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# Damage identification based on strain and translational measurements with substructure approach

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ABSTRACT: Damage identification of in-service engineering systems is important from the viewpoint of safety and maintenance. Using structural identification algorithms, the physical or modal parameters of a structure can be identified from measured data. Based on changes of these critical structural parameters, damage can then be quantified. However, to identify a large scale system as a whole, numerical convergence is a challenging problem due to the large number of unknown parameters to be identified. The substructure method provides an attractive alternative to improve identification accuracy and efficiency by performing identification in smaller subsystems of manageable size. The response of substructure is determined by the external excitation and interface motion, the latter including measurements of translational and rotational responses. Nevertheless, measuring rotational response is usually expensive or difficult. In this study, a substructural damage identification strategy is proposed to avoid the rotational measurements, by using measured strains and translational accelerations on the interface. In a numerical example of a simply supported beam, the damage extents of three different substructures are evaluated with a modified genetic algorithm method. The results show that the proposed method performs well in identifying structural damage.

## 1 INTRODUCTION

Damage identification and health monitoring for structural systems has attracted increasing attention in the industry and research community. By analyzing the measurement signals, structural identification aims to determine physical parameters of a structure such as stiffness, mass or modal parameters such as natural frequency, mode shape and modal damping ratio. Then by comparing the identified parameters for the undamaged and damaged states, the location and even the extent of damage can be revealed. However, convergence becomes very difficult for a large system as it involves many degrees of freedom (DOFs) and unknown parameters. An alternative approach is to focus on a part of the structure of concern by the virtue of substructure approach. Various research works have been reported on substructural identification. The earliest work on substructural identification was reported by Koh et al. (1999) who identified the stiffness and damping coefficients using the extended Kalman filter. The substructural approach was found to perform much better than the complete structural identification in terms of accuracy and efficiency. Based on substructural approach, the natural frequencies and mode shapes are used as input patterns to the neural network for element-level identification in a truss and a frame structure (Yun et al. 2000). Based on a genetic algorithm (GA) approach, a time domain substructure identification method was used to identify stiffness of several structural systems including a long span truss structure (Koh et al. 2003a). A frequency domain substructure method was established to identify parameters without the need of interface measurements by using different sets of measurements in the substructure (Koh et al. 2003b). Recently, a new substructural analysis method (Law et al. 2009) was proposed to simultaneously identify local damage of a substructure and its interface forces represented in terms of orthogonal functions with unknown coefficients. Thereafter, the damaged substructure is assessed with the reconstructed interface forces through measurements in an adjacent healthy substructure (Law et al. 2010). These substructure methods show that the interface forces play an important role to determine substructural response. However, the interface forces are usually difficult to obtain especially for beam, plate and shell structures since measurement of rotational response is difficult in practice. To address this issue, a procedure is proposed to obtain rotational accelerations from measured strains and translational accelerations. Hence, the present identification is studied with interface rotational accelerations recovered from measured strains and translational (linear) accelerations. With a modified genetic algorithm approach based on a search space reduction, the damage can be qualified by comparing the identified substructural stiffness parameters for the undamaged and damaged states. The proposed strategy is demonstrated for a simply supported beam in successfully detecting the structural damage within three different substructures.

## 2 DAMAGE IDENTIFICATION STRATEGY

### *2.1 Substructure method*

Generally, a multi-DOF dynamic system can be described as

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{P(t)\} \tag{1}$$

Where $[M]$, $[C]$ and $[K]$ are mass matrix, damping matrix and stiffness matrix of the structural system, respectively. $\{\ddot{u}(t)\}$ , $\{\dot{u}(t)\}$ and $\{u(t)\}$ represent the acceleration, velocity and displacement time signals when the structure is excited by dynamic force $\{P(t)\}$ .

The equation of motion for a substructure extracted from the whole system yields

$$\begin{bmatrix} M_{rj} & M_{rr} \end{bmatrix}\begin{Bmatrix} \ddot{u}_j(t) \\ \ddot{u}_r(t) \end{Bmatrix}+\begin{bmatrix} C_{rj} & C_{rr} \end{bmatrix}\begin{Bmatrix} \dot{u}_j(t) \\ \dot{u}_r(t) \end{Bmatrix}+\begin{bmatrix} K_{rj} & K_{rr} \end{bmatrix}\begin{Bmatrix} u_j(t) \\ u_r(t) \end{Bmatrix}=P_r(t) \tag{2}$$

where subscript '$r$' denotes internal DOFs of the substructure while subscript '$j$' represents the interface DOFs. The concept of "quasi-static displacement" vector (Koh et al, 2003b) is adopted to eliminate the requirement of displacement and velocity since acceleration measurement is preferred over displacement and velocity. Displacements for internal DOFs are expressed as the sum of quasi-static displacements $u_r^s(t)$ and relative dynamic displacements $u_r^*$, i.e.

$$u_r(t)=u_r^s(t)+u_r^*(t) \tag{3}$$

Since damping force is usually small compared to inertia force in civil engineering structures, the velocity dependent part in the interface force is assumed to be negligible. Thus Eq. (2) can be written as

$$[M_{rr}]\{\ddot{u}_r^*(t)\}+[C_{rr}]\{\dot{u}_r^*(t)\}+[K_{rr}]\{u^*{}_r(t)\}=\{P_r(t)\}-\left([M_{rj}]+[M_{rr}][r]\right)\{\ddot{u}_j(t)\} \tag{4}$$

Where $[r]=-[K_{rr}]^{-1}[K_{rj}]$ is called the influence coefficient matrix relating internal DOFs to interface DOFs under the quasi-static condition.

*2.2 Rotational measurement recovery method*

Within an element, the displacement $u$ can be divided into deformation $d$ and rigid-body motion $r$ as follows

$$u = d + r \tag{5}$$

The rigid-body motion can be written as

$$r = \Phi_\alpha \alpha \tag{6}$$

where $\Phi_\alpha$ and $\alpha$ are the elemental rigid-body modes and associated rigid body motion amplitude. Within an element, the displacement-strain relation is

$$s = Su \tag{7}$$

Since the rigid body motion does not induce any strain, substituting Eq.(5) into Eq.(7)

$$s = Su = S(d+r) = Sd \tag{8}$$

Therefore, the deformation can be obtained by taking pseudo inverse operation due to the rank deficiency of matrix $S$ , i.e.

$$d = \Phi_s s = \left(S^T S\right)^{-1} S^T s \tag{9}$$

Hence, the displacement within an element can be expressed as

$$u = \Phi_s s + \Phi_\alpha \alpha \tag{10}$$

For a beam, plate or shell element, the displacement $u$ involves translational $u_w$ and rotational motion $u_\theta$, and rearranging Eq.(10) gives

$$\begin{Bmatrix} u_w \\ u_\theta \end{Bmatrix} = \begin{bmatrix} \Phi_{sw} \\ \Phi_{s\theta} \end{bmatrix} s + \begin{bmatrix} \Phi_{\alpha w} \\ \Phi_{\alpha\theta} \end{bmatrix} \alpha \tag{11}$$

With the measured translational motion and strain, the rotational displacement and rigid body motion amplitude can be obtained by solving Eq. (11), as follows.

$$\begin{Bmatrix} \alpha \\ u_\theta \end{Bmatrix} = \begin{bmatrix} -\Phi_{\alpha w} & 0 \\ -\Phi_{\alpha\theta} & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \Phi_{sw} \\ \Phi_{s\theta} \end{bmatrix} s - \begin{bmatrix} I \\ 0 \end{bmatrix} u_w \right\} \tag{12}$$

Differentiating Eq. (12) twice with respect to time, rotational acceleration can be obtained by measured strain and translational acceleration

$$\begin{Bmatrix} \ddot{\alpha} \\ \ddot{u}_\theta \end{Bmatrix} = \begin{bmatrix} -\Phi_{\alpha w} & 0 \\ -\Phi_{\alpha\theta} & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \Phi_{sw} \\ \Phi_{s\theta} \end{bmatrix} \ddot{s} - \begin{bmatrix} I \\ 0 \end{bmatrix} \ddot{u}_w \right\} \tag{13}$$

The translational acceleration $\ddot{u}_w$ can be measured directly by using accelerometers. Nevertheless, strain signal with noise needs to be differentiated first, which will inevitably induce some errors. To reduce such errors, Savitzky-Golay differentiation algorithm is adopted in view of its good performance (Luo et al, 2005). It is noted that similar idea has been reported by Reich et al. (2001) with displacement measurement, although which is not readily measured in practice.

### *2.3 Search algorithm*

Genetic algorithms (GA) are developed based on Darwin's theory of natural selection and survival of the fittest. In the past decades, GA has been widely studied and applied in many fields in engineering and science. In many complex problems, GA outperforms other traditional methods since most of traditional methods require a good initial guess, are sensitive to noise and often converge to local optima. An improved GA method based on a search space reduction method (SSRM) was proposed to increase the accuracy and reliability of identification by reducing the search space (Perry et al. 2006a). The method was shown to achieve significant improvement in identification accuracy as compared to a standard GA, by narrowing the search space adaptively based on the statistics of results obtained. The proposed strategy uses the improved GA method as the search engine to minimize the difference between the simulated and measured internal acceleration in the substructure.

## 3 NUMERICAL INVESTIGATION AND RESULTS

To assess the performance of the proposed strategy, numerical simulation is conducted on a simply supported beam with known structural parameters under intact and damaged states. The dimension of simply supported beam is 960 mm long, 50 mm wide and 3 mm high. It is modeled by 16 identical beam elements as shown in Figure 1. There are two DOFs (translation and rotation) at each intermediate node, while only rotation is considered at the two supporting nodes. Young's Modulus and density of the beam are $2.1\times10^{11}$N/m$^2$ and 7862 Kg/m$^3$, respectively.

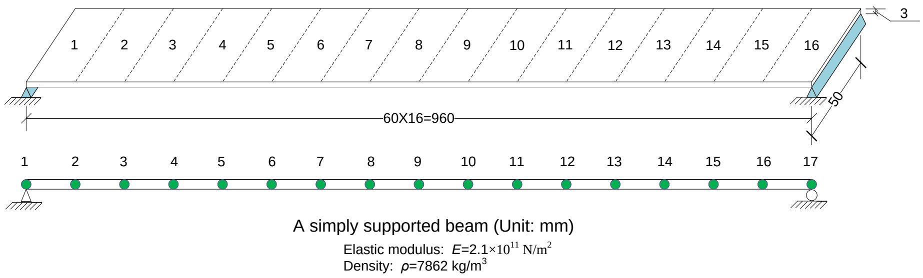


Figure 1. Simply supported beam model

Damage in the simply supported beam is simulated by reducing the beam width as indicated in Figure 2, such that the effective beam width in element 6 and 10 is reduced from 50 mm to 24 mm resulting 52% reduction of flexural stiffness. A random excitation acts at node 13. Damping effect is taken into account with Rayleigh damping scheme by assuming 5% critical damping for the first two modes. Simulation of the structure response to a given excitation is carried out by Newmark's constant acceleration method for 0.2 s. The sampling rate is considered to be 10,000 samples/s.

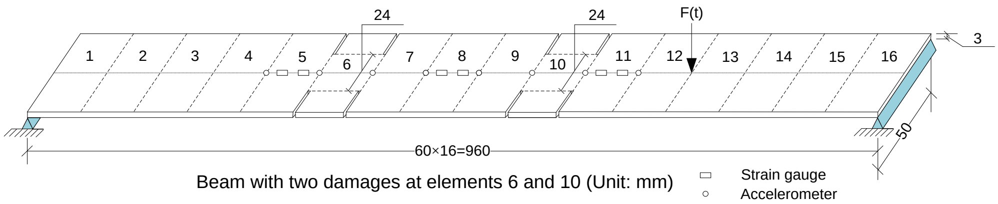


Figure 2. Damage scenarios and sensors placements

Three different substructures are investigated, denoted as S1, S2 and S3 shown in Figure 3. To recover the interface rotations of these substructures, the strain responses are assumed to be measured in the 5th, 8th and 11th elements. The strain measurements in elements 5 and 8 are used to recover interface rotation accelerations for $S_1$, element 8s and 11 for $S_2$, and element 5 and 11 for $S_3$. Translational acceleration responses are assumed to be available at nodes 5-12. To account for the effects of noise, the simulated strain and acceleration responses are assumed to be contaminated with zero-mean Gaussian white noise. Three different noise levels are considered: 2%, 5% and 10%.

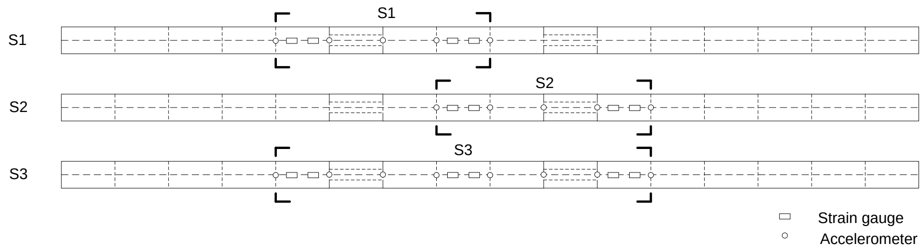


Figure 3. Three different substructures

In addition to unknown flexural stiffness values, damping parameters $\alpha$ and $\beta$ are treated as unknown resulting in 6, 6 and 9 unknown parameters, respectively, for S1, S2 and S3. The fitness function needed in GA is defined as

$$f = \sum_{i=1}^{M} \frac{\sum_{n=1}^{L} \left| \ddot{u}_m(i,n) - \ddot{u}_e(i,n) \right|^2}{\sqrt{\sum_{n=1}^{L} \left| \ddot{u}_m(i,n) \right|^2 / L}} \tag{14}$$

where subscripts $m$ and $e$ denote measured and estimated quantities, respectively; $L$ is the number of time steps and $M$ is the number of measurement sensors used. With the proposed damage identification strategy, not only the measured internal translational accelerations but also some recovered internal rotational accelerations are taken into account in the fitness function. The search limit is taken as 0.5-2.0 times the exact value for each unknown parameter.

To quantify the damage, measurements for the undamaged and damaged states are required. With the measured strain and translational acceleration respectively for the undamaged and damaged states, the rotational accelerations at the interface are respectively recovered for each state. Then the flexural stiffness for the undamaged and damaged substructure is respectively identified by using an improved GA method based on SSRM. The extent of the damage in the beam is calculated as the reduction of flexural stiffness relative to the undamaged value, i.e. the damage index of element *i* is

$$D_i = \frac{EI_{i\_u} - EI_{i\_d}}{EI_{i\_u}} \times 100\% \tag{15}$$

where $EI_{i\_u}$ and $EI_{i\_d}$ are the flexural stiffness values for the undamaged and damaged states. For the two damaged elements in the numerical example, the $EI_{i\_u}$ and $EI_{i\_d}$ values are 23.625 $N \cdot m^2$ and 11.34 $N \cdot m^2$, respectively. Therefore, the damage index is 52% in elements 6 and 10.

In Figures 4.a-c, the exact as well as the identified damage indices for each element in S1, S2 and S3 under different noise level are plotted for comparison. In addition, the mean and maximum of identification errors for undamaged and damaged states under different noise level are listed in Table 1.

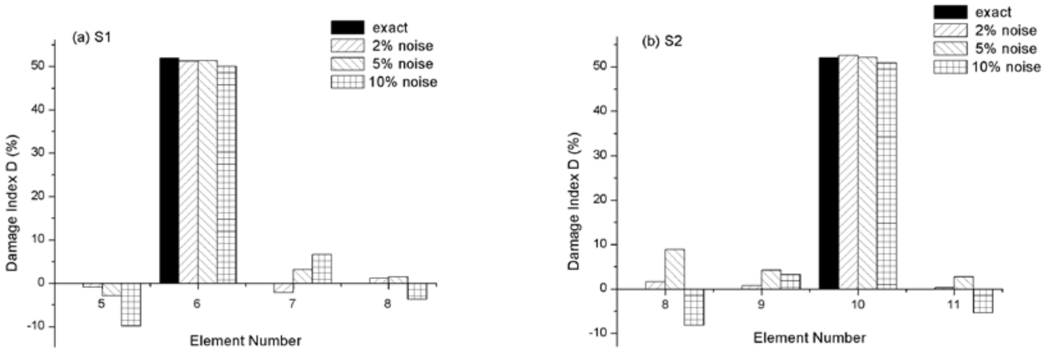


Figure 4.a-b. The exact and identified damage indices in S1 and S2

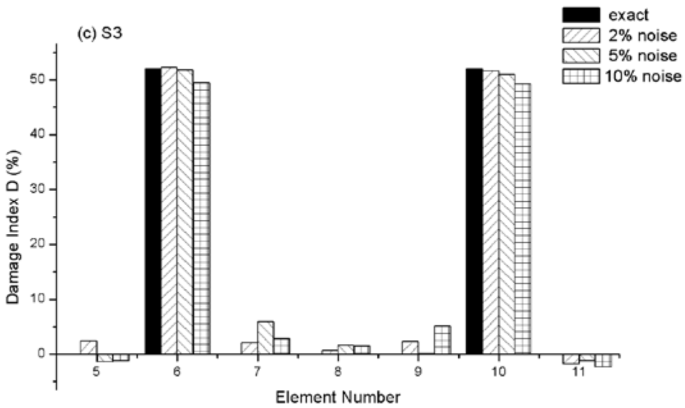


Figure 4.c. The exact and identified damage indices in S3

Table 1. The identification error for undamaged and damaged beam

| State | Noise level | S1 Mean error (%) | Max error (%) | S2 Mean error (%) | Max error (%) | S3 Mean error (%) | Max error (%) |
|---|---|---|---|---|---|---|---|
| Undamaged | 2% | 2.70 | 4.34 | 2.33 | 4.08 | 1.61 | 3.07 |
| | 5% | 3.43 | 5.10 | 4.42 | 7.81 | 4.34 | 8.91 |
| | 10% | 7.50 | 12.59 | 8.45 | 12.65 | 5.46 | 13.31 |
| Damaged | 2% | 1.85 | 2.65 | 2.79 | 4.00 | 2.18 | 3.87 |
| | 5% | 3.12 | 5.10 | 3.29 | 5.89 | 2.82 | 6.16 |
| | 10% | 6.68 | 10.53 | 7.66 | 13.19 | 4.96 | 11.45 |

From the identified results in Figures 4.a-c, the proposed method gives fairly accurate identification in terms of the location and extent of damage under different noise levels. For the two damaged elements, the damage indices of element 6 in S1, element 10 in S2 are identified as 49.46% and 49.26%, which just deviate 4.88% and 5.27% from the exact value 52% in the worst case. It indicates that the severity of damage in damaged elements can be very accurately identified with proposed strategy. For these undamaged elements, when measurements are contaminated by 10% noise, the maximum identified false damage is 10.84%, which is acceptable for practical application. The identification errors summarized in Table 1 presents that the flexural stiffness of these three substructure in this simply supported beam can be reasonably evaluated even the are polluted by 10% noise.

## 4 CONCLUSION

A damage identification strategy is proposed to avoid rotational measurements, by the use of the measured strains and translational accelerations on the interface. The substructure approach is adopted in this strategy to keep the identification system size manageable. The interface rotational accelerations are recovered from measurements of strains and translational accelerations. With the proposed strategy, the recovered rotations include not only the interface rotations but also some internal rotations. The recovered interface rotations enable the substructure forward analysis while the identified accuracy is improved by including some of the recovered internal rotations in the fitness function. Reasonably accurate identification results in the numerical study show that the proposed strategy performs well in identifying damage through the reduction in flexural stiffness values.

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