0 = 2.256$ *MPa* and $G_f^{II} = 0.958$ *N/mm*, are assumed in the following numerical simulations for the mechanical quantities which are relevant for both the bond-slip relationships, according to the average values identified by the cited authors for the specimens A11, A12 and A13, tested under monotonic actions. Regarding the softening branch, it can be consistently derived by the three aforementioned values and taking into account the two expressions in eq. (7), which connect the β exponent and the k_S slope characterising the exponential and the linear softening relationships, respectively. Moreover, the unit value is considered for the damage parameter α_d.

Fig. 4 compares the results (in terms of force-slip relationship) obtained in the cyclic test labelled as "A14" by Ko & Sato (2007) with the corresponding numerical simulations obtained by assuming the exponential expression (2.a) for the softening branch. The agreement between experimental and numerical results is rather satisfactory, especially if it is kept in mind that no fine tuning of the relevant mechanical parameters was performed in this paper, but they were simply assumed in accordance to the values identified by Ko & Sato (2007) on monotonic tests. However, the higher residual slip which affected the actual experimental observations with respect to the resulting numerical simulation points out a possible limit of the proposed fracture model which needs to be further assessed in the future stages of the validation procedure.

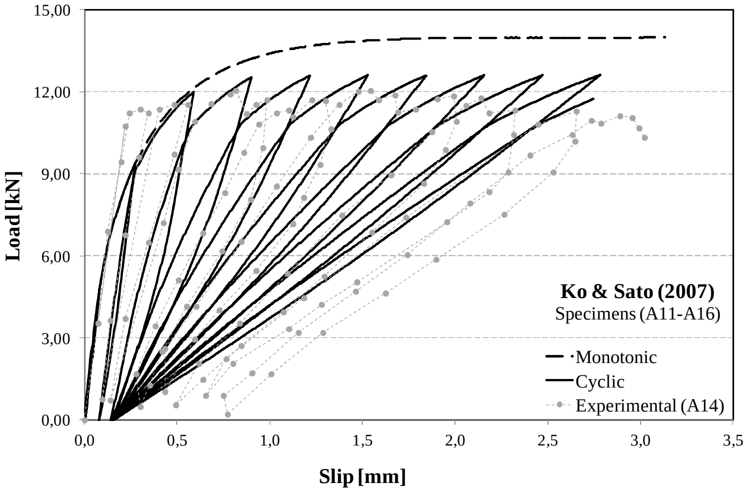

Figure 4.Load-slip response under monotonic and cyclic actions of FRP strips glued on concrete (Ko & Sato, 2007) – Exponential softening.

Finally, Fig. 5 proposes a similar comparison based on the analyses carried out by assuming a linear softening branch for the bond-slip relationship. It is clear that such an assumption, generally accepted to simulate the monotonic response of FRP strips glued to concrete, is less fit for simulating the cyclic behaviour of their adhesive interface, as it results in an overestimation of damage and, then, in a significant difference in terms of both maximum forces and ultimate slips.

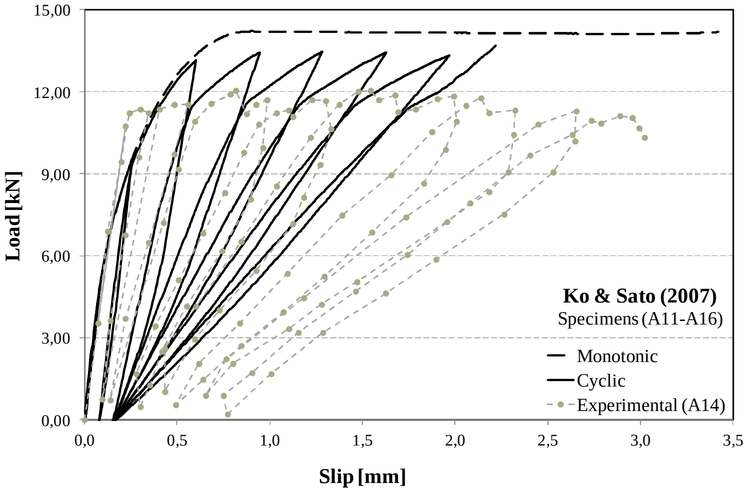

Figure 5.Load-slip response under monotonic and cyclic actions of FRP strips glued on concrete (Ko & Sato, 2007) – Linear softening.

4 CONCLUDING REMARKS

This paper presented a contribution to the analysis of the cyclic behaviour of FRP-to-concrete interface. Particularly, the proposed model has been formulated within the framework of Fracture Mechanics and assumed two alternative expressions for the softening branch of the bond-slip relationship. The closed-form expressions obtained for determining the key damage-related quantities are among the novel and most attractive features of the present formulation. The comparison between some experimental results available in the literature and the numerical simulations performed by means of the present model highlighted the predictive potential of the latter. Moreover, such experimental comparisons pointed out the higher accuracy obtained by assuming an exponential softening branch, with respect to the linear one, generally accepted for simulating the response under monotonic actions. This observation is the starting point for the future development and validation of the present model.

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