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# Integrated Evolutionary Optimization Framework for Finite Element Model Identification

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ABSTRACT[i]**:** An integrated evolutionary optimization method is developed for finite element (FE) model identification and structural damage detection. The method is formulated to optimize FE model parameters such that the difference between the observed and analyzed responses is minimized. It is implemented as a generalized framework by coupling the well-developed FE analysis software with the evolutionary optimization technique. The implemented framework is designed by using Extensible Markup Language (XML) to ensure its compatibility and extensibility. A competent genetic algorithm (GA) is employed to search for the optimal and near-optimal solutions. Each solution is evaluated by the defined error function taking into account both static and modal responses. The approach is tested with an example of damage detection of truss structure. The results obtained show that the proposed method is effective at detecting damage and that the framework is generic at facilitating FE model parameter identification.

## 1 INTRODUCTION

Safe and reliable infrastructure is critical to the well being of human society. Civil infrastructures such as bridges and buildings are largely constructed for half a century in the developed countries, infrastructure conditions are badly deteriorated over time. According to ASCE infrastructure report card (ASCE 2009), over 500,000 highway bridges with a span length greater than 25 feet, more than 26% of the bridges are either structurally deficient or functionally obsolete. Therefore, timely detecting the defects and the accumulated damages is essential for infrastructure owners and engineers to take preventive or proactive actions in order to avoid disasters such as I-35 bridge collapse on Mississippi river in 2007, and also ensure that an infrastructure system is able to perform its current and future service as designed. This is usually achieved by undertaking structural health monitoring (SHM), which is defined as the process of tracking structural integrity and assessing the nature of damage in an infrastructure system. SHM is an integration of multiple components including data acquisition, fusion, cleansing, information condensation and model development for purposes of determining structure integrity and performance. One of the primary tasks is to develop effective method and software tool for damage detection.

## 2 BRIEF REVIEW

Over last decades, three types of damage detection methods have been investigated for SHM, namely physics-base methods, data-driven methods and statistics-based method (Doebling et al. 1996). Physics-based method is to infer the physical characteristics of a structural system by solving an inverse problem, which correlates the corresponding mathematic model, e.g. finite element model, with the monitored responses. The induced characteristics of a calibrated FE model, such as the change of properties (e.g. section area, boundary condition etc.) of a civil structure, are attempted to represent possible deterioration or damage.

Many methods are developed for updating FE model parameters. Baruch and Bar Itzhack (1978) and Wei (1990) proposed that stiffness and mass matrices of FE models could be adjusted in one step without iteration. In general, FE model updating can be formulated as an optimization problem of finding the optimal values of FE model parameters by minimizing the discrepancy between the modeled and the observed values of structural responses (Friswell and Mottershead 1995). Soh and Dong (2001) proposed an inverse problem solving for material uncertainty of Young's modulus. One uncertain Young's modulus was assigned to be identified for all the elements of the same material. Two examples including a steel plate and pavement quality problems were demonstrated. One Young's modulus parameter was optimized for the steel plate example while two Young's modulus parameters, one for each of two layers, were optimized for the composite pavement problem. Genetic algorithm (GA) was also applied to identify a set of stiffness reduction factor (SRF) as the indicator of structural damages for the experimental aluminum cantilever beam and one-span steel portal frame (Hao and Xia 2002). SRF is defined as the ratio of stiffness reduction to the initial stiffness. One finite element is assigned with one SRF to be identified. Stiffness reduction seems to be more meaningful to indicate structural damage than Young's modulus variation, but both types of parameters are not directly related to the physical property of structural damage.

In addition to selection of appropriate damage parameters, the definition of objective functions, which evaluate the calibrated FE models quantitatively, is critical in parameter estimation in that they affect the performance of search process. Wang et al. (2007a; 2007b) defined the objective functions (error functions) for bridge FE model calibration based on static and dynamic responses separately. Sanayei et al. (2006) proposed a multiresponse error function by normalizing the error function matrix with the initial value of the matrix, based on the initial parameter values.

Although several stand-alone FE model updating programs, e.g. PARIS (Sanayei 1997), were developed, the modeling capacity of these programs is inadequate for several reasons. They are limited to the types of built-in finite elements in the stand-alone FE updating programs, which are also lacking of Computer-Aided Engineering (CAE) environment. It is difficult to conduct FE modeling along with model updating, especially for the structures with complex configurations and boundary conditions. Moreover, in some cases, FE models created at the stage of structural design can be used for model updating as a nominal model. The well-developed commercial structural analysis software (e.g. STAAD.Pro and SAP2000) provides application programming interfaces (API) (Bentley 2007; Computers and Structures 2005), by which an application program can perform FE analysis and access results. To take advantage of API module of structural analysis software and existing FE model from design, object-oriented design patterns were employed in the development of the software framework. In this paper, structural damage is represented with a set of meaningful geometric parameters, a finite element model identification method is formulated by a unified error function taking into account both static and modal responses. The method is implemented as the generic framework (Xu and Wu 2009) that takes advantages of the available structural analysis package and software design patterns.

## 3 FINITE ELEMENT MODEL IDENTIFICATION

### *3.1 Identification formulation*

The task of FE model calibration is to optimize the material and geometric parameters such that the difference between the observed structural responses and the model analyzed values is minimized. To develop a unified and generic framework for parameter estimation, both static and modal responses need to be considered and assumed to be equally important for the estimation. Each of static test and vibration mode reveals a part of characteristics of a structure. Moreover, the error functions are independent from the value of a specific response. For example, the error of responses from two measured degrees of freedom (DoF) should contribute equally to the objective functions, no matter what the values are. Therefore, the mean absolute percentage error (MAPE), measuring the relative error in each response, is introduced to avoid the situation that the error from one specific response dominates the objective function. In addition, the MAPE gives an apparent indication of how good a model is relative to the corresponding real structure.

The error function based on static respnses is defined to measure the discrepancy between the static FE analysis and the observed static responses, given as:

$$E_S = \frac{1}{NM}\sum_{i=1}^{N}\sum_{j=1}^{M}\left|\frac{U_{ij}^A - U_{ij}^O}{U_{ij}^O}\right| \tag{1}$$

where $U_{ij}$ is the displacement or strain at DoF $i$ for load case $j$, $N$ is the number of DoF and $M$ is the number of load cases. The superscript $A$ denotes the analytical responses, while $O$ denotes the observed responses. To calibrate FE models or detect structural damages accurately, especially small damages, it requires that objective function be sensitive to the changes of properties or dimensions of a structure. The modal flexibility (Berman and Flannelly 1971), which has been used in structural damage detection (Pandey and Biswas 1994), is a sensitive index of changes in structures. It increases with occurrences of structural deterioration or damage. Thus, the modal-based error function is defined as:

$$E_M = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\frac{\mathbf{F}_{ij}^A - \mathbf{F}_{ij}^O}{\mathbf{F}_{ij}^O}\right| \tag{2}$$

Where $\mathbf{F}$ represents flexibility matrix defined as:

$$\mathbf{F} = \mathbf{\Phi}\mathbf{\Lambda}^{-1}\mathbf{\Phi} \approx \sum_{i=1}^{m}\frac{\mathbf{\Phi}_i\mathbf{\Phi}_i^T}{\omega_i^2} \tag{3}$$

in which $\mathbf{\Phi} = [\mathbf{\Phi}_1, \mathbf{\Phi}_2, ..., \mathbf{\Phi}_n]$ is the mode shape matrix, $\mathbf{\Lambda}$ is the diagonal matrix of squared natural frequencies, $diag(\omega_i^2)$, $\omega_i$ is the $i$ th natural frequency, $\mathbf{\Phi}_i$ is the $i$ th mode shapes, $n$ is the number of measured DoF in modal responses and $m$ is the number of the modes used to approximate mode shapes.

The mode shapes in Eq.(3) need to be normalized to unity with respect to a mass matrix as $\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{I}$. Theoretically, all the mode frequencies and shapes are required to obtain the accurate flexibility matrix of a structure. In practice, only several lower frequency modes can be measured due to the limit of techniques and complexity of structures. It is not a major issue since the contributions from higher frequency modes to the flexibility matrix is small when comparing to lower modes, as shown in Eq.(3). Hence, a good estimate of the flexibility matrix can be obtained with only a few of lower frequency modes.

Following the definition of two separate error functions, a unified objective function is defined as:

$$E_T = \frac{E_S + E_M}{2} \tag{4}$$

where subscript $T$ denotes the total model error including static and modal error. If observed static or modal data is not available, then the value of the corresponding objective function is set to zero. The FE model identification is essentially an optimization problem. In this application, the GA is used to maximize the fitness (i.e. score) function instead of minimizing the error function. Thus a fitness function in the framework is defined as reciprocal of the average of static and modal error given as:

$$F = \frac{1}{E_T} \tag{5}$$

Mathematically, the value of fitness approaches infinite when the error is very small, while it approaches zero if the error is quite large. In practice, the value of the total model error is usually in a range that is not too large or too small, it is unlikely that the value of fitness has the problem of overflow. Let vector $\vec{X}$ represent the set of FE model parameters, the FE model updating problem is formulated as follows.

Search for $\quad (\vec{X}) = (x_1, x_2, x_3, ..., x_N)$ (6)

Maximize $\quad F$ (7)

Subject to $\quad x_i^{\min} \le x_i \le x_i^{\max}$, $i = 1, ..., N$ (8)

Where $N$ is the maximum number of model parameters, $x_i$, $i = 1, ..., N$, is the $i$-th parameter, $x_i^{\min}$ and $x_i^{\max}$ are the minimum and maximum limits of the $i$-th parameter respectively. The optimization model as formulated by Eq. (6) – Eq. (8) is a typical parameter identification problem. It is solved by using the competent genetic algorithm (GA) that has been applied to the leakage detection and model calibration of water distribution systems (Wu 2009).

### *3.2 Solution method*

As shown in Figure 1, a nominal FE model data is imported along with the observed structural responses and the specified parameters to be estimated. It is important to select a set of significant parameters in order to obtain the accurate FE models within the context of damage detection. The selected damage parameters need to be physically related to the source of errors in FE model identification and reasonably meaningful to the possible damages of a civil infrastructure system Also, the responses of FE model must be sensitive to these parameters in order to effectively search for the optimal solution. After input data is processed, the GA randomly generates an initial population of trial solutions, each of them represents a set of FE model parameters and is passed to FE solver, which performs the analysis and produces the structural responses for the corresponding solution. The calculated responses are compared with the monitored responses, the static and modal response errors are computed as Eq.(1) – (3), together with fitness value as Eq.(4) and (5). The fitness values are passed back to GA module, where a new population of solutions is generated by emulating the principles of natural selection and genetic reproduction. Thus one generation is completed. Generation after generation, the FE model parameters are expected to be optimized.

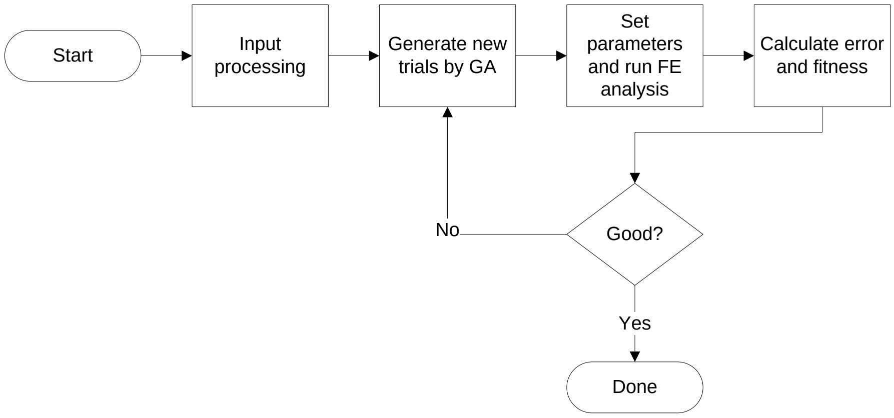


Figure 1 Data flow of finite element model updating method

# 4 FRAMEWORK PROTOTYPE

The solution method is prototyped as generic and extensible framework, which is designed in the context of object oriented programming and Microsoft .net technology. More details for the solution framework design and implementation are given as follows.

## *4.1 Prototype design*

Figure 2 shows the structure of software framework architecture. It illustrates the static relationship of high-level classes. Class ParaEstimation is responsible for implementation of the workflow. Classes StaticError and ModalError inherited from class ObjectiveFunction calculate the error of the static and modal response stored in classes StaticResponse and ModalResponse. The FESoftware class is a wrapper for a specific API module of FE analysis software. Similar to FESoftware class, the GA class is also a wrapper. It keeps the interfaces unchanged no matter which specific genetic algorithm is utilized.

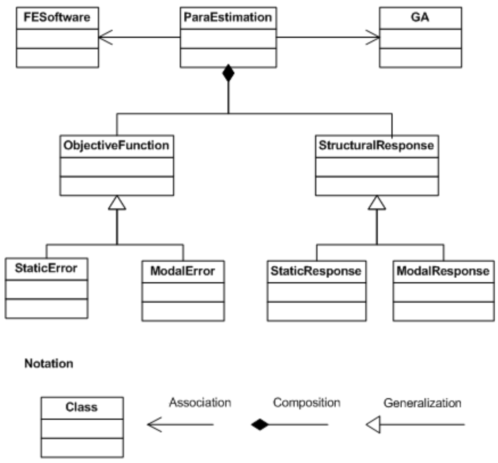


Figure 2 Static structure of the software framework

## *4.2 Design patterns*

For different applications, there may be different nominal FE models or structural analysis software available. Unfortunately, no standard of the API of FE analysis software exists. Each FE analysis package has its own specification of API. To make the main part of the framework reusable and take advantage of the available nominal models and software, the object adapter design pattern (Gamma 1995), a structural pattern, was adopted in the framework. As illustrated in Figure 3, the adaptee, the APIModule class, provides existing interfaces of specific FE analysis software to be integrated. The FESoftware class, which is the adapter, wraps the interface of adaptee to the interfaces defined in the interface class FESoftwareOperation. In this way, the ParaEstimation class calls the functions in the FESoftware class instead of the API module of FE analysis software directly. The FESoftware class makes calls to an instance, which is contained in the FESoftware class as a member, of the utilized FE analysis software. If a different FE analysis software is used as analysis engine or the API of current software is updated to a new version, only FESoftware class needs to be modified accordingly.

The siminar architecture is designed for integrating GA otpimization method. As shown in Figure 4, the framework is flexible at coupling with different GA library when the representation of solution domain (e.g. an array of bits) of the integrated GA library doesn't match the required representation (i.e. a set of uncertain parameters) of parameter estimation. Hence, the adapter design pattern enables a versatile framework prototype for FE model updating.

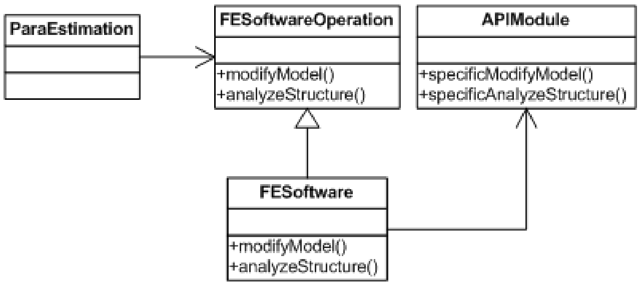


Figure 3 Object adapter pattern for FE analysis software

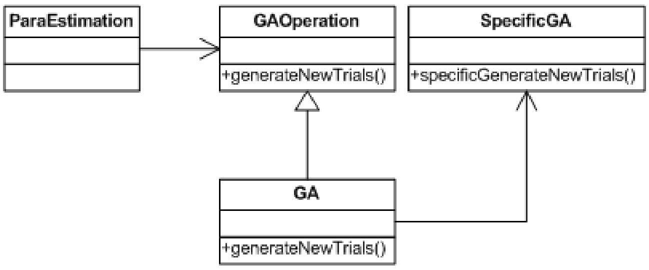


Figure 4 Object adapter pattern for GA

## 5 APPLICATIONS

To test the performance of the implemented method and framework, a planar truss structure with seven members, as shown in Figure 5, was used as the example for application of the FE model identification method.

### *5.1 Example*

The truss is simply supported at the bottom with all the members made of steel pipes. To simulate the damaged structure, the pipe element No.6 is reduced from healthy condition diameter of 0.0675m to 0.05m, while all the other members are assumed to be in healthy condition with the original dimension. The damaged structure scenario was analyzed by using FE solver built in STAAD.Pro. The results of five modal responses for the damage scenario were extracted from STAAD.Pro and used as artificially monitored data.

To detect the damage in the truss structure, seven uncertain parameters of section diameter were selected to be optimized. Assuming no pre-knowledge is available for which member or how many members were damaged in the truss structure, the integrated method was applied to identify the correct cross-sectional diameters for all seven bars. The bar(s) with the reduced diameter is deemed as the damaged element.

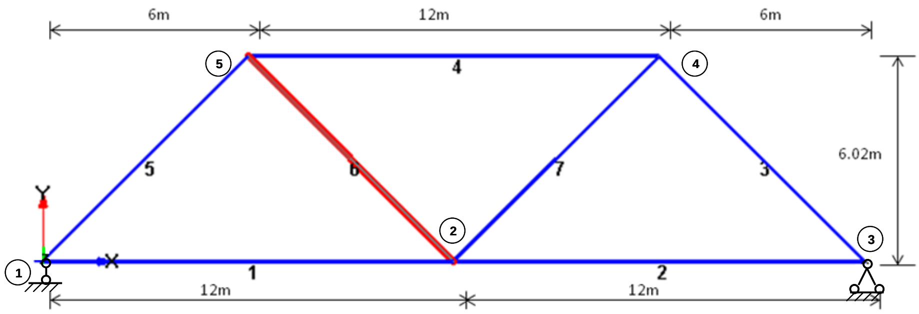


Figure5 Dimension of example truss structure

*5.2 Results*

To apply the FE model updating optimization method, the upper bound of the cross-sectional diameter was set to 0.068m the same as the original diameter while the lower bound of the diameter was set to 0.045m, which is smaller than the diameter of 0.05m under the damaged condition. Structural damages usually lead to reduction of effective cross-sectional area. In real application of parameter estimation, the value of lower bound of uncertain parameters depends on a variety of factors such as site location, surrounding environment and history records.

The population size of GA optimizaiton was set to fifty-five. As elaborated in the previous section, each GA solution, representing one possible damage solution, was analyzed by calling STAAD.Pro FE solver and the corresponding modal responses of the first five modes were compared with the observed responses, and the fitness value was calculated by Eq. (2) - (5).

Figure 6 presents the snapshots of the optimized cross-sectional diameters. From solution trial 33, the diameter of member No. 6 is consistently identified as the smallest among all the solutions. Optimal or near-optimal solution was obtained at trial 1568, the optimized cross-sectional diameter of member No. 6 was 0.051m , which is very close to the damaged scenario diameter of 0.051 m. The diameters of the other members identified are 0.068 m for member No. 1, 4, 5 and 7, 0.065 m for member No. 2 and 0.062 m for member No. 3 They are very close to the healthy condition diameter of 0.0675 m. Figure 7 illustrates the comparison of mode frequencies between the damage scenario and the optimized FE model. The largest difference comes from mode 4 with relative error of 3.07% and smallest relative error of 0.58% was achieved for mode 3.

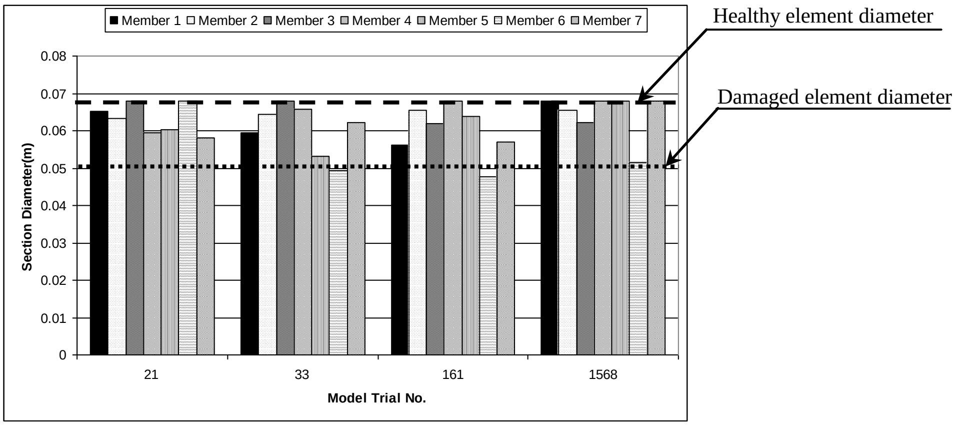


Figure 6 Snapshots of GA solution trials for the identified member diamters of the truss strsucture

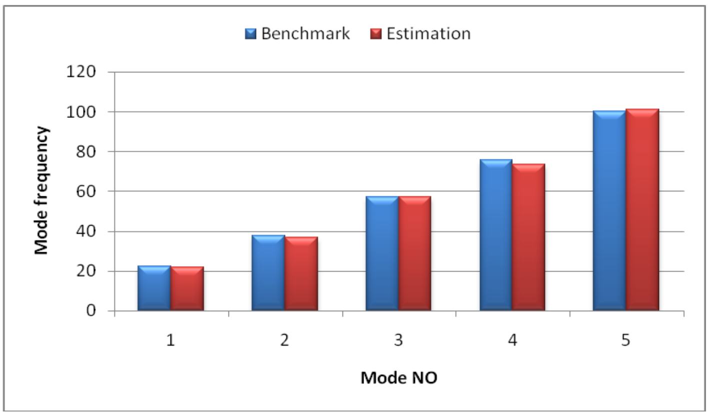


Figure 7 Comparison of mode frequencies between the FE model estimated and the observed benchmark

## 6 CONCLUSIONS

A FE model parameter identification method and a software framework have been developed for SHM in this paper. The method provides an integrated approach for updating FE model. The model parameters can be any combination of material and geometric attributes as desired. The competent genetic algorithm has been applied to optimize the FE model identification. The integrated approach enables engineers to construct sound analytical model for the existing structural systems and also detect possible damages of civil structural systems. For reusability of the developed tool, the design pattern was applied for integrating both FE analysis software and the GA library in the prototyped framework. It ensures the adaptability to the available nominal FE analysis software and GA optimization technique. the results obtained for the testing case demonstrate that the method holds a great deal of promise for effective FE model identification of large civil structural systems. It is worthwhile further developing the solution method and verifying the effectiveness in practice.

## 7 REFERENCES

Baruch, M., and Bar Itzhack, I. Y. (1978). "Optimal weighted orthogonalization of measured modes." *AIAA Journal*, 17(8), 927-928.

ASCE (2009) <http://www.infrastructurereportcard.org/>.

Bentley Systems, Inc. (2007). *STAAD.Pro 2007 technical reference manual*, Exton, PA, <http://www.bentley.com/>.

Berman, A., and Flannelly, W., G. (1971). "Theory of incomplete models of dynamic structures " *AIAA Journal*, 9(8), 1481-1487.

Computers and Structures, Inc. (2005). *CSI analysis reference manual for SAP2000, ETABS, and SAFE*, Berkeley, CA, <http://www.csiberkeley.com/>.

Doebling, S. W., Farrar, C. R., Prime, M. B., and Shevitz, D. W. (1996). "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review." Technical Report, LA--13070-MS, Los Alamos National Lab., NM

Friswell, M. I., and Mottershead, J. E. (1995). *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic Publishers: Dordrecht.

Gamma, E., Helm, R., Johnson, R., and Vlissides, J. (1995). *Design patterns: Elements of reusable object-oriented software*, Addison-Wesley, Reading, Mass.

Hao, H., and Xia, Y. (2002). "Vibration-based Damage Detection of Structures by Genetic Algorithm." *Journal of Computing in Civil Engineering*, 16(3), 222-229.

Pandey, A. K., and Biswas, M. (1994). "Damage Detection in Structures Using Changes in Flexibility." *Journal of Sound and Vibration*, 169(1), 3-17.

Sanayei, M., Bell, E. S., Javdekar, C. N., Edelmann, J. L., and Slavsky, E. (2006). "Damage Localization and Finite-Element Model Updating Using Multiresponse NDT Data." *Journal of Bridge Engineering*, 11(6), 688-698.

Sanayei, M. (1997). *PARIS-PARameter Identificafication System*, Tufts University. Medford, Mass.

Soh, C. K., and Dong, Y. X. (2001). "Evolutionary Programming for Inverse Problems in Civil Engineering." *Journal of Computing in Civil Engineering*, 15(2), 144-150.

Wang, X., Swanson, J. A., Helmicki, A. J., and Hunt, V. J. (2007a). "Development of Dynamic-Response-Based Objective Functions for Finite-Element Modeling of Bridges." *Journal of Bridge Engineering*, 12(5), 552-559.

Wang, X., Swanson, J. A., Helmicki, A. J., and Hunt, V. J. (2007b). "Development of Static-Response-Based Objective Functions for Finite-Element Modeling of Bridges." *Journal of Bridge Engineering*, 12(5), 544-551.

Wei, F.-S. (1990). "Mass and stiffness interaction effects in analytical model modification." *AIAA JOURNAL*, 28(9), 1686-1688.

Wu, Z. Y. (2009) "A unified Approach for Leakage Detection and Extended Period Model Calibration of Water distribution Systems", *Urban Water Journal*, 6(1), 53-67.

Xu, G. and Wu, Z. Y. (2009) "Finite Element Model Identification for Structure Health Monitoring", *Technical Report*, Bentley Systems, Incorporated, Watertown, CT, USA.

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