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A Non-Dimensional Design Procedure for Flexural Strengthening of R/C Beams Using FRP

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ABSTRACT: The use of FRP in strengthening R/C structures has increased dramatically in the past two decades. This increase was due to the better understanding of the system behavior as a result of the large number of research projects that were carried out around the world. It was also due to the many advantages of the system, such as its light weight and ease and speed of construction over the available alternatives such as steel jacketing. This increase in demand for the system required a complete set of design procedures to guide the designer in the design process. One comprehensive guideline for the design of externally bonded FRP systems for strengthening of R/C structures is the ACI 440.2R-08 document. In this document a step-by-step procedure for the design of such a system is presented. This procedure, however, is iterative and lengthy and involves finding the depth of the neutral axis based on compatibility of strains for an assumed area of FRP. Therefore, if the assumed FRP area was either small or large for the new required ultimate moment of the beam, a new FRP area is assumed and another set of trial-and-error procedure is performed to find the depth of the neutral axis. The objective of this research is to develop a non-iterative design procedure for strengthening of R/C beams for flexure using FRP. Sample non-dimensional design charts will be presented along with a design example to illustrate the proposed non-iterative design procedure.

1.0 INTRODUCTION

For the last two decades, the use of externally bonded carbon fiber reinforced polymer (CFRP) composite sheets and laminates to strengthen reinforced concrete (RC) beams in flexure has gained wide and rapid acceptance in the civil engineering construction community. The fact that the FRP products are attractive materials in strengthening and rehabilitation of structural members is due to their high strength to weight ratio, resistance to corrosion, lightweight, and ease of instalation and application. In addition, numerous research projects [Meier & Kaiser (1991), Meier, (1995), Arduini et al. (1997), Norris et al. (1997), Spadea et al. (1998), Ross et al. (1999), Fanning & Kelly (2001), Alagusundaramoorthy et al. (2003), Rasheed et al. (2006), and Bahn & Harichandran (2008)] were carried out throughout the world that demonstrated the effectiveness of externally bonded CFRP composites to improve the flexural capacity of RC beams. To support the strengthening revolution using FRP composites, the American Concrete Institute (ACI) released the ACI 440.2R-08 (2008) document to provide guidelines for the design and analysis of externally bonded FRP systems for strengtheneing of R/C structural members.

The existing design process for flexural strengthening of RC members as presented in the ACI 440.2R-08 (2008) document includes an iterative and lengthy step-by-step procedure to find the depth of the neutral axis for an assumed area of FRP based on compatibility of strains and equilibrium of forces. Upon finding the correct location of the neutral axis, the ultimate moment capacity of the member is computed. If the assumed area of FRP reinforcement was small or large, a new FRP area is assumed and another set of trial-and error procedure is performed to find the depth of the neutral axis and the corresponding new ultimate moment capcity of the member is calculated. For an optimum design, the trial-and-error process terminates when the moment capcity of the flexural member is equal to or slightly greater than the applied design moment.

The objective of this paper is to develop a non-iterative design procedure that facilitates the design process for flexural strengthening of RC beams using externally bonded FRP sheets or laminates. A set of non-dimensional parameters and procedures for the design of such a system is proposed. Sample non-dimensional design charts that require no iterations are presented. A design example is also presented to illustrate the use of the proposed design charts.

2.0 CURRENT DIMENSIONAL FLEXURAL STRENGTH PROCEDURE

The design procedure for the ultimate load capacities of strengthened singly reinforced rectangular sections with FRP laminates or strips in flexure is presented in Section 10.2.10 of the ACI 440.2R-08 (2008) document. The ACI 440 design procedure is a trial-and-error approach and based on strain compatibility and equilibrium of internal forces and the governing failure mode. The distribution of internal strain and stress at the ultimate limit state for a rectangular singly reinforced concrete section is presented in Fig. 1.

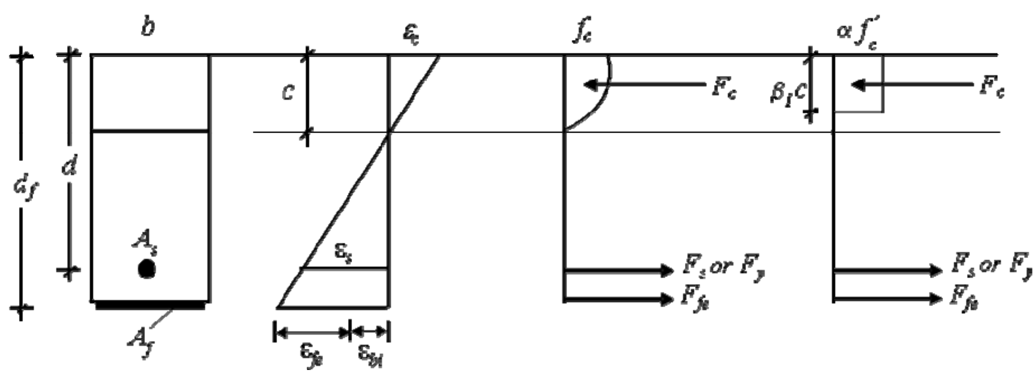


Figure 1: Internal strain and stress distribution at ultimate limit state [ACI 440.2R-08 2008]

The different symbols presented in Figure 1 are defined as follows:

A_s = area of steel reinforcement
A_f = area of FRP external reinforcement
b = beam width
d = distance from extreme compression fiber to centroid of tension reinforcement
d_f = effective depth of FRP reinforcement
c = distance from extreme compression fiber to the neutral axis
ε_c = strain level in concrete at ultimate limit state
ε_s = strain level in the steel reinforcement at ultimate limit state

ε_{fe} = effective strain level in FRP reinforcement at ultimate limit state
ε_{bi} = strain level at the soffit of the concrete substrate at time of FRP installation
f_c = compressive stress in concrete
F_c = resultant compressive force in concrete attained at ultimate limit state
F_s = tensile force in steel reinforcement attained at ultimate limit state
F_y = yield force in steel reinforcement attained at ultimate limit state
F_{fe} = effective tensile force in FRP reinforcement attained at ultimate limit state
β_1 = ratio of depth of equivalent rectangular stress block to depth of neutral axis
α = multiplier on $f_c^{'}$ to determine intensity of an equivalent rectangular stress distribution for concrete

The flexural strength of a section depends on the controlling failure mode. The possible flexural failure modes for a R/C section stengthened with externally bonded FRP laminates or sheets [ACI 440.2R-08 2008] are:

- Concrete crushing in compression before steel yielding in tension
- Yielding of steel in tension followed by concrete crushing
- Yielding of steel in tension followed by rupture of the FRP laminate
- Cover delamination
- FRP debonding from the concrete substrate

Concrete crushing is assumed to occur if the compressive strain in the concrete reaches its maximum usable strain of 0.003. In addition, rupture of the FRP laminate is assumed to occur if the attained strain in the FRP reaches its design rupture strain ε_{fu}. Furthermore, FRP debonding or concrete cover delamination is assumed to occur if the effective strain in the FRP reinforcement ε_{fe} exceeds the FRP strain debonding limit, ε_{fd}.

The current design procedure to calculate the required FRP flexural area is based on a trial-and-error approach that involves the following steps:

1. Assuming an area of FRP reinforcement
2. Assuming the depth of the neutral axis, *c*
3. Calculating, using strain compatibility, the strain level in each material
4. Calculating the associated stress level in each material
5. Checking equilibrium of internal forces; if the internal forces are not in equilbrium, the depth of the neutral axis, *c* is revised and steps 1-4 are repeated.
6. Calculating the ultimate moment capacity of the member upon finding the correct location of the neutral axis
7. Checking if the ultimate moment capacity of the member is equal to the required ultimate moment; if the ultimate moment capacity is smaller or considerably larger than the required moment, a new area of FRP reinforcement A_f is assumed and steps 1-7 are repeated. The trial and error procedure terminates when the moment capcity of the flexural member is equal to or slightly larger than the required design moment.

The design moment capacity of the FRP-strengthened section is computed using Eqs. (1-3) as follows:

$$\phi M_n = \phi \left[M_{ns} + \psi_f M_{nf} \right] \tag{1}$$

$$M_{ns} = A_s f_s \left(d - \frac{\beta_1 c}{2} \right) \tag{2}$$

$$M_{nf} = A_f f_{fe}\left(d - \frac{\beta_1 c}{2}\right) \tag{3}$$

where,
ϕ = strength reduction factor as defined in ACI 318-05 (2005)

$$\phi = 0.9 \ for \ \varepsilon_s \geq 0.005$$

$$= 0.65 + \frac{0.25(\varepsilon_s - \varepsilon_{sy})}{0.005 - \varepsilon_{sy}} \qquad \text{for} \quad \varepsilon_{sy} \leq \varepsilon_s \leq 0.005$$

$$= 0.65 \qquad \text{for} \quad \varepsilon_s \leq \varepsilon_{sy}$$

M_{ns} = contribution of steel reinforcement to nominal flexural strength
ψ_f = 0.85; FRP strength reduction factor for flexure
f_s = stress in steel reinforcement
f_{fe} = effective stress in the FRP attained at section failure

3.0 PROPOSED NON-DIMENSIONAL DESIGN FORMULATION

A simplified non-dimensional design procedure is essential in promoting the design and construction of R/C beams strengthened with FRP sheets in flexure. Referring to Figure 1, the following non-dimensional parameters are defined:

$$\Omega_1 = \frac{c}{d} \tag{4}$$

$$\Omega_2 = \frac{d_f}{d} \tag{5}$$

$$\rho_s = \frac{A_s}{bd} \tag{6}$$

$$\rho_f = \frac{A_f}{bd_f} = \frac{nt_f w_f}{bd_f} \tag{7}$$

where,
n = number of layers of FRP reinforcement
t_f = thickness of FRP reinforcement
w_f = width of FRP reinforcement

For a given section (given Ω_2 and ρ_s), the designer initially assumes a value for Ω_1 and ρ_f. The effective strain ε_{fe} relationship can be obtained from the internal strain distribution diagram presented in Figure 1 using similar triangles as follows:

$$\varepsilon_{fe} = \varepsilon_{cu}\left(\frac{d_f - c}{c}\right) - \varepsilon_{bi} \tag{8}$$

where ε_{cu} = 0.003

Substituting the corresponding non-dimensional parameters defined above, Eq. (8) becomes

$$\varepsilon_{fe} = \varepsilon_{cu}\left(\frac{\Omega_2}{\Omega_1} - 1\right) - \varepsilon_{bi} \tag{9}$$

The failure mode of the strengthened member is defined as follows:

$if \;\; \varepsilon_{fe} = \varepsilon_{cu}(\frac{\Omega_2}{\Omega_1} - 1) - \varepsilon_{bi} \le \varepsilon_{fd}$ $\Rightarrow$ concrete crushing controls and $\varepsilon_c = \varepsilon_{cu} = 0.003$

$if \;\; \varepsilon_{fe} = \varepsilon_{cu}(\frac{\Omega_2}{\Omega_1} - 1) - \varepsilon_{bi} \ge \varepsilon_{fd}$ $\Rightarrow$ FRP failure (delamination or debonding) controls

where $\varepsilon_{fd} = 0.083\sqrt{\frac{f_c^{'}}{nE_f t_f}} \le 0.9\varepsilon_{fu}$ is the strain in the FRP at which debonding may occur, and ε_{fu} is the ultimate strain for the FRP reinforcement. If FRP failure controls, the strain level of the concrete in compression ε_c will be less than 0.003 and should be determined using Eq. (10).

$$\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\frac{c}{d_f - c} \tag{10}$$

Substituting the corresponding non-dimensional parameters, Eq. (10) becomes

$$\varepsilon_c = (\varepsilon_{fe} + \varepsilon_{bi})\frac{\Omega_1}{\Omega_2 - \Omega_1} \tag{11}$$

and

$$\beta_1 = \frac{4\varepsilon_c^{'} - \varepsilon_c}{6\varepsilon_c^{'} - 2\varepsilon_c} \tag{12}$$

$$\alpha_1 = \frac{3\varepsilon_c^{'}\varepsilon_c - \varepsilon_c^2}{3\beta_1 \varepsilon_c^{'2}} \tag{13}$$

where,

$$\varepsilon_c^{'} = \frac{1.7 f_c^{'}}{E} \tag{14}$$

where E is the concrete modulus of elasticity.
Similarly, the strain level in the steel reinforcement is determined using Eq. (15).

$$\varepsilon_s = (\varepsilon_{fe} + \varepsilon_{bi})\left(\frac{1 - \Omega_1}{\Omega_2 - \Omega_1}\right) \tag{15}$$

The strength reduction factor ϕ is determined according to the strain level in the steel reinforcement ε_s as defined in the previous section. The associated stress levels in the steel and FRP reinforcement are determined using Eqs. (16-17).

$$\begin{aligned} f_s &= E_s \varepsilon_s \quad \text{if} \quad \varepsilon_s \le \varepsilon_y \\ &= f_y \quad \text{for} \quad \varepsilon_s \ge \varepsilon_y \end{aligned} \tag{16}$$

$$f_{fe} = E_f \varepsilon_{fe} \tag{17}$$

Using equilibrium of forces, the neutral axis location c is determined using Eq. (18).

$$c = \frac{A_s f_s + A_f f_{fe}}{\alpha_1 \beta_1 f_c^{'} b} \tag{18}$$

The corresponding non-dimensional parameter Ω_1 is determined using Eq. (19) by dividing Eq. (18) by *d* and substituting the corresponding non-dimensional parameters

$$\Omega_1 = \frac{c}{d} = \frac{\rho_s f_s + \rho_f \Omega_2 f_{fe}}{\alpha_1 \beta_1 f_c^{'}} \tag{19}$$

If the value of Ω_1 obtained from Eq. (19) is not equal to the initially assumed value, a new value of Ω_1 is assumed and the entire design process is repeated. Thus the following constraint equation could be added to the current design process

$$(\Omega_1)_{calculated} - (\Omega_1)_{assumed} = 0 \tag{20}$$

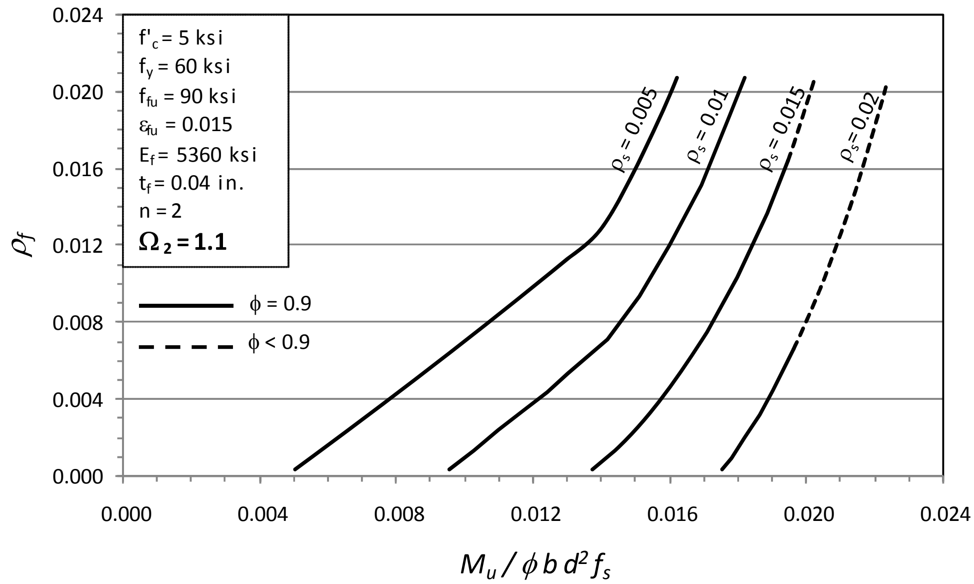
Once Eq. (20) is satisfied, the design moment strength could be determined. By substituting the non-dimensional parameters and with some algebraic manipulation of Eqs. (1-3), the following non-dimensional equation could be derived to determine the moment capacity of the strengthened member

$$\frac{M_{\mathrm{u}}}{\phi\, b d^2 f_s} = \left[\rho_s \left(1 - \frac{\beta_1 \Omega_1}{2} \right) + \psi_f \rho_f \Omega_2 \frac{f_{fe}}{f_s} \left(\Omega_2 - \frac{\beta_1 \Omega_1}{2} \right) \right] \tag{21}$$

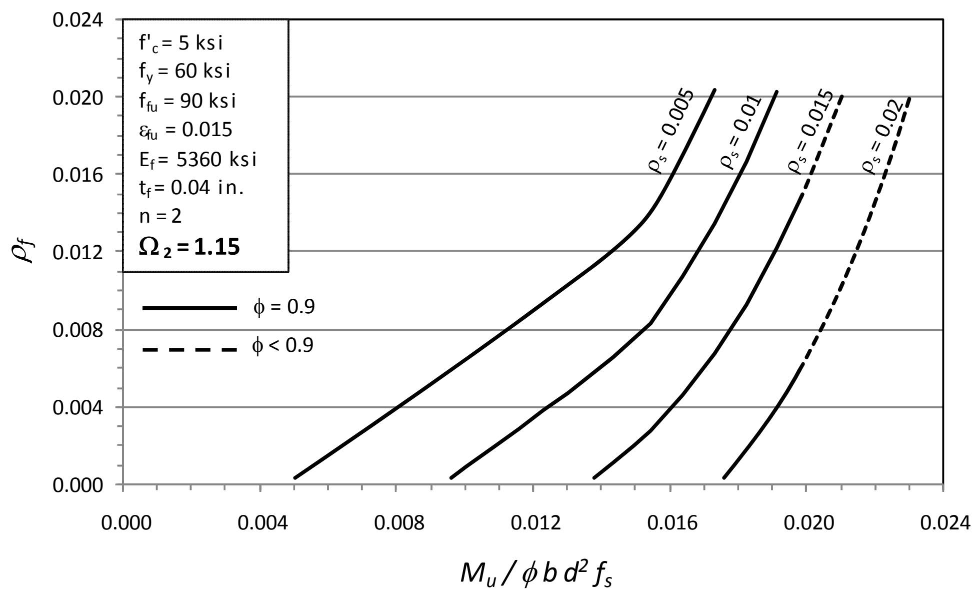
4.0 SAMPLE NON-DIMENSIONAL DESIGN CHARTS

A simplified design procedure was devised to eliminate lengthy and iterative calculations in the design of FRP systems in flexure. This design procedure is based on the non-dimensional design equations derived above. Although the design procedure based on the non-dimensional equations is iterative, this procedure was used to develop non-dimensional design charts that will eliminate any iterations in the design process.

In order to illustrate the applicability of the proposed design procedure, Microsoft Excel's Solver feature was used to calculate the relative neutral axis depth Ω_1 by varying the FRP reinforcement ratio ρ_f for a certain set of parameters such as $f^{'}_c$, f_y, f_{fu}, ε_{fu}, E_f, t_f, n, ρ_s, and Ω_2. Sample design charts are shown in Figure 2(a) and (b). It should be noted that the curves in these figures were terminated when the FRP reinforcement ratio ρ_f becomes practically not feasible. Also, within this range the steel stress f_s is always equal to the yielding stress f_y.



(a)



(b)

Figure 2: Non-dimensional design charts for a certain set of input parameters. Note: $f_s = f_y$

5.0 DESIGN EXAMPLE

The same example presented in the ACI 440.2R-08 (2008) document, section 15.3, will be designed using the developed non-dimensional design charts. The beam is a singly reinforced concrete beam with the following data:

Beam properties	FRP properties
Beam width b = 12 in.	Thickness per ply t_f = 0.04 in.
Beam depth d_f = 24 in.	Number of layers n = 2
Effective depth d = 21.5 in.	FRP ultimate tensile strength f_{fu} = 90 ksi
Concrete strength f_c' = 5 ksi	FRP rupture strain ε_{fu} = 0.015
Steel yield strength f_y = 60 ksi	FRP modulus of elasticity E_f = 5360 ksi
Steel area A_s = 3 in.^{2}	

The ultimate moment capacity with FRP is 327 k-ft.

Solution:
Design charts presented in Figure 2 can be used to design this beam since material properties used to develop these charts are the same as those used in this example. First, the following terms need to be calculated:

1. ρ_s = 3/(12 x 21.5) = 0.0116
2. Ω_2 = 24/21.5 = 1.116
3. Assume that ϕ = 0.9

$$\frac{Mu}{\phi\, bd^2 f_y} = \frac{327x12}{0.9x12x21.5^2 x60} = 0.0131$$

4. Enter Figure 2(a) (Ω_2 = 1.1) with x-axis = 0.0131 and find y-axis by interpolating between ρ_s = 0.01 and 0.015 => ρ_f = 0.0035 (note that ϕ is 0.9, assumption is OK)
5. Enter Figure 2(b) (Ω_2 = 1.15) with x-axis = 0.0131 and find y-axis by interpolating between ρ_s = 0.01 and 0.015 => ρ_f = 0.003 (note that ϕ is 0.9, assumption is OK)
6. By linear interpolation for Ω_2 = 1.116 we get ρ_f = 0.00334
7. Area of FRP = A_f = 0.00334 x 12 x 24 = 0.96 in.^{2}
8. Width of FRP = w_f = 0.96 / (2 x 0.04) = 12 in. (same as assumed in ACI 440.2R-08 (2008) document).

The actual required moment in the ACI 440.2R-08 (2008) example was in fact 294.4 k-ft. The ACI document calculations were started by assuming that the FRP width is 12 in. as a first iteration. The corresponding moment was 327 k-ft which is greater than 294.4 k-ft, so no more iterations were carried out due to the lengthy calculations. One advantage of the developed design charts is that the designer can find the exact area of FRP required for a certain moment with minimum calculations. In this case, if the above calculations were repeated for the actual required moment of 294.4 k-ft, the required width of FRP would be 6 in. only.

6. CONCLUSIONS

The following conclusions are made:

1. The ACI 440.2R-08 (2008) document design procedure to design externally bonded FRP systems for strengthening of R/C structures can be modified into a non-dimensional design procedure.
2. The non-dimensional design procedure can be presented in non-dimensional design charts. A sample of the non-dimensional design charts is presented in this paper.
3. A design example was shown to illustrate the effectiveness of the non-dimensional design charts. The lengthy calculations of the ACI 440.2R-08 (2008) document can lead to significantly larger area of FRP as was shown in the example.
4. The currently proposed design procedure requires the development of a complete set of design charts each specifically for certain types of materials and steel location. More research is underway to develop unified design charts and procedures that can incorporate all the variables involved in the design.

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