

## State Representation Methodology (SRM) for Bridge Condition Assessment in SHM

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**ABSTRACT:** This paper introduces a new concept of “State Representation Methodology (SRM)” which is a kind of bridge condition assessment method for structural health monitoring system (SHM). There are many methods for system identification from the simplicity comparison of damage index to the complicated statistical pattern recognition algorithms in SHM. In these methods, modal analysis and parameters identification or many defined indices are common-used for extracting the dynamic or static characteristics of a system. However, there is a common problem: due to the complexity of a large size system with high-order nonlinear characteristics and severe environment interference, it is impossible to extract and quantify exactly these modal parameters or system parameters or indices as the feature vectors of a system in damage detection in an easy way. The SRM considered a more general theory for the non-parametric description of system state.

### 1 INTRODUCTION

In recently, structural health monitoring (SHM) has received an increasing attention in the field of deteriorating civil infrastructures, such as bridges, highway networks, etc [1]. Meanwhile, the condition assessment is becoming one of the most important issues in this field. A large size structure such as bridges produces a huge amount of monitoring data for every day, every month and every year. It becomes also an important thing that how to detect damages from such huge number of monitoring data, that is a big challenge for analysis to discover the damage information of a target structure. Furthermore, there are not only many kinds of sensor data, but also many undetermined factors in the system, such as the dynamic loads, ever-changing climate condition, and severe noise interference. At the same time, we cannot exactly find a model to describe any structure and a variety of data interrelationship in design stage or during any operating period. In generally, the large structure can be viewed as a system, and the large-scale systems theory or any other new methods can be employed to research the system. It is obvious that those large systems are a high-order nonlinear system. The nature exists a great of complex systems, many of them have some similar or not. At least, the description of a system state is an essential problem, and even a philosophical methodology problem [2].

**Fig. 1** shows that how important the condition assessment in SHM research. It is easy to understand from this figure that the practical condition assessment is still a basic problem in SHM research, although there are many literatures to study on the topics for SHM. In this paper, it will be focusing on to try to easy explanation of the analysis methods of condition assessment in SHM with comparison between traditional methods and a new proposed method which called “Sate Representation Methodology (SRM)”[3],[4]. Then the main content of the paper is consisted from the basic theory

(concept) of “State Representation Methodology (SRM)” to its applications to practical “Condition Assessment” for bridge structures.

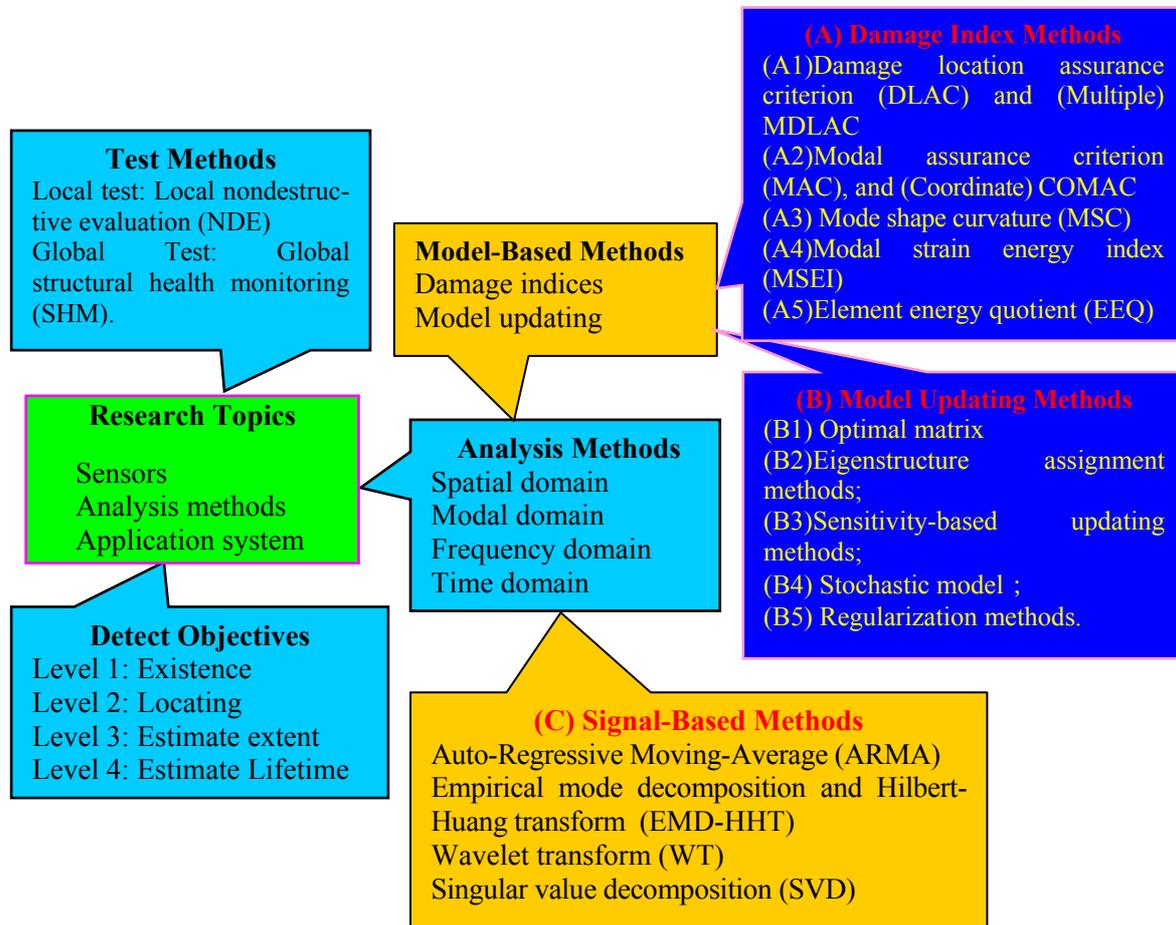


Figure 1. Data analysis methods for condition assessment in SHM research

## 2 SRM CONCEPT BASED ON KERNEL FUNCTION METHOD

### 2.1 What is the SRM?

Here, we will be focusing on the structural health monitoring system (SHMS). Especially, how do we describe the probability state from the structure monitoring data? In another view, can we know more about what’s happened & when the monitored signals have been changing, and what is the relationship between the system state and the sensor’s signals? All of those about detecting and locating are very important for a practical SHMS. In this paper, we will be able to give a new idea and systemic methods to describe and assess the structural state which called as the system State Representation Methodology (SRM) [2],[3]. Based on theoretical research of the SRM, we developed a systemic method to describe damage feature and state of structural system.

In a complex system such as bridge structures, it comes some common and basic questions as follows:

- (1) What is the present state of a system ?
- (2) How is it possible to build a model to describe the system state ?

- (3) How is it possible to compare the state change of the system ?
- (4) How is it possible to extract the state features of the system ?
- (5) What kind of damages are occurring in the system ?
- (6) How is it possible to locate a damage of the system ?

2.2 *What is the State of a System?*

The state of a system is interpreted as the overall response to its internal and external factors which essentially depend on the response of structure itself or structural properties and nature environment. Then, the quantitative assessment of the system state is the description for system responding to the exciting factors. If the responses satisfy the suitable values, the system state is considered as normal state, otherwise as abnormal(damaged) state. In usually circumstances or under normal use conditions, the system is in stable state, this means its state should be a constant, or fluctuation near a steady state. In general sense, we therefore assume that it is a steady random variable and it is usually obeyed the normal distribution.

**Assumption:** The state of a system is a function of the system response to environmental exciting.

Obviously, this assumption has so natural sense. Nevertheless, we cannot use the response data directly in time domain because it is obvious time-varying signal. Therefore, we usually translate into the frequency domain or other transformation domain. This process is called features extracting. **Fig. 2** shows the procedure of feature extracting, in where  $x$  is called the system feature vector,  $\zeta$  is the state variable,  $\lambda$  is the system structure alias parameter. Therefore the state of a system can be described by the variable  $\zeta$ , and it should be a function about the system features that can be extracted from the responses to its various factors. In fact, the successful use of the SRM depends on the experimenter's ability to develop a suitable

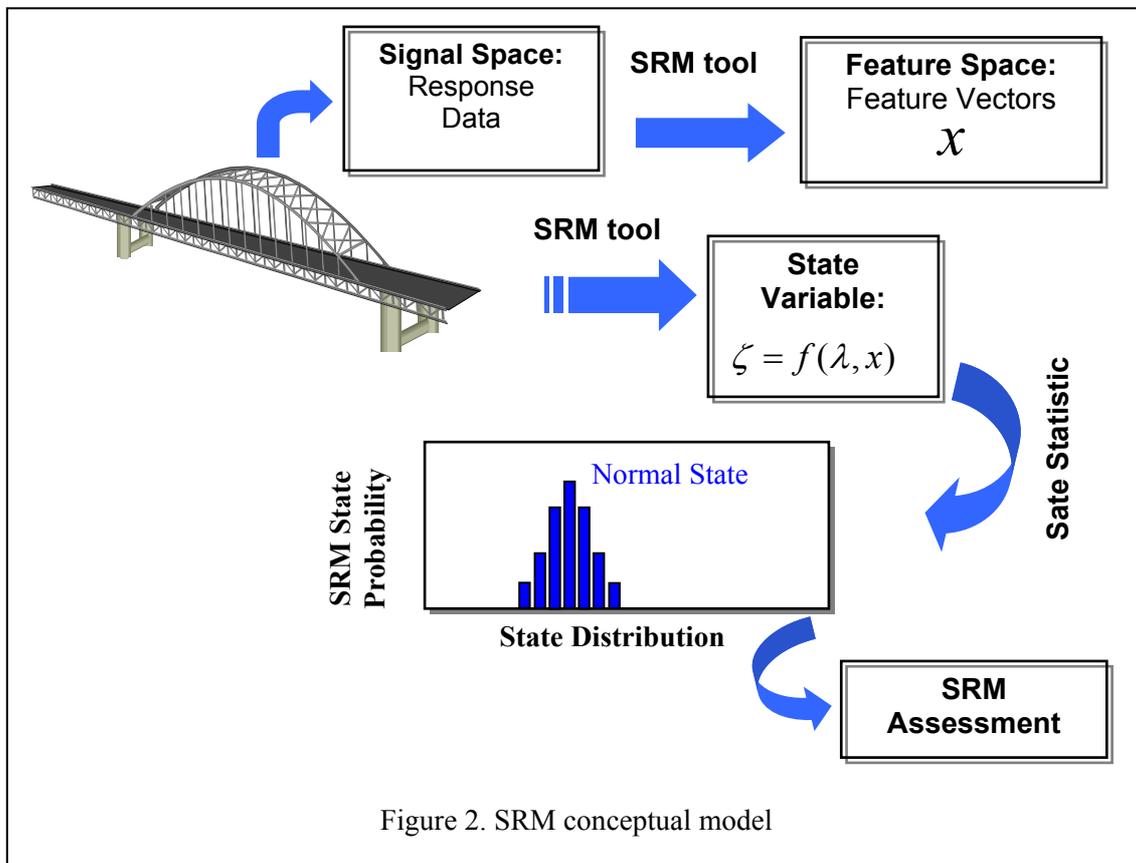


Figure 2. SRM conceptual model

approximation for the system state function  $f(\cdot)$ , which it is called “State Representation Function (SRF)”. The SRF can be written as,

$$\zeta = f(\lambda, x) = 1 \quad (1)$$

The SRM tools mentioned in **Fig. 2** will be introduced in Ref. [4].

### 2.3 System State Approximating

**Fig. 3** shows a basic idea for state variable expression by approximating methods. The following will analyze the main idea of the Kernel Function expression.

Let  $w$  is constant vector in  $R^n$ , called as system state support vector which is related to the system structure parameters, we take the first-order model to approximate the Eq. (1) as the following:

$$1 = \langle w, x \rangle = \sum_{k=1}^n w(k)x(k) \quad (2)$$

Use the Least Squares Estimators (LSE) method, it is easy to get the weight vector  $w$ ; if the system has only one response feature vector  $h$  in its initial feature space ( $IFS$ ), i.e. we assume that  $h \in IFS \subset R^n$ , the least squares solution of Eq. (2) should be:

$$w = \frac{h}{\|h\|^2} \quad (3)$$

However, we respectively take  $m$  feature vectors of  $IFS$ , the least squares solution respectively should be:

$$w_i = \frac{h_i}{\|h_i\|^2}, i = 1, 2, \dots, m \quad (4)$$

Now let us consider every response feature vector  $h \in IFS$  as a projector of the current system state along with each feature direction, then we give them with a weight:  $\lambda_i \geq 0$  and

$$\lambda_i \in (0, 1), \sum_{i=1}^m \lambda_i = 1, w = \sum_{i=1}^m \lambda_i w_i = \sum_{i=1}^m \alpha_i h_i, \alpha_i = \frac{\lambda_i}{\|h_i\|^2} \geq 0$$

Then,

$$1 = \langle w, x \rangle = \langle \sum_{i=1}^m \alpha_i h_i, x \rangle = \sum_{i=1}^m \alpha_i \langle h_i, x \rangle \quad (5)$$

One can define a function included parameter vector  $\alpha$ , as:

$$f(\alpha, x) = \sum_{i=1}^m \alpha_i \langle h_i, x \rangle \quad (6)$$

Eq. (6) is called the first-order representation function of system state, it's a linear operator, where vector  $\alpha$  is also called as system alias parameter or system state parameter, which is relative with the system structure. Note that Eq. (6) can be rewritten as,

$$f(\lambda, x) = \sum_{i=1}^m \lambda_i \frac{\|x\|}{\|h_i\|} \langle \frac{h_i}{\|h_i\|}, \frac{x}{\|x\|} \rangle, \sum_{i=1}^m \lambda_i = 1, \lambda_i \geq 0 \quad (7)$$

If  $\|h_i\| = \|x\|$  is always assumed, then we have,

$$f(\lambda, x) = \sum_{i=1}^m \lambda_i \langle \frac{h_i}{\|h_i\|}, \frac{x}{\|x\|} \rangle, \sum_{i=1}^m \lambda_i = 1, \lambda_i \geq 0 \quad (8)$$

Then,

$$f(\lambda, x) \leq 1 \quad \text{for always} \quad (9)$$

Let  $\bar{w} = \sum_{k=1}^m \lambda_k \frac{h_k}{\|h_k\|}$ , then Eq. (8) can be rewritten as:

$$f(\bar{w}, x) = \langle \bar{w}, \frac{x}{\|x\|} \rangle \quad (10)$$

The inner product  $\langle . \rangle$  can be replaced by a Kernel Function form as follows. In generally, we can redefine  $\langle s, x \rangle, s \in X, x \in X$  as,

$$\langle s, x \rangle = \langle \phi(s), \phi(x) \rangle = k(s, x), s \in X, x \in X \quad (11)$$

where,  $X$  is a vector space. Here  $k(s, x) = \langle \phi(s), \phi(x) \rangle$  is called a Kernel Function. There are many choices for a Kernel Function  $k(\cdot, \cdot)$ . Here, the following Kernel Functions are recommended in [3,5].

$$k(s, x) = \exp\left(-\frac{d(s, x)}{\sigma^2}\right) \quad (12)$$

here,  $d(s, x) = \sum_{i=1}^n \frac{(s_i - x_i)^2}{s_i + x_i}$  or  $d(s, x) = \left(\sum_{i=1}^n |s_i - x_i|^p\right)^{1/p}$

$$k(s, x) = (s \cdot x)^d, (\|s\| = 1, \|x\| = 1) \quad (13)$$

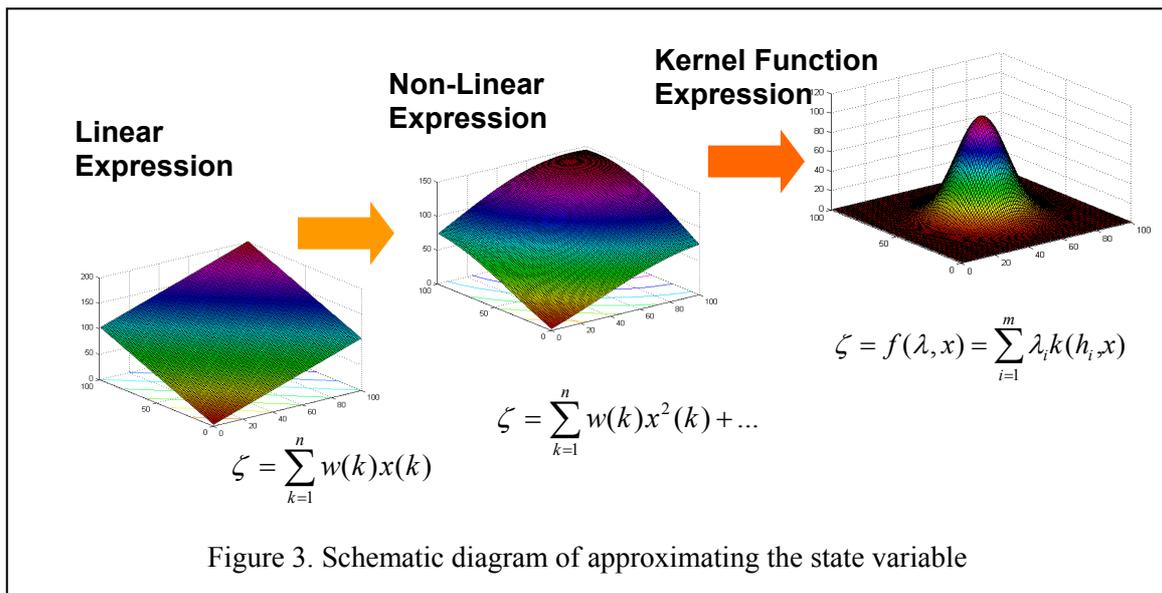
Note that  $\sigma$  is called the SRM scale. In fact, the difference between two any objects is absolute on any scales, but their similarity is relative on any scale. Therefore, scale concept is important to study the difference between two states of a system. Subsequently, we can get state function in the Kernel Function form:

$$\zeta = f(\lambda, x) = \sum_i^m \lambda_i k(h_i, x)$$

In order to satisfy that state should be a constant, we are able to define the following objective function:

$$\min \left\| \left( I - \frac{1}{m} ee^T \right) G \lambda \right\|^2 \quad (14)$$

Subject to  $\sum_i \lambda_i = 1, 0 \leq \lambda_i \leq 1, i = 1, 2, 3, \dots, m$



$$\text{here, } G = \begin{pmatrix} k(h_1, h_1), k(h_1, h_2), \dots, k(h_1, h_m) \\ k(h_2, h_1), k(h_2, h_2), \dots, k(h_2, h_m) \\ \dots \\ k(h_m, h_1), k(h_m, h_2), \dots, k(h_m, h_m) \end{pmatrix}$$

Then, we can get state function as following:

$$\zeta = f(\lambda, x) = \sum_i^m \lambda_i k(h_i, x)$$

Eq. (14) can be illustrated as **Fig. 4**, where  $c$  is constant for a system, and  $c = \frac{1}{m} e^T G \lambda$ . We can get the alias parameter  $\lambda$  of system structure, which is also called as the support vector in SVM.

Let the system be in state  $\zeta$  in current time with feature vector  $x$ , i.e.  $\zeta = f(\lambda, x)$ .  $\zeta$  has different value with different feature vector  $x$  because of the system complexity. We may not know how many state values a system has, but we usually assume  $\zeta$  to be a normal probability distribution. Many statistics methods can be performed to estimate the probability distribution. Therefore, we can use in variety of statistic tests such as  $F$ -test and  $t$ -test etc. to assess the difference between “virgin” state and “damaged” state in the past time. By using a laboratory bridge monitoring system (LBMS) (see [6]) to verify the SRM, experiments show that the SRM is available and steady to express the system state.

#### 2.4 How Differences between the Existing Methods and the SRM?

**Fig. 5** shows that traditional methods based on frequency domain decomposition (FDD) are widely recognized as a simple method [6]. However, it tends to often lead to the loss of sensitivity and accuracy of damage detection, because the power spectrum of the measured responses could not be accurately estimated, particularly for high-damped systems and systems with severe modal interference and high noise. Meanwhile, there are some important questions in it: The natural frequency, corresponding mode shape and damping coefficient are usually changing very slowly when the bridge system is deteriorating in the early stage, in other words, the bridge health condition is always not sensitive with those parameters. At the same time,

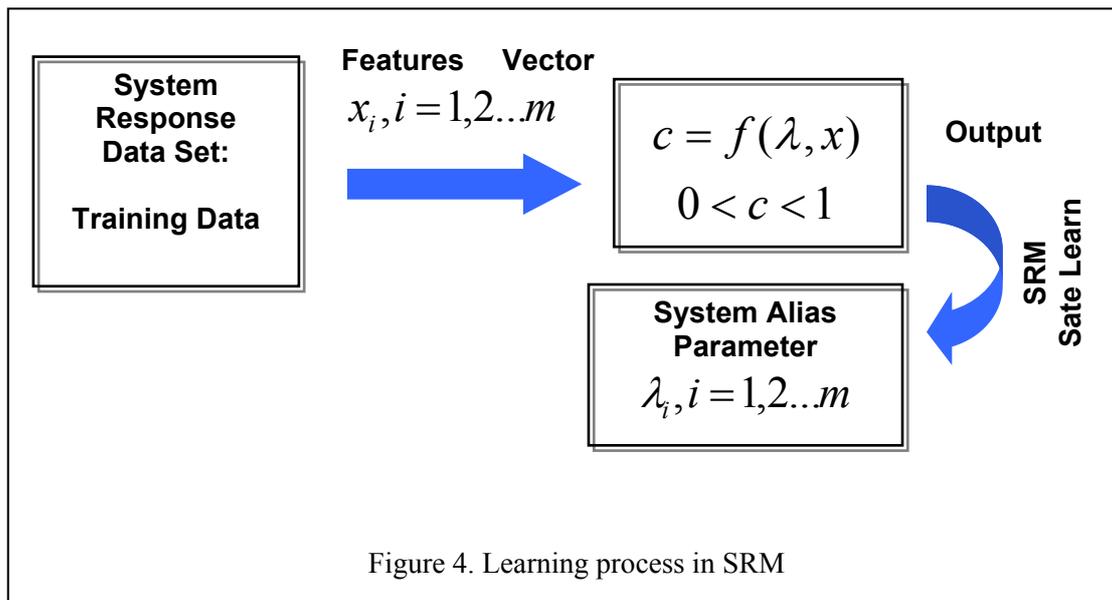


Figure 4. Learning process in SRM

since a complex system include many structural parts, it has many frequency components, and they are interfered each other, so it is very difficult to use FDD to analyze a system in damage detection because it is impossible to get modal parameters exactly. Then we proposed a new method based on an overall view parameter to identify a system. In here, we assume that the system state is a dynamic variable, and a probability method with random factors was introduced as a better method to describe its present state because it is always in many random conditions (factors).

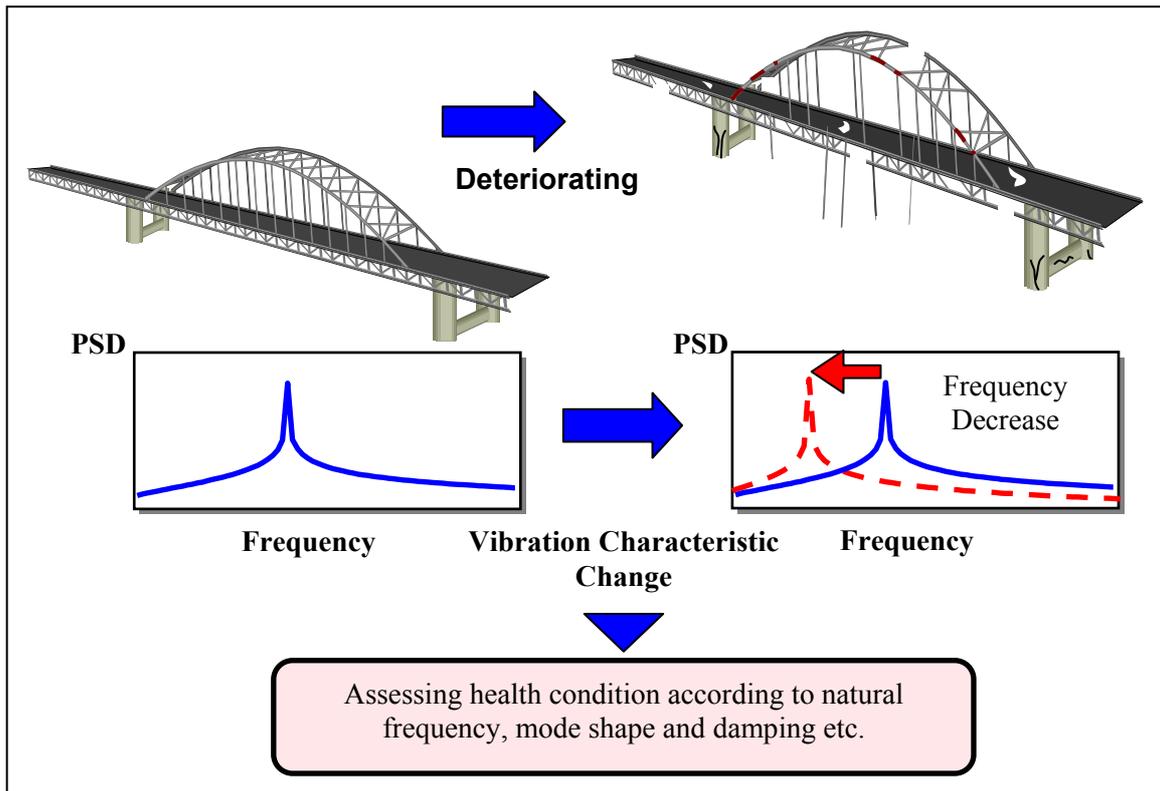


Figure 5. Frequency domain decomposition (FDD) principle

**Fig. 6** shows the main principle of the proposed SRM. At first, it needs to change time domain data into the transformation domain features in the SRM. Then we are able to derive a system state variable in the feature space. Furthermore, we need to make the statistic probability distribution of the derived variable. Finally, the question of “condition assessment” of the present system becomes into a problem of “state assessment”.

As stated above, among various damage detection methods, the vibration-based one is most widely used, the core idea is that vibration responses are directly relative with the physical structure and properties, such as mass, stiffness and boundary conditions, and those changes of vibration responses can be used to characterize the structural damage. Although many intuitive and considerable research efforts have been devoted on it during the past decades, assessing structural damage in large-scale bridges still remains a challenging task for civil engineers. A research [6] shows that many factors can affect the assessed results, such as the insensitive of modal properties to local damage of bridge structures, uncertainty and incompleteness in measurement data, modal variability arising from varying operational and environmental

conditions, and modeling errors in the analytical model etc. Use of physics-based damage detection or model-based damage detection methods often lead to update a large number of damage parameters, especially when the structure has an abundance of structural members. Therefore they have to reduce their parameters. One problem which may arise with this method is that parameter reduction may identify the most damage-sensitive parameters but may fail to locate the damage correctly. At the same time, these same factors often lead to the ill-conditioning of model updating and damage detection problems, where small measurement noises could be magnified, this means that noise or complex factors can always corrupt the analysis accuracies. However, the sensitivity and the stability of a method are simultaneously important to the practical applicability. Meanwhile, many literatures don't consider these facts that the model updating condition is dynamic and our knowledge is an embedded process as the time ongoing. Therefore, not only requires the improving of approaches for model updating and damage detection, but also our methods should be built on a process of observation by means of continuously accumulated data. The SRM gives the method driven by monitoring data or by experience data in inspection. The SRM directly helps to know what is the difference between "virgin" state and "damaged" state after an observed time. A nonparametric structural damage detection methodology based on nonlinear system identification approaches is presented for the SHM.

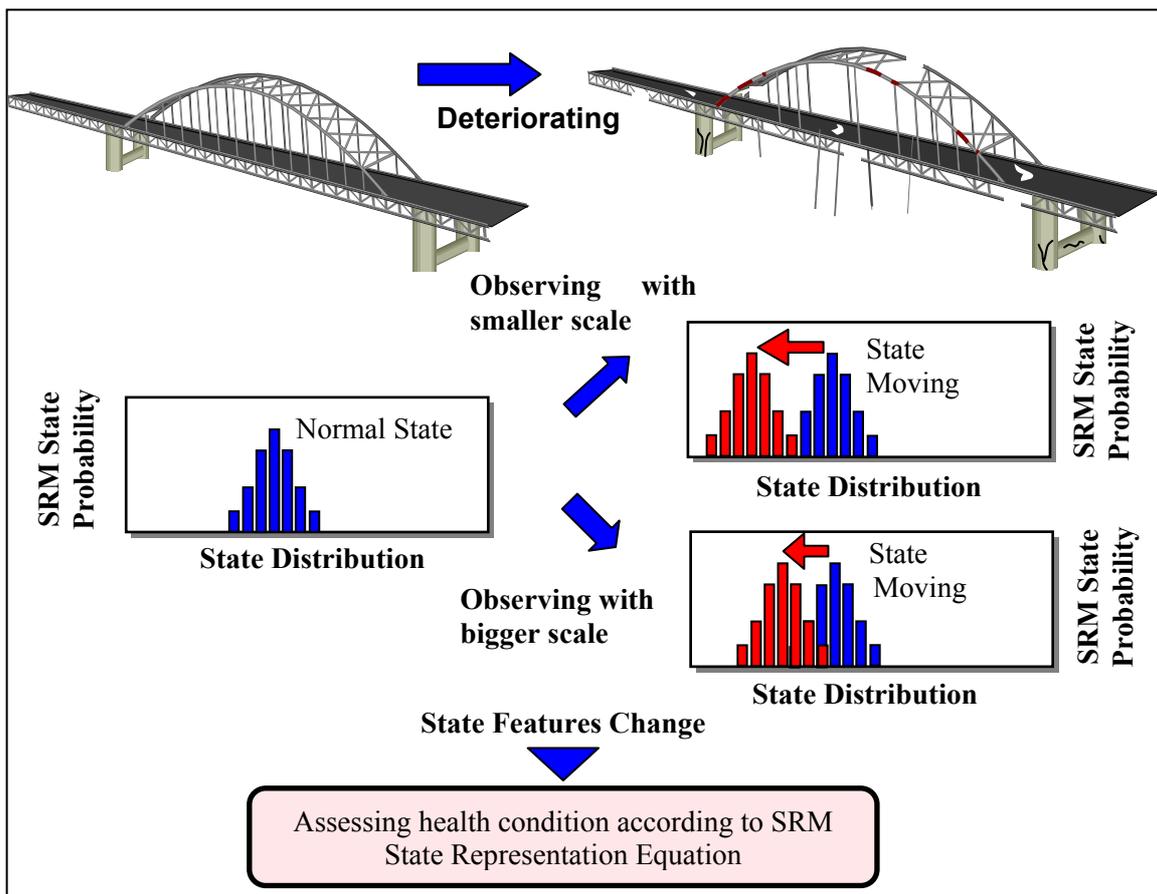


Figure 6. SRM principle

### 3 APPLICATION TO LABORATORY BRIDGE MONITORING SYSTEM

#### 3.1 Outline of a Laboratory Monitoring System by Bridge Model

The bridge model used in this study is a simply supported girder bridge model with three main girders, which is the minimum number of girders needed to obtain necessary information such as load distribution characteristics in the direction perpendicular to the bridge axis. **Fig. 7(a)** shows the bridge model prepared for the purposes of a laboratory bridge monitoring system (LBMS). As a damage effect capable of modeling the effects of many types of damage in an idealized way, a decrease in flexural stiffness was mainly considered[7]. Girder stiffness reduction was introduced by reducing lower flange width such as **Figs. 7(b)**[no-damaged] and **7(c)**[damaged]. A total of six damaged girders with various damage conditions were prepared for impact hammer dynamic tests. These girders were replaced with sound(no-damaged) girders to above mentioned damage conditions.

Single impact load as the dynamic test was applied to four points(B1-D3 in **Fig. 8**) of the bridge model by using an impact hammer. Accelerometers were installed in bottom flange of the main girders as shown in **Fig. 8**(acc1-acc9). To enhance the accuracy of the transfer function, impact loads were applied 10 times at each point.



(a) Overview of Bridge Model



(b) Main girder without damage (c) Main girder with damage in lower flange

Figure 7 Details of bridge model for laboratory bridge monitoring system

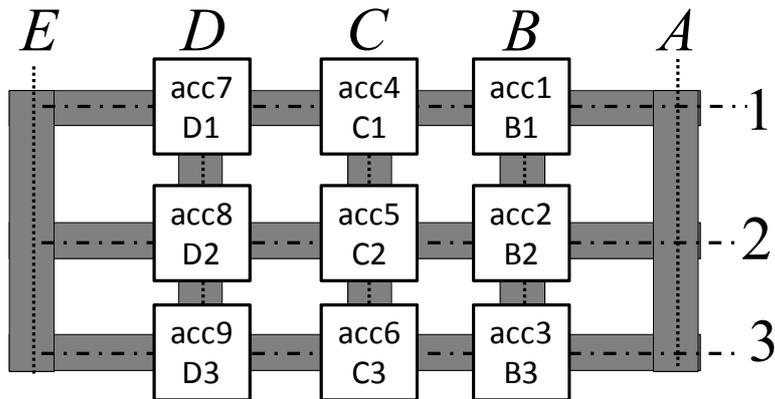


Figure 8 Details of bridge model for laboratory bridge monitoring system

### 3.2 Condition Assessment of Bridge Model based on SRM

**Fig. 9(a)** shows an example of acceleration response of a main girder as the laboratory monitoring data. Based on the time domain data, the SRM algorithm as shown in **Fig. 10** will be applied to get the state variable  $\zeta$  as bridge condition assessment. In the algorithm, it is necessary that the system features need to extract from the complex responses observed data in the system. A new time-frequency analysis tool, called “Frequency Slice Wavelet Transform(FSWT)”[4] which is implemented to transform the time domain data into the time-frequency domain data will be able to powerfully reveal a change of the characteristics in vibration signal. **Figs. 9(b) & 9(c)** show an example of Fourier spectrum and 2D map of FSWT with feature extracting grids on an impact hummer test data, respectively.

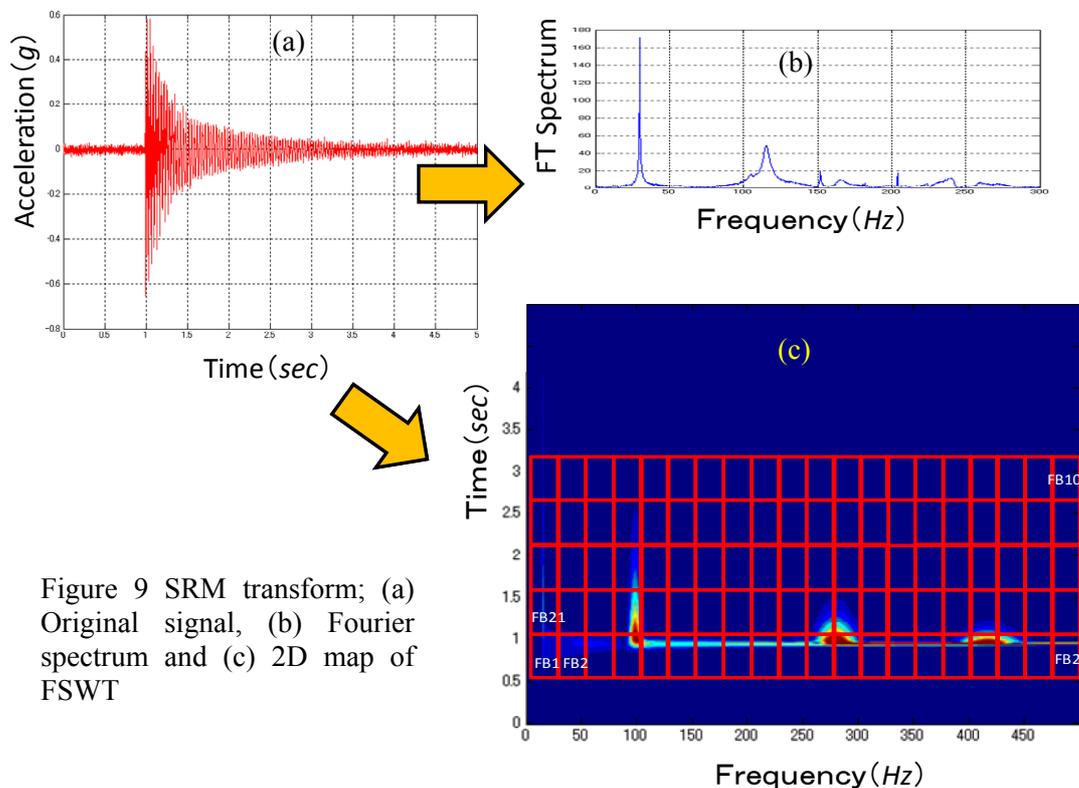


Figure 9 SRM transform; (a) Original signal, (b) Fourier spectrum and (c) 2D map of FSWT

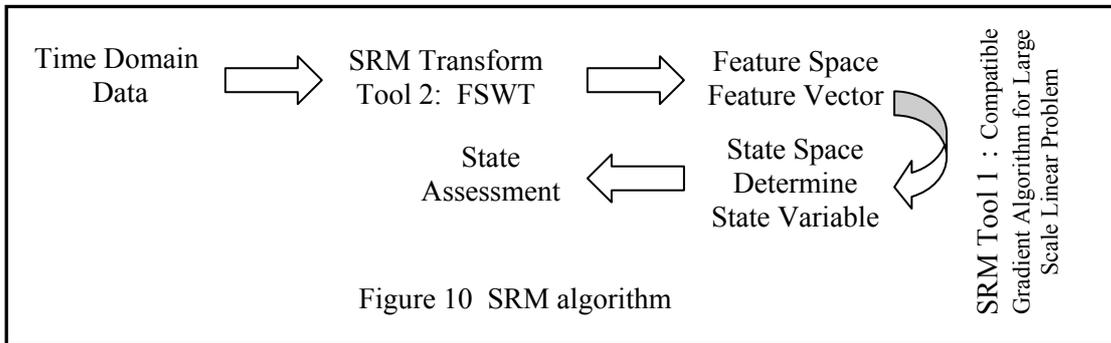


Figure 10 SRM algorithm

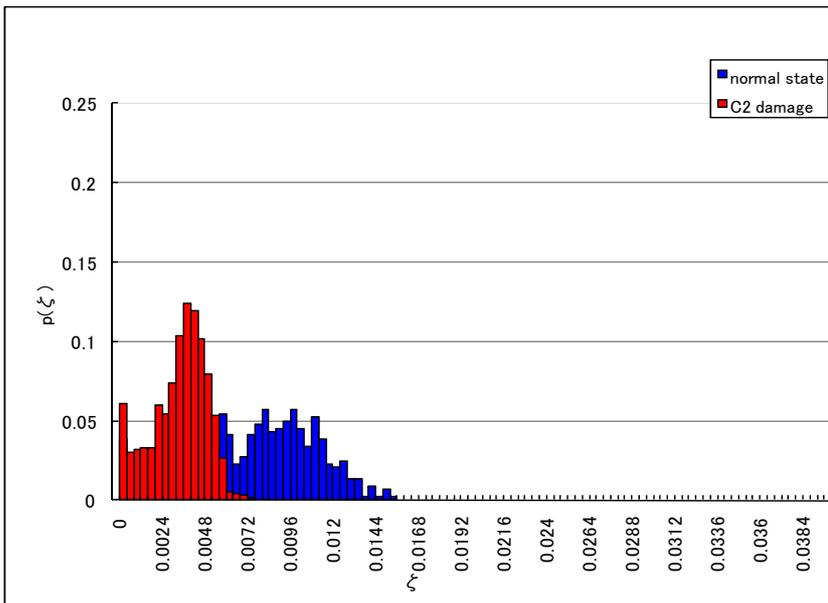


Figure 11 Comparison between C2 damaged and no-damaged (normal) girders on SRM state probability distribution

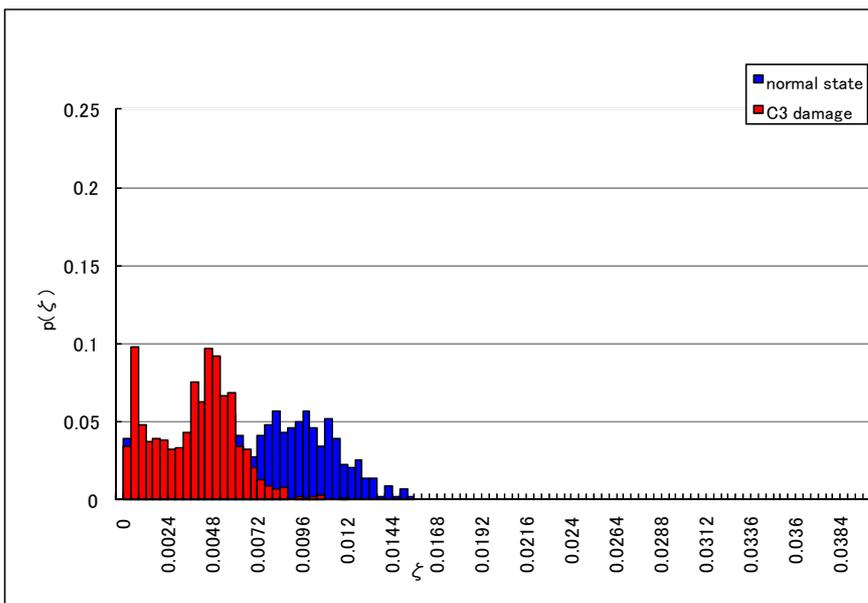


Figure 12 Comparison between C3 damaged and no-damaged (normal) girders on SRM state probability distribution

Based on the SRM algorithm as shown in **Fig. 10**, we will be able to get the distribution of state variable  $\zeta$ , that is the state probability distribution, as shown in **Figs. 11** and **12**. In here, **Fig. 11** shows the results of comparison between C2 damaged (symmetry state; see **Fig. 8**) and no-damaged girders on the SRM state probability distribution. As the same manner, **Fig. 12** shows the results of comparison between C3 damaged (asymmetry state; see **Fig. 8**) and no-damaged girders on the SRM state probability distribution. It is clear that damages like girder stiffness reduction tend to not only move away the peak value of SRM state probability distribution from their normal (original) state (ex.  $\zeta = 0.0096 \rightarrow 0.0036$  for C2 damage &  $\zeta = 0.0096 \rightarrow 0.0048$  for C3 damage) but also change the parameters related to state variable. Then, it is found that based on these distributions, we will be able to recognize the condition changes between the current state and previous state (normal state) in a deteriorating bridge.

#### 4 CONCLUDING REMARKS

This paper introduced the details of a newly proposed “State Representation Methodology (SRM)” and its application to bridge condition assessment based on the bridge monitoring data. The SRM is a novel tool that can provide some ideas and algorithms for data mining in the bridge monitoring system. The state of a system such as bridge structure could be obtained by a state variable  $\zeta$  that calculate from a State Representation Equation (SRE). A Kernel function method which plays an important role in the Support Vector Machines (SVM) was applied to get solutions of the SRE. In the computation of the SRE, it needs to be changed into a Large-Scale Linear Constraint Problem (LSLCP). And a new time-frequency analysis tool, called Frequency Slice Wavelet Transform (FSWT), was able to powerfully reveal a change of the characteristics in vibration signal. Finally, an application example in the laboratory bridge monitoring system was presented so as to demonstrate how to apply the SRM to practical problems.

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