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Features of Nonlinear Vibration-Based Structural Health Monitoring

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ABSTRACT: For the last two decade, there has been a growing interest to use the vibration-based Structural Health Monitoring (SHM) as a global assessment method for existing infrastructures. This technique provides a tool for assessing inaccessible structure areas. The vibration-based technique includes different approaches which can be classified as linear or nonlinear. The former one is faced by various obstacles preventing it from going beyond research topics in civil engineering fields. Accompanied with the linear, the nonlinear method overcomes some of these disadvantages, for which the existence of a datum for the intact structure is a necessity. This datum is usually not available for existing infrastructure as mostly all the codes do not enforce collecting the structure dynamic response just after construction as a datum for future monitoring. However, the need of a reference can be eliminated in the nonlinear approach by detecting special features occurring only due to the nonlinear structure behaviour in the presence of damages. The aim of this paper is to highlight some of these features such as sub-harmonic and supper-harmonic components, which could be used as indicators for existence of degradation. Firstly, an analytical investigation is carried out in which a concrete post with breathing crack is modelled using MATLAB. Secondly, the system stiffness is determined based on the breathing crack situation either it is opened, partially opened, or fully closed. Then the system dynamic response is determined for different level of deterioration illustrating the corresponding nonlinear features.

1 INTRODUCTION

Bridges are considered an important asset for either developed or developing countries. Bridges have a direct impact on the national economy. Helmicki et al. (1999) estimated that inadequacies in the nation's transportation system in USA can reduce the annual growth rate by as much as 1%. Edward et al. (2006) mentioned that according to the National Bridge Inventory in United States, there are approximately 593,885 bridges. The authorities managing these enormous infrastructures face difficulties to stand on their exact situations. The used inspection technique usually is visual inspection carried by experts. The output of this process depends mainly on the expert's experiences. The evaluation and the results from such inspections are questioned. An automatic technique is vital to ensure the reliability of the assessment outputs.

In the last few decades, many researches had been carried out aiming to use the changes in the structure dynamic characteristics as damage indicators. The early works in this field used the changes in the fundamental natural of frequencies to define the damage existence. For example, the earliest work done in this approach is Cawley and Adams et al. (1979). Bicanic et al. (1997) developed a technique to predict the damage existence using only the changes in the natural frequencies. However, it had been confirmed in previous works that the frequency's changes in bridges are usually inadequate to define the damage existence, as they could be due to another effects such as environment or ambient traffic.

Other authors proposed different dynamic features such as: mode shapes, mode shape curvatures, dynamic flexibility and others feature extractions. A full review about structure health monitoring can be found in Los Alamos National Laboratory Report number (LA-13976-MS, 2003). These vibration-based techniques assumed the existence of a linear relation between the structure dynamic responses and the degradation. This assumption can not be satisfied in all damage types such as fatigue crack. This type should be considered as a breathing crack which would have a nonlinear dynamic response. As a result, the vibration-based technique can be classified as a linear and nonlinear which are discussed below.

2 LINEAR VIBRATION-BASED SHM

Farrar and Jauregui et al. (1998) carried a destructive modal test on The I-40 Bridges over the Rio Grande in Albuquerque. They concluded that the changes in the natural frequencies of the first and second modes were 7% and 4% respectively after introducing significant damage. The authors of this paper carried out analytical investigation on a single cell box-girder bridge. The changes in the first fundamental natural frequencies for a single span and a continuous two-span bridge were 14% and 7% respectively after considerable stiffness reduction was introduced along the two webs and the soffit slab. In spite of being significant changes, they may not be distinguished from those due to the environmental effects, which could be within the range of 5%.

In such analytical studies, the damaged zones are usually modelled as permanent stiffness reductions in structural elements. However, this method is inadequate to represent the actual behaviour of a breathing crack such as fatigue crack. This type of crack has nonlinear effects in the structure performance. Friswell and Penny discussed the different used crack modelling techniques et al. (2002).

3 NONLINEAR VIBRATION-BASED SHM

A breathing crack can not be modeled by either a constant stiffness reduction or by decreasing the elements' cross sections at damage location. This returns to the fact that the stiffness at the breathing crack location changes according to the structure response. In other words, if the applied loads can gradually close the crack, the stiffness shall be recovered.

As the stiffness is not constant and varied continuously with respect to the response, the system will have a nonlinear behavior. Friswell and Penny demonstrated such behavior using a single degree of freedom model solving numerically the equation of motion. However, in their model, a bilinear stiffness is used ignoring the smooth behavior in the closing-opening process, as it was for illustrating purposes.

Benfratello et al. (2007) used a bilinear system in modeling a cantilever beam, which includes multi cracks. His model included only the intact stiffness and that of the full opened crack. Pungo et al. (2000) investigated the nonlinear beam vibration with multi cracks, in which he assumed that the cracks are closing smoothly from fully, open to fully close. Due to this nonlinear behavior, its dynamic responses should be nonlinear. However, the degree of nonlinearity relates to the level of degradation. If this nonlinearity is monitored, a deep understand could be achieved. In a similar work carried out by Bovsunovsky et al. (2007), a fracture crack and its effect on the system dynamic response was investigated by examining two specimens. A finite element model was created in which the equations of motion were resolved with step-by-step in time using Runge-Kutta method. The aim was to determine the non-linearity features of the vibration responses such as super and sub-harmonic resonances. The super harmonic frequency is defined as multiplies of the system fundamental natural frequency as shown in Figure 1.

In this paper, an illustrative single degree of freedom model is used, in which the stiffness of the spring is changing smoothly between the intact and the degraded stiffness value, as it will be explained in the numerical study. The nonlinearity components such as the super and sub-harmonics are depicted with the different degradation levels.

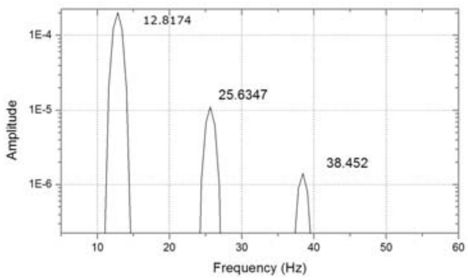

Figure 1: Supper-Harmonic for Nonlinear System

4 NUMERICAL STUDY

In this section a numerical study is presented using a simplified structure system which can be easily modeled by a single degree of freedom system. It has been taken into consideration the material nonlinearity initiated due to the presence of different degradation levels.

The objectives of this numerical study can be summarized as follows: (i) highlighting the importance of modeling the cracks in an appropriate method; (ii) presenting the material nonlinearities to be considered for degradation presentation; (iii) investigate the dynamic nonlinear vibration features.

The structural element under consideration is a concrete post with light reinforcement, hanged from the top. It has a squared cross section, each side is 0.1 meter. Its height is assumed to be one meter. A mass of 2000 kg is hanged at the bottom end. This structure can be represented as a single degree of freedom system by ignoring the post's own weight. Figure 2-a) illustrates the structural system considered, and Figure 2-b) shows the equivalent single degree of freedom.

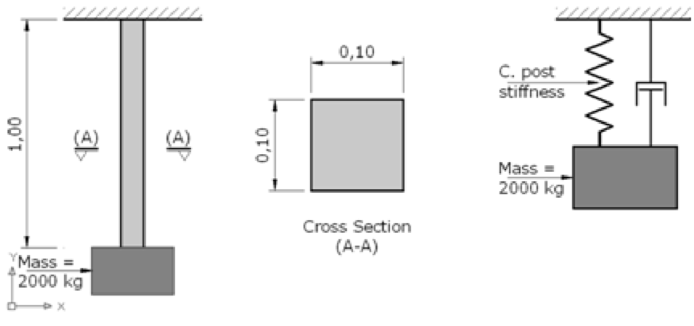

Figure 2. Concrete post used in the numerical study, (a) Concrete post's geometric dimensions, (b) Single degree of freedom model represents the concrete post model

In the model, the damping and the mass are assumed to be constant. However, the stiffness is assumed to be related to the degradation level introduced in the structural system. In addition, the stiffness has a smooth change from the fully open to the fully close position. The relationship between the stiffness and the response is calculated based on the concrete tension softening theory.

As the concrete is a brittle non-homogenous material, it had been observed that the concrete tensile resistance does not vanish by the micro crack formation, and it has a stress tensile-strain softening. The concrete still contributes in the structure tensile resistance as long as the crack width is less than the critical width. It is reported that the concrete strain softening was first observed by L'Hermite in 1959 (ACI 1999). In a work done by Reinhardt, he proposed a simple equation in which an empirical factor ($k = 0.31$) is estimated. Equation (1) shows the equation suggested by Reinhardt.

$$\frac{\sigma}{f_t} = 1 - \left(\frac{\varepsilon}{\varepsilon_o}\right)^k \tag{1}$$

Elmorsi et al. used a tension softening relation developed by Stevens et al. (1998). The same relations are used in this study to calculate the stiffness degradations. Equation (2) illustrates the proposed expression for the tensile stress-strain relationship envelope. The concrete tensile stress-strain relationship is shown in Figure 3

$$\frac{f_t}{f_{cr}} = (1-\alpha)e^{-\lambda_t(\varepsilon_t - \varepsilon_{cr})} + \alpha \tag{2}$$

$$\lambda_t = \frac{270}{\sqrt{\alpha}} \qquad \lambda_t \le 1000 \tag{3}$$

$$\alpha = 75(\rho_s / d_b) \tag{4}$$

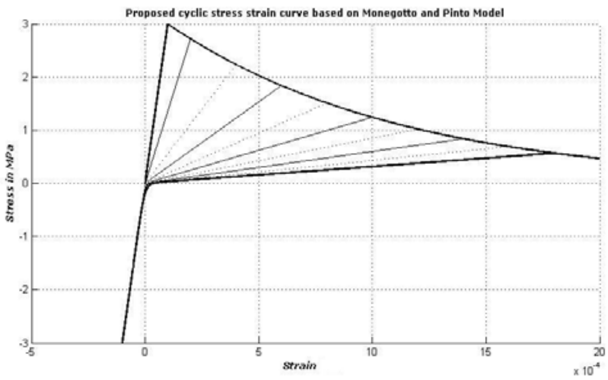

Figure 3. Stress vs. Strain relationship for both monotonic and cyclic loading

For cyclic tensile stress-strain relations, Elmorsi used a relationship developed by Monegotto and Pinto. The used expression is a simple one that defines the process of crack opening and closing. The closing of the existence cracks will take place only if the compression strain exceeds the strain of the focal point. The compression stress at this point is assumed to be 0.1 of the maximum compression stress. The equations of the curve connecting between the unloading tensile stresses and the compression stresses are expressed as follows:

$$f^* = b\varepsilon^* + \frac{(1-b)\varepsilon^*}{\left(1+\varepsilon^{*R}\right)^{1/R}} \quad (5)$$

$$b = \frac{f_{un}/\varepsilon_{un}}{f_{fp}/\varepsilon_{fp}} \quad (6)$$

$$R = R_o - \frac{a_1\varepsilon_{un}}{a_2 + \varepsilon_{un}} \quad (7)$$

$$f^* = \frac{\left(f_{fp} - f_c\right)}{f_{fp}} \quad (8)$$

$$\varepsilon^* = \frac{\left(\varepsilon_{fp} - \varepsilon_c\right)}{\varepsilon_{fp}} \quad (9)$$

Where R_o, a_1, and a_2 are assumed to be 20, 18.5 and 0.0015, respectively. Figure 3 and Figure 4 illustrate the stress-strain curves calculated using Monegotto and Pinto suggested model.

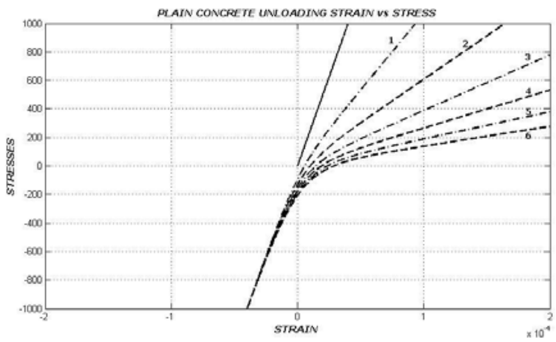

Figure 4. Stress vs. Strain curve for cyclic loading for different degradation level

4.1 Stiffness vs. Response

The previous section demonstrates the equations of the monotonic tensile stress envelope and the cyclic unloading tensile stress-strain relation, based on which the Young’s Modulus can be calculated. Then the stiffness of the cracked structure is determined at different degradation levels. Figure 4 demonstrates the gradual change in Young’s Modulus reaching the intact situation if the compression stress exceeded the focal compression stress under the applied load. It is expected that the stiffness of the system will be gradually change from the degraded situation occurring under tension stresses to the intact stiffness after exceeding the focal strains.

As a result, all the stiffness curves are divided into three different zones as shown in Figure 5. The first zone is the upper limit of the stiffness values, which has a constant value. The second middle zone decreases gradually from the upper limit to the lower limit value. The third zone is the lower limit of the degraded stiffness. The transverse zone between the upper and lower value is fitted using the curve fitting function in MATLAB. In this step, the fitting equations are determined to be used later in describing the spring’s stiffness in the single degree of freedom system through the dynamic analysis. Pulling excitation will be considered in the numerical analysis. In the following paragraphs, the results of the numerical work will be discussed. Figure 5 shows the stiffness vs. the displacements.

4.2 Pulling Excitation

The source of excitation is pulling vertically the free end of the concrete post and suddenly release the pulling force applied on the structural system. This approach typically can be practically done by cutting suddenly the pulling tendon. The excitation method is a similar concept to which is used in the post tension slabs, in which the tendons embedded in the slab concrete is pulled and released. The same step was followed in the numerical analysis by introducing a specified displacement at the edge of the hanged mass. By solving the dynamic equation of the system using the ode45 built in MATLAB, the system's acceleration can be resolved.

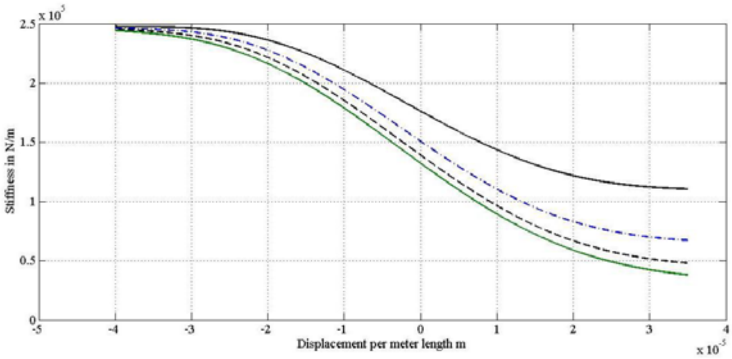

Figure 5. Stiffness vs. deflection at different degradation levels

After determination of the corresponding accelerations to the different stiffness deterioration levels, the frequency response curves could be determined using Fast Fourier Transform (FFT) analysis. Figures (5, 6, 7, 8, 9, and 10) illustrate the frequency response curves. Figure (6) shows the fundamental natural frequency of the intact concrete post without introducing any type of damage. However, the other figures illustrate the fundamental natural frequency of the deteriorated concrete post and the corresponding supper-harmonic components. It can be noticed that the number of the supper harmonic frequencies increase by the increasing of the percentage of deterioration. The same result had been demonstrated by Sinou et al. 2007 who investigated these features for a steel rotor shaft.

4.3 Results

By using Fast Fourier Transformer (FFT), the Frequency Response Function (FRF) can be plotted for the intact system, as well as for the degraded system. From figure (6), the fundamental natural of frequency can be depicted as 55.82 Hz. The fundamental natural frequency can be concluded using the peak method after plotting the response curve. It should be mentioned that it is expected to have only one peak value for the single degree of freedom system. Moreover, it is expected that after initiation of cracks the system's stiffness should be reduced and the peak value of the FRF curve should be shifted to a lower value. Table (1) illustrates the values of the intact and deteriorated frequencies, as well as the percentage of change corresponding to each degradation level.

However, the FRF curves show other peak values at the multipliers of the deteriorated natural frequencies. These peaks which are called super-harmonic frequencies return to the consideration of the nonlinear behavior of the system during oscillation. Table (2) summarized the depicted super-harmonic frequencies at different degradation levels. The number of super-harmonic frequencies increases with the deterioration percentage.

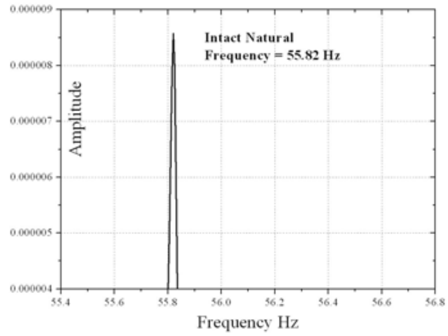

Figure 6: Intact System

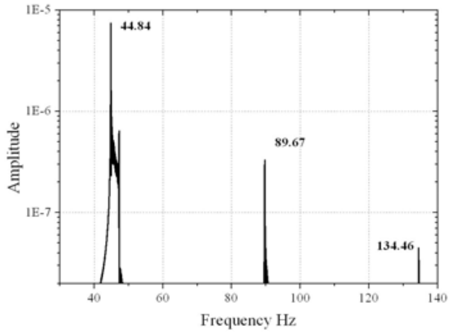

Figure 7: Degradation Level (1)

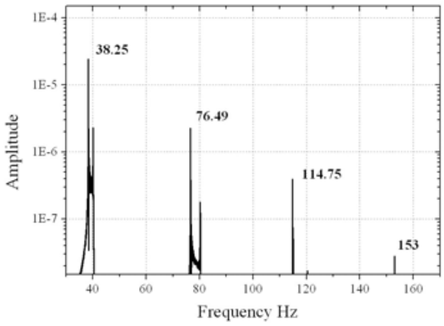

Figure 8: Degradation Level (2)

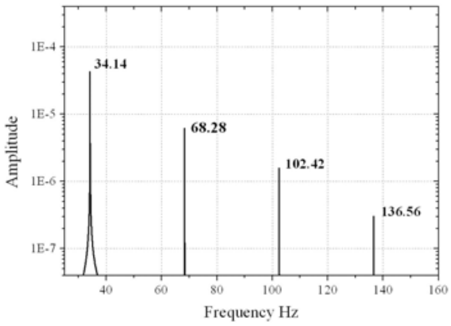

Figure 9: Degradation Level (3)

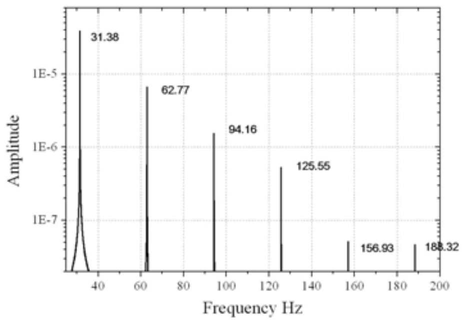

Figure 10: Degradation Level (4)

Table 1. Percentage of changes in Frequency

Degradation Level	1	2	3	4
Stiffness Reduction %	55	72	80	85
Natural Frequencies in Hz	44.84	38.25	34.14	31.38
Changes in Frequencies %	20	32	38	43

Table 2. Depicted Super-harmonic Components

Degradation level	Frequency	X2	X3	X4	X5	X6
1	44.84	89.67	134.46	--	--	--
2	38.25	38.25	76.49	114.75	153	--
3	34.14	34.14	68.28	102.42	136.56	--
4	31.38	31.38	62.77	94.16	125.55	156.93

5 CONCLUSION

From the previous numerical study, it can be concluded that non linear modeling of the system after crack propagations produces new features such as super-harmonic components. These features would not appear if the crack is defined in a simplified way with permanent stiffness degradation or any other modeling process which would neglect the breathing crack existence.

Moreover, the number of the super-harmonic components is directly proportional to the deterioration level. It is noticeable that the higher the degradation level is the higher the number of the super-harmonic components. Their values would be integer multipliers of the natural frequency after degradation. As a result, the presence of such super-harmonic frequencies would be an indicator of the damage existence in the infrastructures.

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