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Transformation Matrix Method for Evaluation of Damage Detection in Structural Elements and Preparation the Algorithms

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1 ABSTRACT

In this paper, a method to detect structural damage is evaluated. Damage is expressed as the loss of stiffness of the structural elements. The method is called the "transformation matrix method", because it is based on the transformation matrix that reduces the global stiffness matrix of a structure to a condensed state with the primary degrees of freedom which correspond to the rigid body movements of the slabs of the storeys.The method fundamentally considers the contribution of each element of the building to the performance of the whole structure.

This method allows the locating and assessing of the damage magnitude of structural elements by considering the contribution of each of them to the overall performance of the structure, is applicable of two and three dimensional building frames with several storeys with one or several damaged elements.

Effects of uncertainties in the experimental measurements of the dynamic characteristics and in the precision of the numerical representation of the structure on this method proposed are evaluated. Results show the good agreement between the estimated damage computed with the proposed method and the true value of damage.

Keywords: damage detection, transformation matrix, structural damage, damage evaluation

2 INTRODUCTION

Structural damage in buildings is difficult to measure.Currently, several damage indices are used to carry out this task (Powell and Allahabadi 1988; Williams and Sexsmith 1995); nevertheless, the problem is still not solved. Several notable studies to locate and assess structural damage are based on different criteria: sensitivity methods (Hassiotis and Jeong 1995; Topole and Stubbs 1995; Stubbs and Kim 1996); methods based on the difference between stiffness and flexibility matrices (Lin 1990); methods of residual forces (Ricles and Kosmatka 1992); methods based on the flexibility matrix (Peterson et al. 1995); artificial neural networks (Ferregut et al. 1995); probabilistic methods (Sohnand Law 1997); and others that use some of the above techniques (Koh et al. 1995; Kahl and Sirkis 1996; Cobb and Liebst 1997).

In this paper, a method to detect structural damage is evaluated. Damage is expressed as the loss of stiffness of the structural elements. The method is called the mtransformation matrix method because it is based on the transformation matrix that reduces the global stiffness matrix of a structure to a condensed state with only the primary degrees of freedom, which correspond to the rigid body movements of the slabs of the storeys. The method fundamentally considers the contribution of each element of the building to the performance of the whole structure.

3 OBJECTIVE AND SCOPE

The main objective of the present paper is to evaluate the transformation matrix method for locating and assessing damage in structural elements of frame buildings.

4 HYPOTHESIS

The global stiffness matrix $[K_d]$ of an analytical model of any structure corresponding to a damage state can be represented as an assembly of the stiffness matrices of the substructures considered, that is,

$$[K_d] = \sum_{i=1}^{n_s} (1 - dK_i)\ [K]_i \tag{1}$$

where n_s is the number of elements or substructures in the system; dk_i is a non-dimensional parameter that represents the reduction in the contribution of the stiffness matrix of the ith element to the global stiffness matrix ($0 < dk_i < 1$); $[K]_i$ is the global stiffness matrix of the ith element or sub-structure without damage.
The factor $(1 - dk_i)$ allows one to determine the damage states in the ith substructure, which are defined as those states for which the value of dk_i is greater than a specific value, normally zero. Because the location and magnitude of computed damage is determined according to the dki values, the global stiffness matrix $[K_d]$ of the structure is expressed as a function of dk_i.

5 DETECTION OF DAMAGE IN TWO-DIMENSIONAL MODELED BUILDINGS

The global stiffness matrix of a plane frame is determined from the contribution of the stiffness matrices, in a global coordinate system, of each of the structural elements that constitutes it. If these elements are considered as substructures, using the notation of eq. [1], the stiffness matrix $[K]_j$ of the jth plane frame corresponding to a damage state can be written as:

$$[K_d]_j = \sum_{i=1}^{n_{ej}} (1 - dK_i)[K]_{ij} \tag{2}$$

where n_{ej} is the number of elements in the jth frame and $[K]_{ij}$ is the stiffness matrix in global coordinates of the ith element of the jth frame. Developing eq. [2], we obtain:

$$[K_d]_j = \sum_{i=1}^{n_{ej}} [K]_{ij} - \sum_{i=1}^{n_{ej}} dK_i [K]_{ij} \tag{3}$$

Observing that the first sum of eq. [3] corresponds to the global stiffness matrix $[K_{wd}]j$ of the original jth frame without damage, this equation can be written as:

$$[K_d]_j = [K_{wd}]_j - \sum_{i=1}^{n_{ej}} dK_i [K]_{ij} \tag{4}$$

Equation [4] shows that the global stiffness matrix corresponding to a damage state can be calculated as the difference between the stiffness matrix of the original structure without damage and a matrix that contains the variation due to damage in the structure. The lateral stiffness matrix corresponding to the damage state of the jth frame is:

$$[\overline{K}_d]_j = [T_d]_j^T [K_d]_{ij} [T_d]_j \tag{5}$$

where $[\overline{K}_d]_j$ is the lateral stiffness matrix of the damaged jth frame and $[T_d]_j$ is the transformation matrix associated with the damage state of the jth frame. Substituting eq. [4] into [5], we obtain

$$[\overline{K}_d]_j = [T_d]_j^T [K_{wd}]_j [T_d]_j - \sum_{i=1}^{n_{ej}} dK_i [T_d]_j^T [K]_{ij} [T_d]_j \tag{6}$$

It is convenient to recall that the transformation matrix is a function of the damage state, because it is obtained from the submatrices that result from the partition of $[K_d]_j$, that is,

$$[T_d]_j = \begin{bmatrix} [I] \\ -[C]^{-1}[B]^T \end{bmatrix} \tag{7}$$

$$[K_d]_j = \begin{bmatrix} [A] & [B] \\ [B]^T & [C] \end{bmatrix} \tag{8}$$

As a first approximation, assuming that the transformation matrix $[T_d]_j$ for the damage state does not differ from the corresponding undamaged state $[T_{wd}]_j$, an iterative procedure to detect damaged elements modifies eq. [6] as follows:

$$[\overline{K}_d]_j = [T_{wd}]_j^T [K_{wd}]_j [T_{wd}]_j - \sum_{i=1}^{n_{ej}} dK_i [T_{wd}]_j^T [K]_{ij} [T_{wd}]_j \tag{9}$$

In this equation, the triple matrix products are equal to the lateral stiffness matrix $[\overline{K}_{wd}]_j$ of the frame without damage, thus:

$$[\overline{K}_d]_j = [\overline{K}_{wd}]_j - \sum_{i=1}^{n_{ej}} dK_i [T_{wd}]_j^T [K]_{ij} [T_{wd}]_j \tag{10}$$

If the product $[T_{wd}]_j^T[K]_{ij}\,[T_{wd}]_j$ is denoted as $[\overline{K}]_{ij}$, the following equation is obtained:

$$[\overline{K}_d]_j = [\overline{K}_{wd}]_j - \sum_{i=1}^{n_{ej}} dK_i [\overline{K}]_{ij} \tag{11}$$

The lateral stiffness matrix of the damaged structure is symmetric and of size $m \times m$; thus the number of its independent terms, n_{it} ,is $m(m + 1)/2$; graphically,

$$\begin{bmatrix} 1 & 2 & 3 & \cdots & m \\ & m+1 & m+2 & \cdots & 2m-1 \\ & & 2m & \cdots & 3m-3 \\ & & & \ddots & \vdots \\ & & & & n_{it} \end{bmatrix}$$

Developing eq. [11] for the tth term of each matrix in this equation, we obtain:

$$\overline{K}_{wd_t} - \overline{K}_{d_t} = \sum_{i=1}^{n_{ej}} dK_i \overline{k}_{ij_t} \tag{12}$$

or, in matrix form,

$$\{\overline{K}_{wd}\} - \{\overline{K}_d\} = [S_K]\,\{dk\} \tag{13}$$

where $\{\overline{k}_{wd}\}$ is a $n_{it} \times 1$ vector that includes the independent terms of the lateral stiffness matrix of the undamaged structure; $\{\overline{k}_d\}$ is a $n_{it} \times 1$ vector containing the independent terms of the lateral stiffness matrix corresponding to the damaged structure; $[S_k]$ is a $n_{it} \times n_{ej}$ matrix with the $\overline{k}_{ij}$ terms; and {dk} is a $n_{it} \times 1$ vector that contains the stiffness degradation values of the structural elements.

A state of damage associated with a lateral stiffness matrix is obtained when the system of equations [13] is solved. If the new corresponding transformation matrix is computed, a new approximation for eq. [9] is defined. The algorithm that allows the carrying out of this procedure is later described in detail.
Equation [13] represents a problem of linear least squares with constraints; these problems are of the following form:

$$\{b\} \cong [A]\ \{x\} \tag{14}$$

subjected to constraints such as:

$$\{b_l\} \leq [C]\ \{x\} \leq \{b_u\} \tag{15}$$

$$\{x_l\} \leq\ \{x\}\ \leq \{x_u\} \tag{16}$$

where $\{b\}$ is the right-hand-side vector of the system; matrix $[A]$ contains the coefficients of the system of equations; $\{x\}$ is the vector of the unknowns of the system; $\{b_l\}$ and $\{b_u\}$ are vectors with the lower and upper limit values of $\{b\}$, respectively; [C] is the matrix with the constraint coefficients; and $\{x_l\}$ and $\{x_u\}$ are vectors with the lower and upper limit values of $\{x\}$, respectively.
By means of a change in variables $\{y\}$; $[C]\{x\}$, these systems of equations are solved, and a solution for least squares with the lower and upper limit values of vectors $\{x\}$ and $\{y\}$ are obtained. The system of equations $[C]\{x\}-\{y\}= \{0\}$ is a set of constraints of equality, which are considered in linear programming as in the weighted penalty method. In the computer program developed in the present research, the subroutine Linearly Constrained Least Squares(LCLSQ) included in the Mathematical and Statistical Libraries (IMSL 1995) was used.
Equation [15] allows the inclusion of uncertainties in the values of the independent terms of the damaged structure lateral stiffness matrix. For example, these values can be disturbed to simulate the effect of the loss of precision in the measurements and to introduce the resulting interval to the problem. On the other hand, because possible damage in the elements is restricted to values between 0 and 1, eq. [16] is used to limit {dk}.

6 SYMMETRIC STRUCTURES

When a structure is symmetric in stiffness and in loads, there are structural elements that cause the same variation over the lateral stiffness matrix for the same damage level, i.e., their stiffness matrices are identical and thus they can take on values in the closed interval from zero to the number of symmetric elements. This assumption implies that the number of unknowns of the problem is reduced. This fact can be useful in many situations, for eq. [13] shares the same limitations with the relationship between the number of equations and unknowns as does the sensitivity matrix method in the detection of damage (Escobar et al. 1998).
It is common that the lateral stiffness matrix of frames can have terms with values of an order that is several times lower than the term with the maximum value of this matrix. If the system of equations [13] is solved with such differences, the precision in the results obtained is reduced. To avoid this problem, the maximum quantity of relatively small value terms is eliminated from the matrix.

7 DAMAGE DETECTION ALGORITHM

To identify the damage state of a structure, it is necessary to establish the specific conditions of the problem that include the elimination of equations and (or) unknowns, as well as the modification of the damage interval, and to follow the next iterative procedure:
1. Matrices $[K]_{ij}$ and $[K_{wd}]_j$ are computed.
2. $[T]$ matrix for the undamaged state is obtained.

3. Matrices $\left[\bar{k}\right]_{ij}$ =[T] ^{T}[K]$_{ij}$ [T] and $\left[k\right]_{ij}$ =[T] ^{T}[K_{wd}]$_{j}$ [T] are computed.
4. Vector $\left\{\bar{k}_{wd}\right\}$ and matrix [S_k] are formed.
5. The system of equations $\left\{\left\{\bar{k}_{wd}\right\}-\left\{\bar{k}_{d}\right\}_{known}\right\}=\left[S_{k}\right]\left\{dk\right\}$ is solved, where $\left\{\bar{k}_{d}\right\}_{known}$ is a vector with terms of the condensed stiffness matrix computed from known modal shapes and vibration frequencies.
6. For the obtained damage vector {dk}, the global stiffness matrix [K_d]$_j$ corresponding to a new transformation matrix [T] is computed.
7. Matrix [K_d]$_j$ is condensed and a vector {k_d }approx is formed.
8. If the difference between {k_d} known and {k_d} approx is less than a tolerance value, the process is halted; if not, the process returns to step 3.

8 CONVERGENCE CRITERIA

To improve the convergence reached by the solution after each iteration, the following criterion was used:

$$e=\min_{x}\left\|\ \left[K\right]-\left[S_{k}\right]\ \right\|^{2} \tag{17}$$

Assuming that initially there is no damage in the structure (step 2 of the algorithm), it is possible to establish an iterative procedure that converges to the damage state defined by the vector $\left\{\bar{k}_{d}\right\}_{known}$. This can be achieved if the transformation matrix used in step 3 of the z + 1 iteration is computed for a fraction of the sum of the damage states obtained in previous iterations z and z – 1, as follows:

$$dK_{Z+1}=\beta\, dK+(1-\beta)\ dK_{Z-1} \tag{18}$$

Different values of the β factor can be used in each iteration. The method locates an optimal value of β in terms of the minimum error computed by using eqs. [17] and [18]. In this way, the transformation matrix shows a gradual change that allows the detection of the damaged elements by successive approximations.
Another way to complete the process is by defining a tolerance for the maximum value obtained when the terms of the vectors $\left\{\bar{K}_{d}\right\}_{known}$ and $\left\{\bar{K}_{d}\right\}_{approx}$ in step 8 are compared. Additionally, since a structural element can present only a damaged or undamaged state, it is possible to improve the direct solution of the system of equations of step 5 by assigning zero damage to the elements with a value of stiffness degradation lower than a specific value. This criterion contributes to the improvement of the location of the damaged elements by using the transformation matrix method proposed.

9 DETECTION OF DAMAGE IN THREE-DIMENSIONAL MODELED BUILDINGS

To determine the stiffness matrix of a three-dimensional model of a building structure, it is necessary to compute the global stiffness matrix of each frame. The stiffness matrix of the three-dimensional model will correspond to the primary degrees of freedom (rigid body displacements of the slabs) obtained from the stiffness matrices of the plane frames that compose it (coupled plane frames). If the notation of eq. [3] is used, the global stiffness matrix corresponding to a damage state of the jth frame turns out to be:

$$\left[K_{d}\right]_{j}=\left[K_{wd}\right]_{j}-\sum_{i=1}^{n_{ej}}dK_{ij}\left[K\right]_{ij} \tag{19}$$

where dk$_{ij}$ is the stiffness degradation of the ith element of the jth frame. The lateral stiffness matrix of each damaged frame and the transformation matrices associated with a damage state are computed from the global stiffness:

$$\left[\bar{K}_{d}\right]_{j}=\left[T_{d}\right]_{j}^{T}\left[K_{d}\right]_{j}\left[T_{d}\right]_{j} \tag{20}$$

Substituting eq. [11] into [12], we obtain:

$$\left[\overline{K}_d\right]_j = \left[T_d\right]_j^T \left[K_{wd}\right]_j \left[T_d\right]_j - \sum_{i=1}^{n_{ej}} dK_{ij} \left[T_d\right]_j^T \left[K\right]_{ij} \left[T_d\right]_j \tag{21}$$

By using compatibility conditions (Ghali and Neville1989), the local lateral stiffness matrix of the damaged jth frame is transformed into the global coordinate system. In this reference the displacement transformation matrix $[C]_j$ relates the lateral degrees of freedom of the jth frame to the primary degrees of freedom of the three-dimensional structure; applying this procedure to eq. [21], we obtain:

$$[C]_j^T \left[\overline{K}_d\right]_j [C]_j = [C]_j^T \left[T_d\right]_j^T \left[K_{wd}\right]_j \left[T_d\right]_j [C]_j - \sum_{i=1}^{n_{ej}} dK_{ij} [C]_j^T \left[T_d\right]_j^T [K]_{ij} \left[T_d\right]_j [C]_j \tag{22}$$

For a damage state, the stiffness matrix $\left[\overline{Kt}_d\right]$ of the three-dimensional model of the structure is obtained by adding the lateral stiffness matrices associated with the global displacements of each frame, that is,

$$\left[\overline{Kt}_d\right]_j = \sum_{j=1}^{N_f} [C]_j^T \left[T_d\right]_j^T \left[K_{wd}\right]_j \left[T_d\right]_j [C]_j - \sum_{j=1}^{N_f} \sum_{i=1}^{n_{ej}} dK_{ij} [C]_j^T \left[T_d\right]_j^T [K]_{ij} \left[T_d\right]_j [C]_j \tag{23}$$

where N_f is the number of frames in the structure. In this equation, the double sum represents the loss of global stiffness of the structure obtained as the contribution of all the elements of each frame. To take into account the contribution of all the elements of the structure, it is convenient to change the double sum. Thus, for each structural element, no matter if it belongs to one or more frames, a unique factor dk will be associated with it. Equation [23] istransformed as:

$$\left[\overline{Kt}_d\right]_j = \sum_{j=1}^{N_f} [C]_j^T \left[T_d\right]_j^T \left[K_{wd}\right]_j \left[T_d\right]_j [C]_j - \sum_{r=1}^{N_r} dK_r \sum_{j=1}^{n_f} [C]_j^T \left[T_d\right]_j^T [K]_{rj} \left[T_d\right]_j [C]_j \tag{24}$$

where N_r is the number of elements in the structure. If the following transformations are carried out, the previous equation can be written as the one obtained when solving the problem of damage detection in plane frames (eq. [10]), that is,

$$\left[\overline{Kt}_{wd}\right] = \sum_{j=1}^{N_f} [C]_j^T \left[T_d\right]_j^T \left[K_{wd}\right]_j \left[T_d\right]_j [C]_j \tag{25}$$

$$\left[\overline{K}_d\right]_r = \sum_{j=1}^{N_f} [C]_j^T \left[T_d\right]_j^T \left[K\right]_{rj} \left[T_d\right]_j [C]_j \tag{26}$$

where $\left[\overline{Kt}_{wd}\right]$ is the original condensed stiffness matrix of the structure without damage and $\left[\overline{K}_d\right]_r$ is the stiffness contribution of the rth element of the structure, obtained by adding the stiffnessess of all the frames that include the element $(r \in j)$. Substituting eqs. [25] and [26] into [24], we obtain:

$$\left[\overline{Kt}_d\right] = \left[\overline{Kt}_{wd}\right] \sum_{r=1}^{N_r} dK_r \left[\overline{K}_d\right]_r \tag{27}$$

From this expression, a linear system of equations is established when writing an equation for the *t*th term that is not zero of each matrix, that is,

$$\overline{kt}_{wd_t} - \overline{kt}_{d_t} = \sum_{r=1}^{N_r} dk_r \overline{kdr}_t \tag{28}$$

or, in matrix notation,

$$\{\overline{kt}_{wd}\} - \{\overline{kt}_{d}\} = [S_k]\ \{dk\} \tag{29}$$

where $[S_k]$ is a matrix formed by the $\overline{k}_{dr}$ terms. Since the displacement transformation matrices $[C]_j$ are independent of the damage state of a frame, the procedure presented above
to solve eq. [27] is analogous to the one used for plane frames. As an initial approximation for the solution, with the previously described algorithm, we can consider that the transformation matrices correspond to the undamaged state.

It is worthwhile to recall that the assumption about symmetrical elements leads to a reduced number of independent elements and to a modification in the damage interval, as already explained. On the other hand, the condensed stiffness matrix can have terms with zero values (Sosa et al. 1998); thus, the number of equations is a function of the size of this matrix, of the connectivity of the structural elements, and of the selection of the structural degrees of freedom.

10 CONCLUSIONS AND RECOMMENDATIONS

A method for locating and estimating structural damage, defined in terms of element-stiffness variations, in analytical models of framed buildings has been presented in this paper.

The method is based on the transformation matrix, which uses the global stiffness matrix of a structure to accomplish a condensation on the primary degrees of freedom.

With the proposed damage detection method, the initial assumption of approximating the transformation matrices for the damaged state as that corresponding to the undamaged state implies that the structural damage was not catastrophic, which means that the limit states of the structure were not exceeded and it is still useful. This fact is improved by introducing an iterative scheme in which this approximation is corrected with the results of the previous one. A method with a similar approach that uses as its initial assumption the vibration modes of the undamaged structure and an iterative procedure of correction is presented by Cobb and Liebst.

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